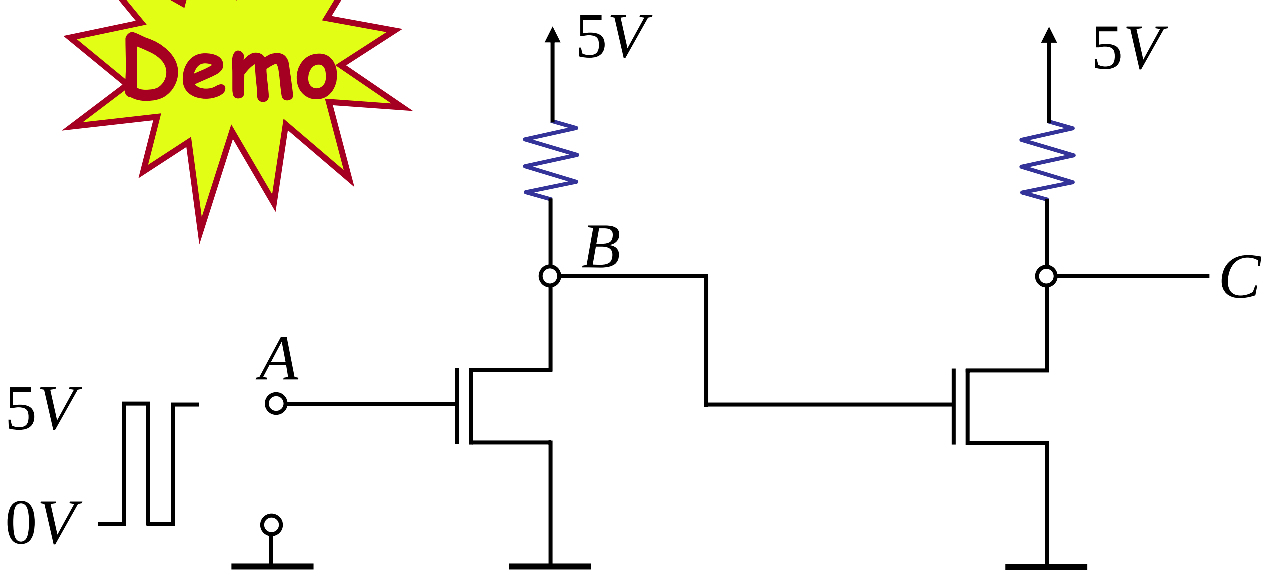
