Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

# 6.002 – Circuits & Electronics
Spring 2007

# Quiz #2

25 April 2007

**Name:**________________________________________________________

- There are 20 pages in this quiz, including this cover page. Please check that you have them all.
- Please write your name in the space provided above, and circle the name of your recitation instructor along with the time of your recitation.
- **IMPORTANT:** The problems in this quiz vary in difficulty; moreover, questions of different levels of difficulty are distributed throughout the quiz. If you find yourself spending a long time on a question, consider moving on to later problems in the quiz, and then working on the challenging problems after you have finished all of the easier ones.
- Do your work for each question within the boundaries of that question, or on the back of the preceding page. When finished, enter your answer to each question in the corresponding answer box that follows the question.
- Remember to include the sign and units for all numerical answers.
- This is a closed-book quiz, but you may use a calculator and your double-sided page of notes.
- You have 2 hours to complete this quiz.
- Good luck!

| | | | | |
|---|---|---|---|---|
| 1A. | 1B. | 1C. | 1D. | |
| | 1E. | 1F. | 1G. | |
| 2A. | 2B. | 2C. | 2D. | |
| | 3A. | 3B. | 3C. | |
| | 3D. | 3E. | 3F. | |
| | | | Final Score: | |



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**Problem 1A: 5 points**

Find the value of $I$ in the circuit shown in Figure 1. Assume that the MOSFET is operating in the saturation region, where $i_{DS} = (K/2)(V_{GS} - V_T)^2$, $K = 2\,\text{mA/V}$, and $V_T = 1\,\text{V}$.

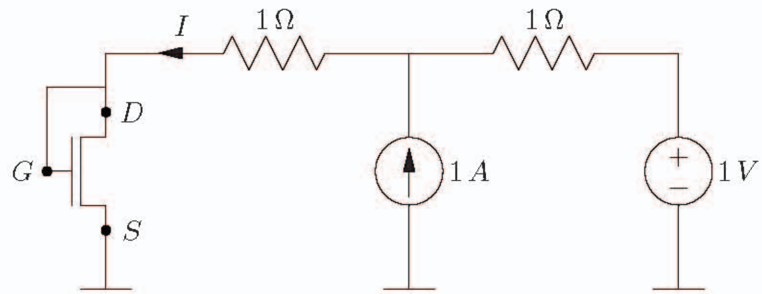


Figure 1.

$I =$



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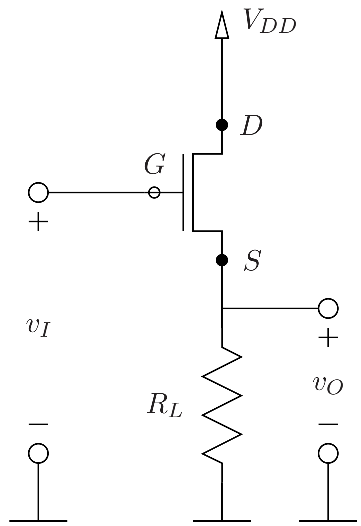
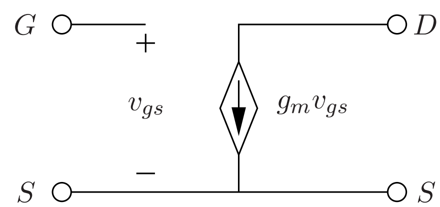
**Problem 1B: 5 points**

Calculate the small-signal gain $v_o/v_i$ of the circuit shown in Figure 2a. Use the small-signal model of the MOSFET shown in Fig 2b. Assume that the MOSFET is operating in the saturation region.




