

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Circuits & Electronics
Spring 2007

Final Exam

21 May 2007

Name: _____

- There are 31 pages in this final, including this cover page. Please check that you have them all.
- Please write your name in the space provided above, and circle the name of your recitation instructor along with the time of your recitation.
- **IMPORTANT:** The problems in this exam vary in difficulty; moreover, questions of different levels of difficulty are distributed throughout the exam. If you find yourself spending a long time on a question, consider moving on to later problems in the exam, and then working on the challenging problems after you have finished all of the easier ones.
- Do your work for each question within the boundaries of that question, or on the back of the preceding page. When finished, enter your answer to each question in the corresponding answer box that follows the question.
- Remember to include the sign and units for all numerical answers.
- This is a closed-book exam, but you may use a calculator and three double-sided pages of notes.
- You have 3 hours to complete this final.
- Good luck!

1A.	1B.	2.	3.	4.	
5.	6.	7.	8.	9.	
10.	11.	12A.	12B.	13.	
14.	15.	16.	17.	18.	
	19A.	19B.	19C.	19D.	
Final Score:					

Problem 1: 8 points

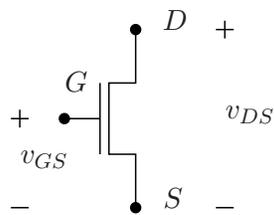


Figure 1.

A MOSFET (shown in Figure 1) operating in the triode region has a characteristic i_{DS} relationship which depends on both v_{DS} and v_{GS} :

$$i_{DS} = K [(v_{GS} - V_T) v_{DS} - v_{DS}^2/2]$$

where $v_{GS} > V_T$ and $v_{DS} \leq v_{GS} - V_T$. The small-signal relationship between i_{ds} , v_{gs} , and v_{ds} can be expressed by an equation of the form

$$i_{ds} = Av_{gs} + Bv_{ds}$$

where A and B are constants. Assume that the device is biased at an operating point (V_{GS}, V_{DS}) .

- (1A) (4 points) Draw the 2-element small-signal model for the device which is operating in the triode region. Express the element values in terms of A and B .

Small-signal equivalent circuit:

- (1B) (4 points) Find the values of the constants A and B in the two-element small signal model for the MOSFET operating in the triode region,

$$i_{ds} = Av_{gs} + Bv_{ds}$$

Formulate your answers in terms of the variables K , V_{GS} , V_T , and V_{DS} .

$A =$

$B =$

Problem 2: 4 points

A and B are 1 bit numbers. Each of them can take on the positive integer values 0 or 1. Implement an adder which takes the two inputs A and B and produces a 1 bit sum output Z. The sum output Z must saturate at 1, which means that if the sum is greater than 1, the circuit must output a 1.

You may use as many inverters, NAND gates and NOR gates as you like in your circuit.

Your adder design:

Problem 3: 4 points

Consider the logic function O described by the following truth table.

A	B	C	$O(A,B,C)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Show how to implement the function O using *only* two-input NAND gates and inverters. (*Hint*: Remember the relations indicated by Figure 2.)

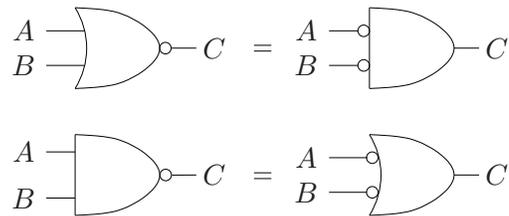


Figure 2.

Problem 3 (continued):

Implementation of O :

Problem 4: 4 points

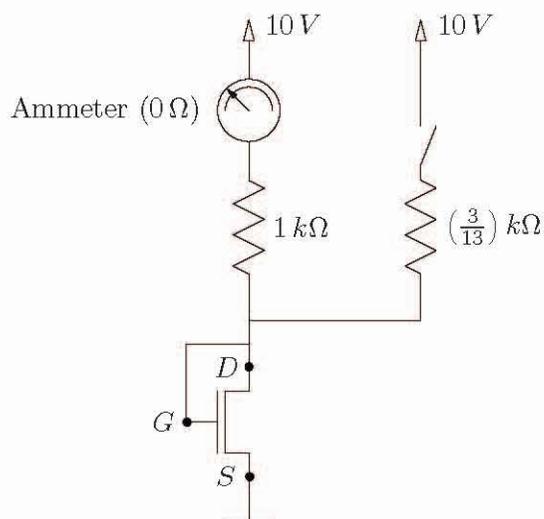


Figure 3.

In the circuit in Figure 3, the current reading from the ammeter is 4 mA with the switch open, and 3 mA with the switch closed, where $i_{DS} = (K/2)(v_{GS} - V_T)^2$. What are V_T and K ?