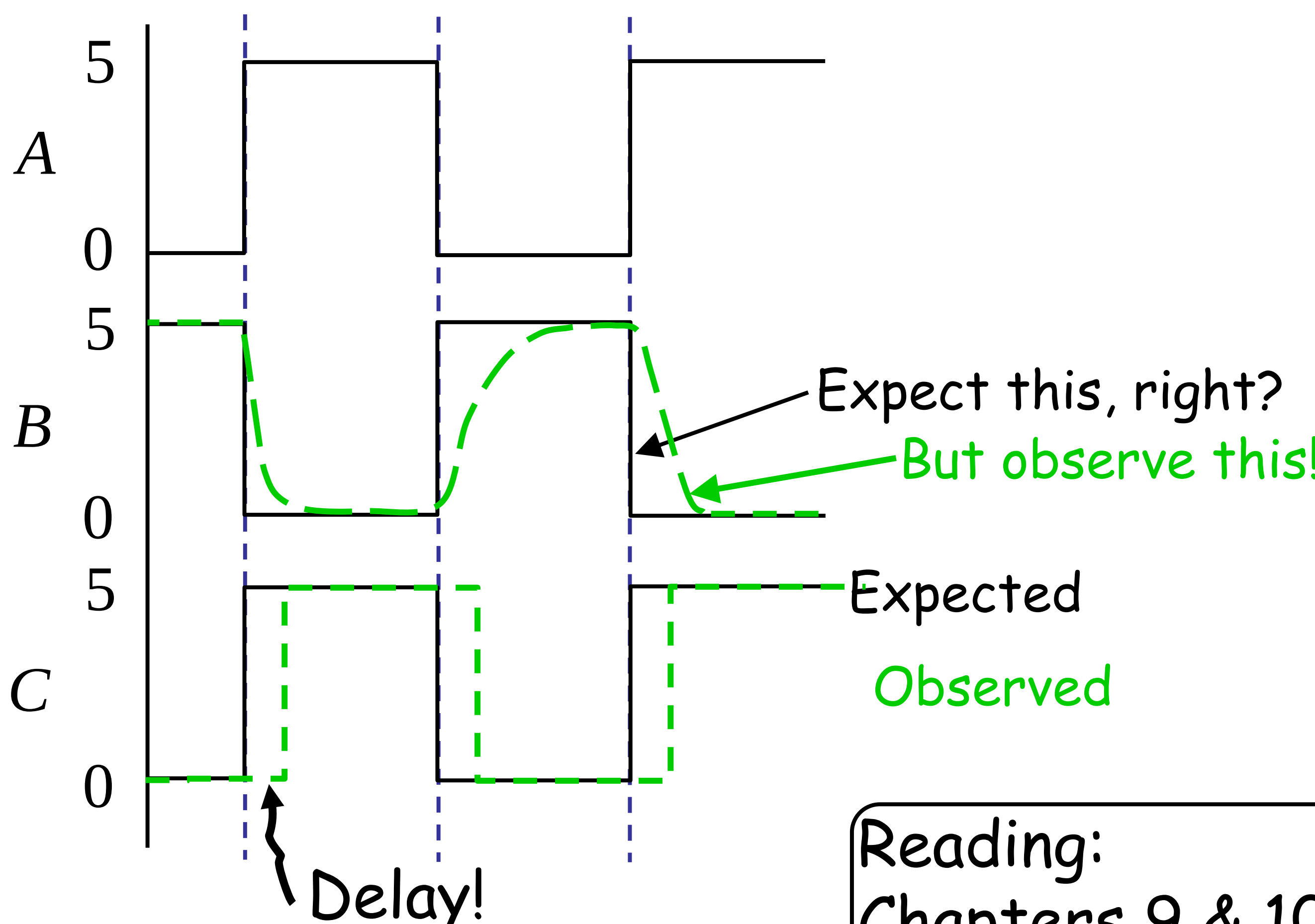
Capacitors and First-Order Systems