(a)




(b)

Figure 2.

$$\frac{v_O}{v_i} =$$



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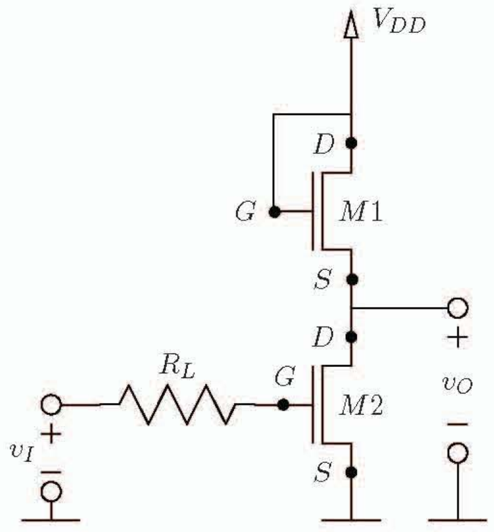
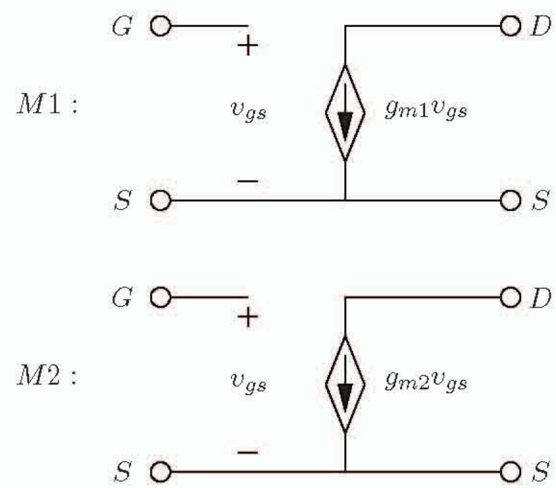
**Problem 1C: 5 points**

Calculate the small-signal gain $v_o/v_i$ of the circuit in Figure 3a in terms of $R_L$, $g_{m1}$, and $g_{m2}$. Assume that both of the MOSFETS are operating in the saturation region, and that the small-signal models of the MOSFETs are as shown in Figure 3b.




(a)




(b)

Figure 3.

$$\frac{v_O}{v_i} =$$



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**Problem 1D: 5 points**

The circuit shown in Figure 4 has been at rest with the switch open for a long time. At $t = 0$ the switch is closed. Sketch the current $i_C$ through the capacitor for $t > 0$ on the axes below. Using the circuit parameters $R_1$, $R_2$, $C$, and $V_O$, indicate on your sketch (i) the initial value $i_C(0^+)$, (ii) the final value $i_C(\infty)$, and (iii) the time constant.

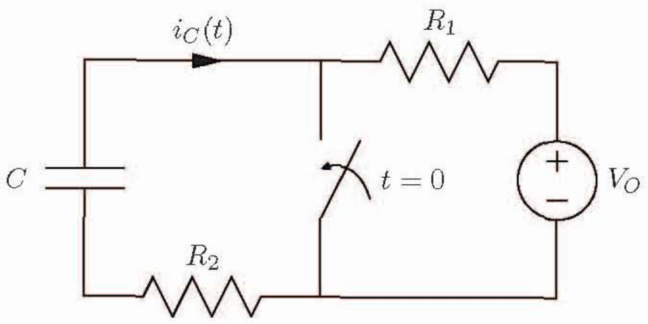
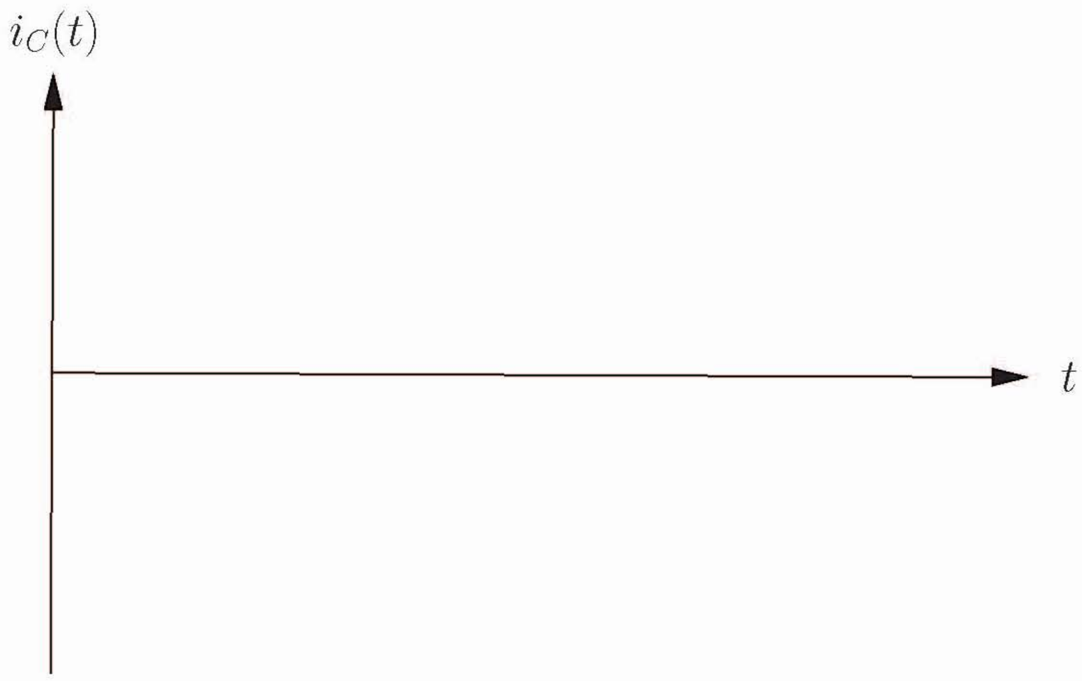



Figure 4.



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**Problem 1E: 10 points**

Consider the circuit illustrated in Figure 5.

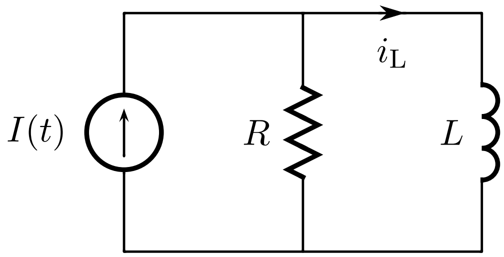


Figure 5.

(a) (5 points) Find the inductor current $i_L(t)$ for $t \geq 0$ in response to the current step $I(t) = I_{step}(t) = I_0 u(t)$. Assume that $i_L(0) = 0$.

$i_L(t) =$ for $I(t) = I_{step}(t)$



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**Problem 1E (continued):**

(b) (5 points) Find the inductor current $i_L(t)$ for $t \geq 0$ in response to the current ramp $I(t) = I_{ramp}(t) = I_0 \alpha t u(t)$, where $\alpha$ is a constant. Again, assume that $i_L(0) = 0$.

| | |
|---|---|
| $i_L(t) =$ | for $I(t) = I_{ramp}(t)$ |



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**Problem 1F: 5 points**

The capacitor in the circuit in Figure 6 has an initial voltage $V_O$ on its terminals at $t = 0^-$ when a step of voltage V$u(t)$ is applied at $t = 0$. Find an expression for the voltage across the capacitor $v_C$ for $t > 0$.

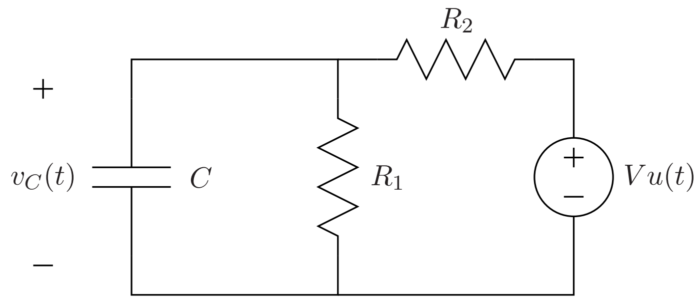


Figure 6.

$v_C(t) =$



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**Problem 1G: 10 points**

Find an expression for the voltage $v_C(t)$ for the circuit in Figure 7, assuming that $v_C(0^-) = 0$ and $i_L(0^-) = 0$. Please express your answer in terms of $\Lambda$, $L$, and $C$.

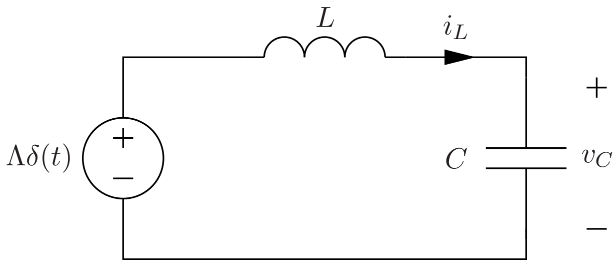


Figure 7.