Motivation

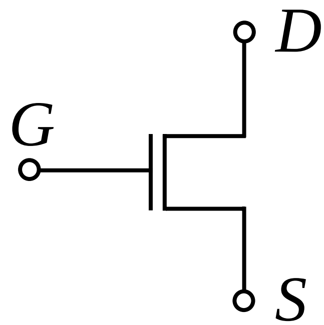
Demo



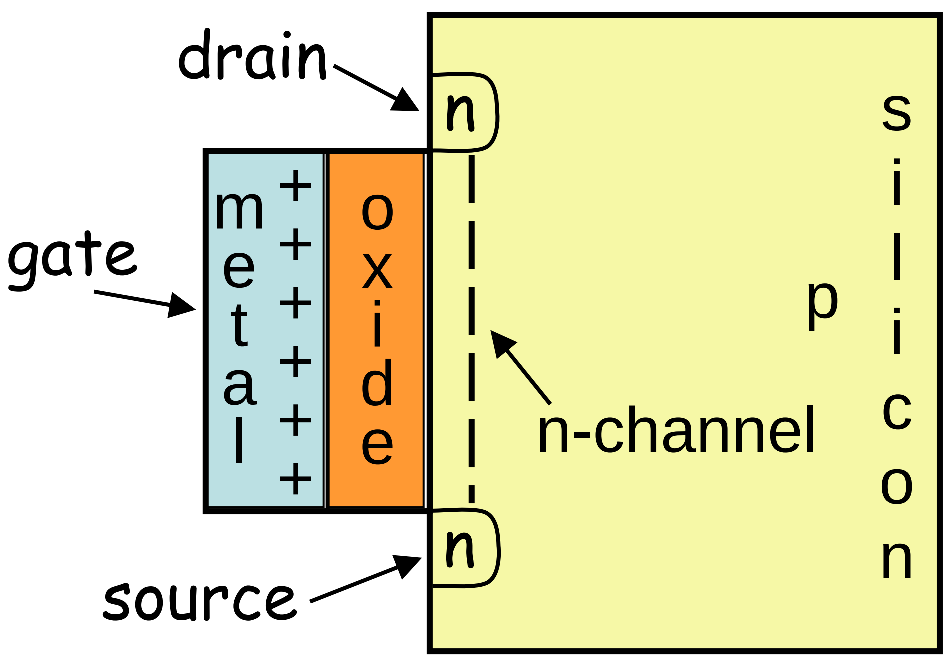


Reading:
Chapters 9 & 10

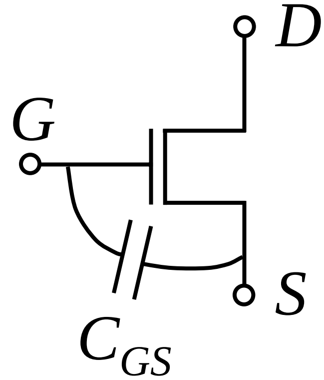
The Capacitor



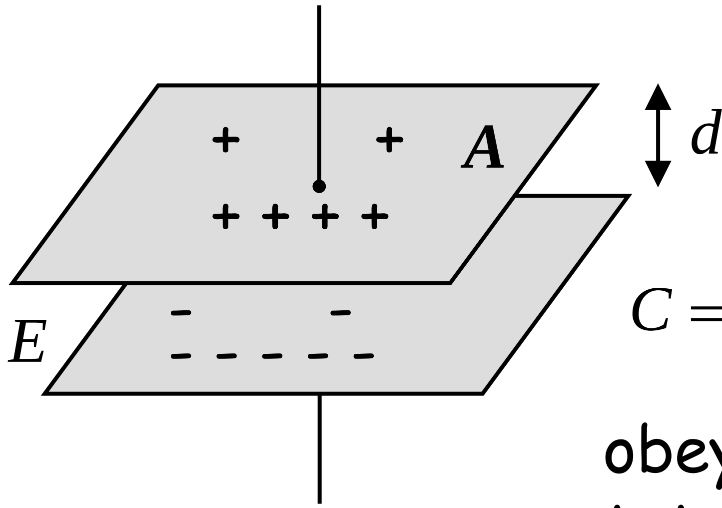
n-channel MOSFET symbol



n-channel MOSFET



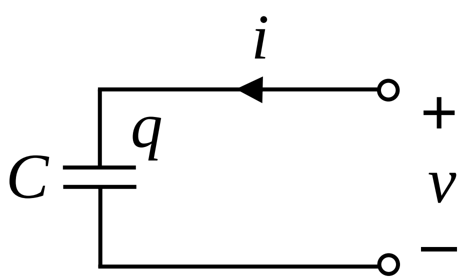
Ideal Linear Capacitor



$$C = \frac{EA}{d}$$

obeys DMD!
total charge on capacitor

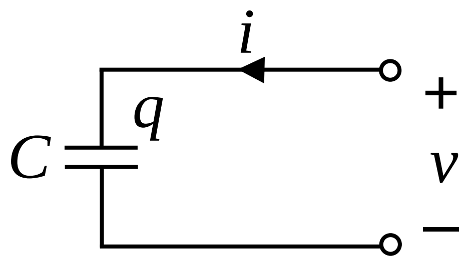
$$= +q - q = 0$$



$$q = C \; v$$

coulombs farads volts

Ideal Linear Capacitor



$$q = C\ v$$

$$i = \frac{dq}{dt}$$

$$= \frac{d(Cv)}{dt}$$

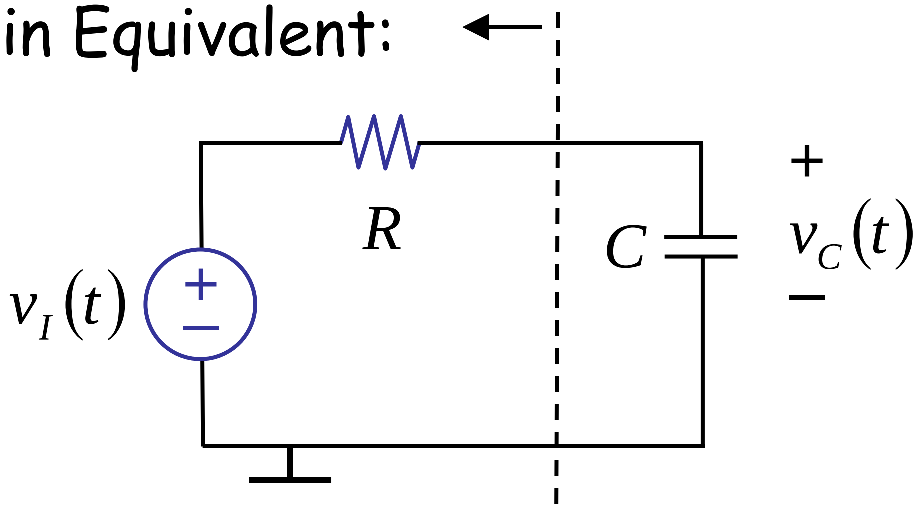
$$= C\frac{dv}{dt}$$

$$\underbrace{\left[E = \frac{1}{2}Cv^2\right]}$$

A capacitor is an energy storage device
→ memory device → history matters!

Analyzing an RC circuit

Thévenin Equivalent:



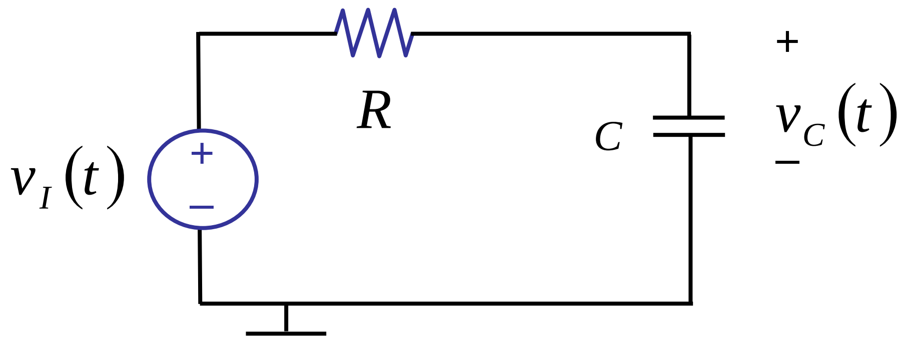
Apply node method:

$$\frac{v_C - v_I}{R} + C\frac{dv_C}{dt} = 0$$

$$RC\frac{dv_C}{dt} + v_C = v_I \quad \begin{cases} t \geq t_0 \\ v_C(t_0) \text{ given} \end{cases}$$

units of time

Let's do an example:



$$v_I(t) = V_I$$

$$v_C(0) = V_0 \quad \text{given}$$

$$RC\,\frac{dv_C}{dt} + v_C = V_I \quad \text{———————} \quad \otimes$$

Example…

$$v_I(t) = V_I$$

$$v_C(0) = V_0 \quad \text{given}$$

$$RC \, \frac{dv_C}{dt} + v_C = V_I \qquad \text{———} \quad \text{(X)}$$

$$\underset{\text{total}}{v_C(t)} = \underset{\text{homogeneous}}{v_{CH}(t)} + \underset{\text{particular}}{v_{CP}(t)}$$

Method of homogeneous and particular solutions:

1. Find the particular solution.
2. Find the homogeneous solution.
3. The total solution is the sum of the particular and homogeneous solutions.