$v_C(t) =$



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**Problem 2: 20 points**

Figure 8a shows a buffer comprising a pair of MOSFETs driven by a source whose Thevenin equivalent circuit is represented by the voltage source $v_{TH}$ and source resistance $R_{TH}$.

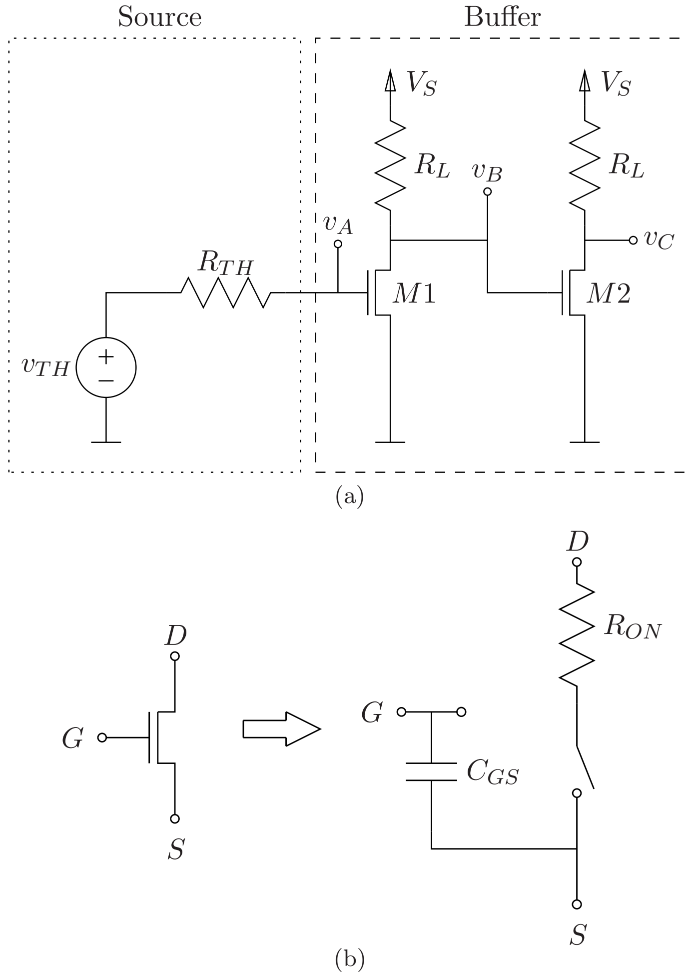


Figure 8.

The MOSFETs $M1$ and $M2$ in the buffer are identical and their behavior is to be modelled by the circuit shown in Figure 8b, where the switch is closed for $v_{GS} > V_T$ and is open when $v_{GS} < V_T$. This problem explores the dynamic operation of the buffer, in particular the delays introduced by the gate-source capacitance $C_{GS}$. Throughout the problem, assume that

$$V_{TH} > V_T > \frac{R_{ON}}{R_L + R_{ON}} V_S$$

$$V_S > V_T$$



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First, consider the case in which the input to the buffer makes a $0 \rightarrow 1$ transition. The voltage source $v_{TH}$ and the output at the drain of $M1$, $v_B$, then exhibit the waveforms sketched in Figure 9.

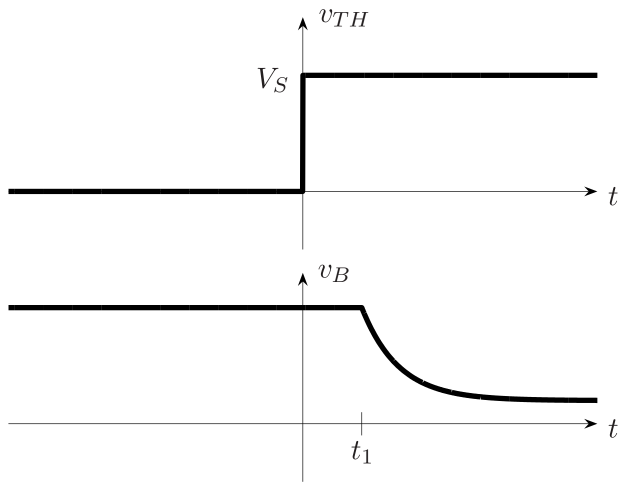


Figure 9.