   Use the initial conditions to solve for the remaining constants.

1 Particular solution

$$RC\frac{dv_{CP}}{dt}+v_{CP}=V_I$$

$\boxed{v_{CP}=V_I}$ works

$$RC\underbrace{\cancel{\frac{dV_I}{dt}}}_{0}+V_I=V_I$$

In general, use trial and error.

v_{CP}: any solution that satisfies the original equation X

2 Homogeneous solution

$$RC\frac{dv_{CH}}{dt}+v_{CH}=0 \quad \text{——— Y}$$

v_{CH}: solution to the homogeneous equation Y

(set drive to zero)

$v_{CH}=Ae^{st}$ assume solution of this form. A, s?

$$RC\frac{dAe^{st}}{dt}+Ae^{st}=0$$

$$RCAs\cancel{e^{st}}+A\cancel{e^{st}}=0$$

Discard trivial $A=0$ solution,

$RCs+1=0$ Characteristic equation

$$\Rightarrow \quad s=-\frac{1}{RC}$$

or

$$\boxed{v_{CH}=Ae^{\frac{-t}{RC}}}$$

RC called time constant τ

③ Total solution

$$v_C = v_{CP} + v_{CH}$$

$$v_C = V_I + Ae^{\frac{-t}{RC}}$$

Find remaining unknown from initial conditions:

Given, $v_C = V_0$ at $t = 0$

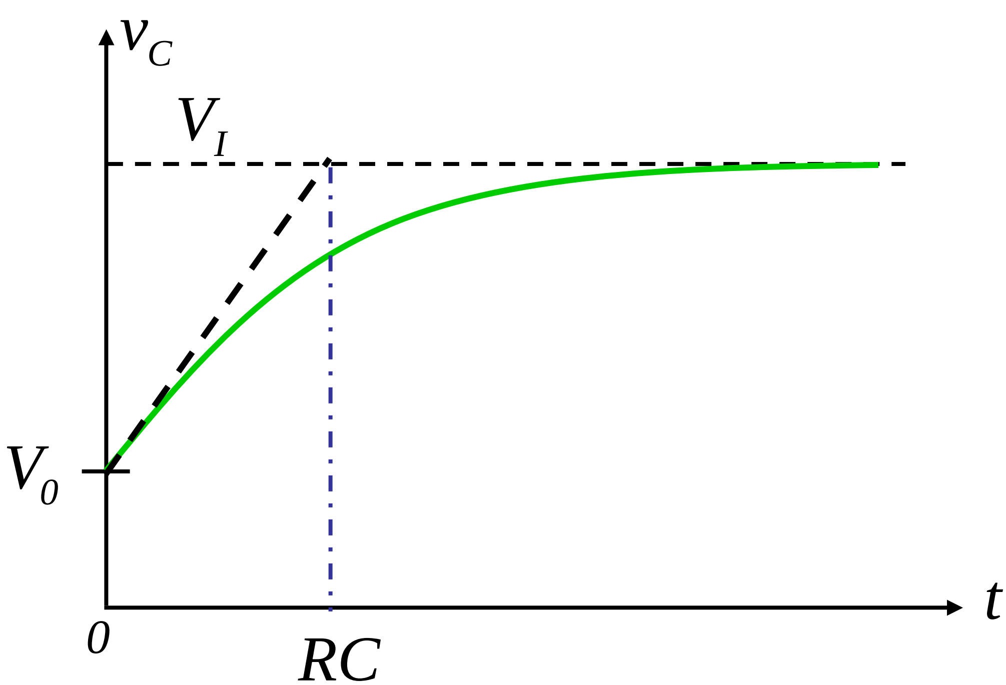
so, $V_0 = V_I + A$

or $A = V_0 - V_I$

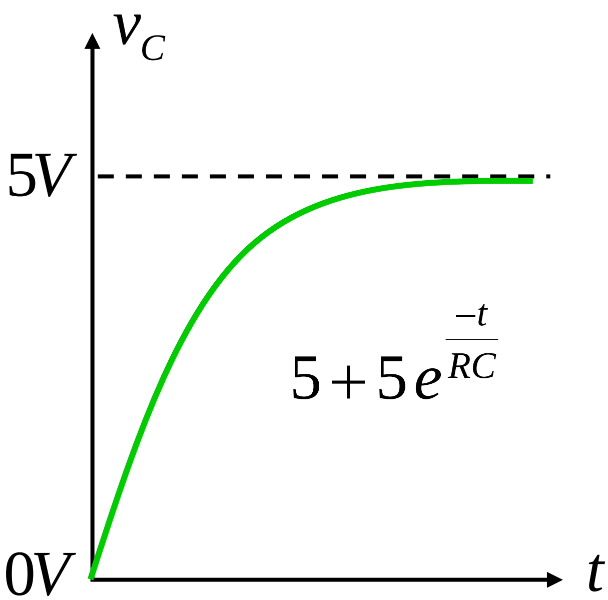
thus $$\boxed{v_C = V_I + (V_0 - V_I)\, e^{\frac{-t}{RC}}}$$

also $$i_C = C\frac{dv_C}{dt} = -\frac{(V_0 - V_I)}{R} e^{\frac{-t}{RC}}$$

$$v_C = V_I + \left(V_0 - V_I\right) e^{\frac{-t}{RC}}$$

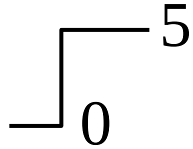


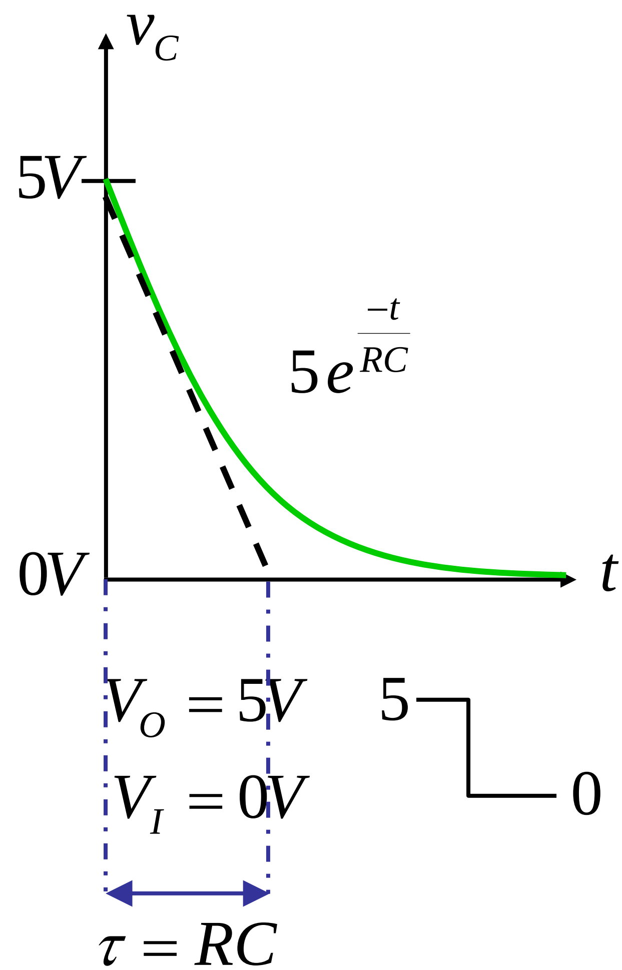
Examples



$V_O = 0V$

$V_I = 5V$





Remember demo