(2A) (5 points) Determine the delay $t_1$ indicated in Figure 9.

$t_1 =$



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(2B) (5 points) The voltage $v_B(t)$ for $t > t_1$ can be expressed by a function of the form

$$v_B(t) = V_1 + V_2 e^{-(t-t_1)/\tau_1}.$$

Determine $V_1$, $V_2$, and $\tau_1$.

$V_1 =$

$V_2 =$

$\tau_1 =$



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(2C) (5 points) Find an expression for the total delay $t_{0\to 1}$ between the $0 \to 1$ transition at the input $V_{TH}$ and the corresponding $0 \to 1$ transition at the output $v_C$.

$t_{0\to 1} =$



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Now consider the case in which the input to the buffer makes a $1 \rightarrow 0$ transition. The voltage source $v_{TH}$ and the output at the drain of $M1$, $v_B$, then exhibit the waveforms sketched in Figure 10.

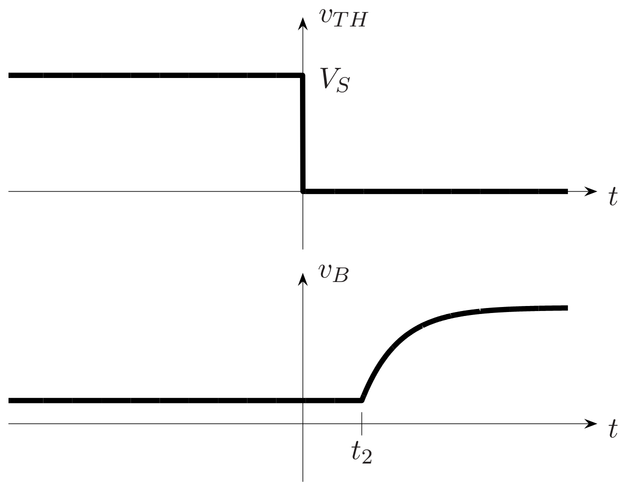


Figure 10.

(2D) (5 points) The voltage $v_B(t)$ for $t > t_2$ can be expressed by a function of the form

$$v_B(t) = V_3 + V_4 e^{-(t-t_2)/\tau_2}.$$

Determine $V_3$, $V_4$, and $\tau_2$.

$V_3 =$

$V_4 =$

$\tau_2 =$



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**Problem 3: 35 points**

Consider the circuit illustrated in Figure 11. Assume that the MOSFET is characterized by $K = 2\,\mathrm{mA/V^2}$, $V_T = 1\,\mathrm{V}$, and is operating in the saturation region.

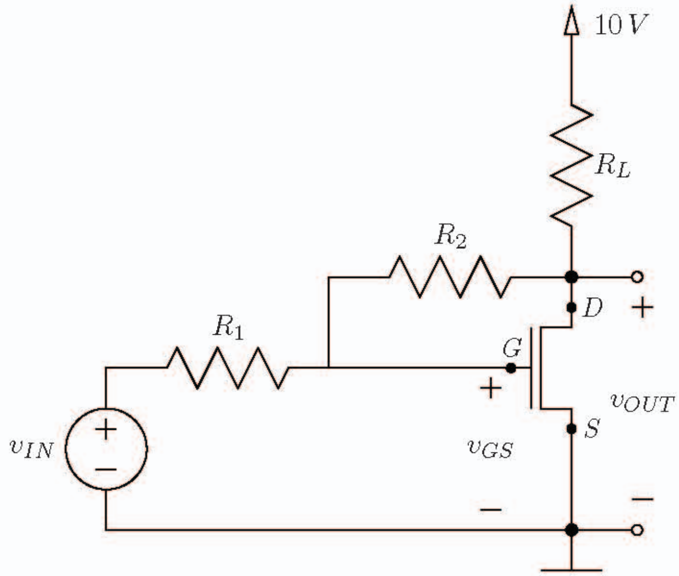


Figure 11.

(3A) (5 points) Express $v_{GS}$ in terms of $v_{IN}$ and $v_{OUT}$.

$v_{GS} =$



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(3B) (5 points) In the laboratory, measurements reveal that $v_{IN} = v_{OUT} = v_{GS} = 5\,V$. Is the MOSFET operating in the saturation region? Explain your answer in a short sentence.

Explanation:



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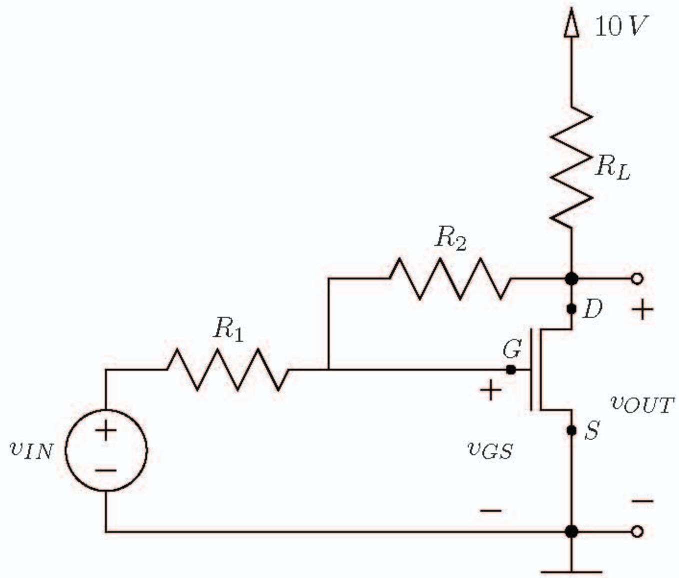


Figure 11.: (repeated for convenience)

(3C) (5 points) What is the value of $R_L$, given that $v_{IN} = v_{OUT} = v_{GS} = 5\,V$?

$R_L =$



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(3D) (5 points) Define the small-signal current through the MOSFET as $i_{ds} = g_m v_{gs}$. What is $g_m$ at the operating point $v_{IN} = v_{OUT} = v_{GS} = 5\,V$?

$g_m =$



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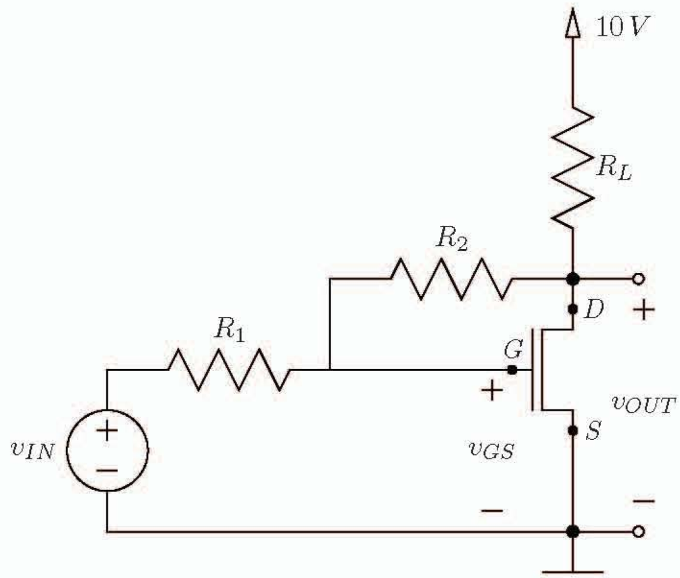


Figure 11.: (repeated for convenience)

(3E) (5 points) Draw the small-signal equivalent model of the circuit in Figure 11, representing the relation between $i_{ds}$ and $v_{gs}$ in the MOSFET by a voltage-controlled current source with a proportionality constant $g_m$.

| Small-signal equivalent circuit: |
|---|
| |



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(3F) (10 points) What is the small-signal gain $v_{out}/v_{in}$? Assume that $R_1 = R_2 = R$, and express your answer in terms of $R$, $R_L$, and $g_m$. Assume that the MOSFET is operating in the saturation region. (*Hint: one way to approach this problem is to set up node equations for the nodes in the circuit, and solve for* $v_{out}$.)

$$\frac{v_{out}}{v_{in}} =$$



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