$V_T =$

$K =$

Problem 5: 4 points

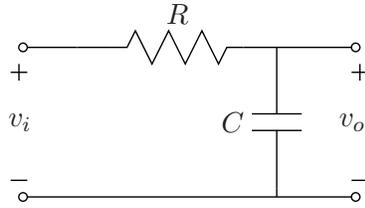
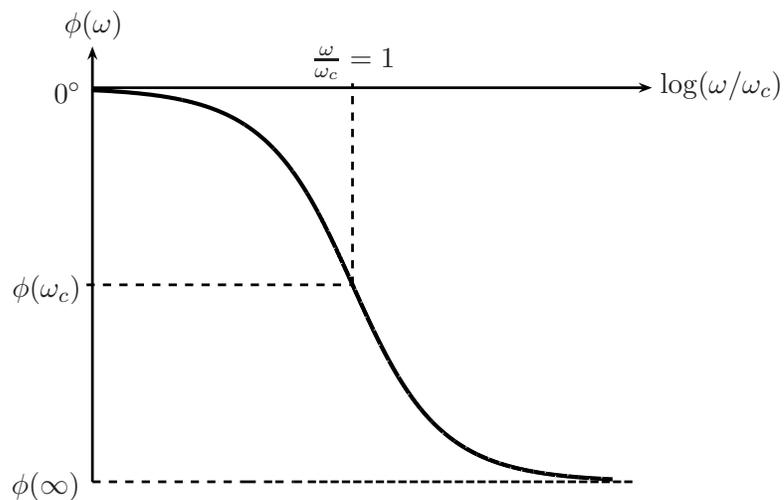
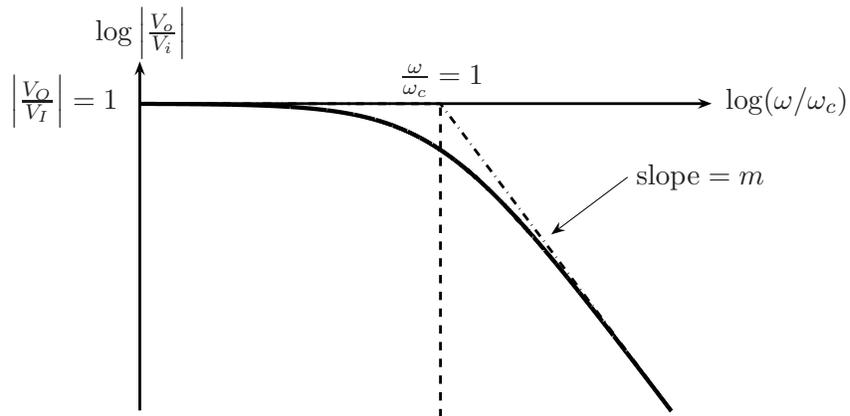


Figure 4.

Let the network depicted in Figure 4 have a frequency response given by

$$\frac{v_o}{v_i} = \frac{1}{1 + j\omega K}$$

where K is positive. The magnitude and phase of this frequency response as a function of ω are illustrated by the two plots below. Find the values of ω_c , m , $\phi(\omega_c)$, and $\phi(\infty)$ in terms of the circuit parameters R and C .



Problem 5 (continued):

$$\omega_c =$$

$$m =$$

$$\phi(\omega_c) =$$

$$\phi(\infty) =$$

Problem 6: 4 points

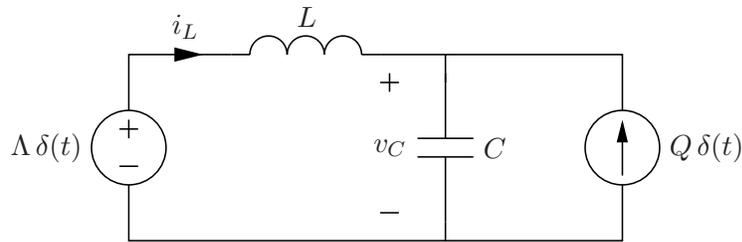


Figure 5.

Consider the circuit in Figure 5 with two inputs $V(t) = \Lambda \delta(t)$ and $I(t) = Q \delta(t)$. The inductor and capacitor have zero initial state, i.e. $v_C(t = 0^-) = 0$ and $i_L(t = 0^-) = 0$. What are the inductor current i_L and the capacitor voltage v_C at $t = 0^+$? At what frequency ω_{osc} will the circuit oscillate?

$$v_C(0^+) =$$

$$i_L(0^+) =$$

$$\omega_{osc} =$$

Problem 7: 4 points

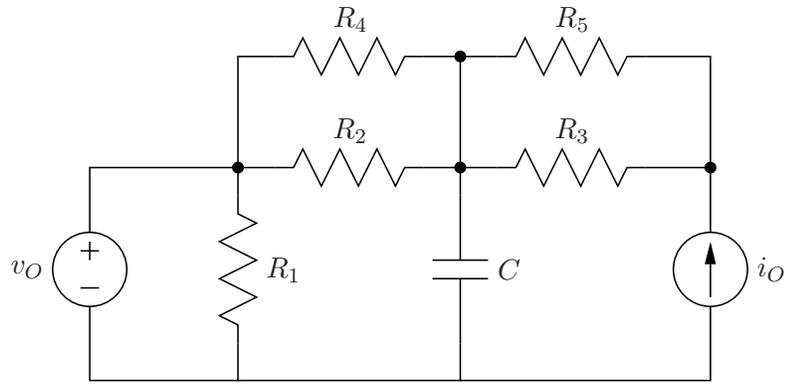


Figure 6.

What is the time constant τ for the circuit depicted in Figure 6?

$\tau =$

Problem 8: 4 points

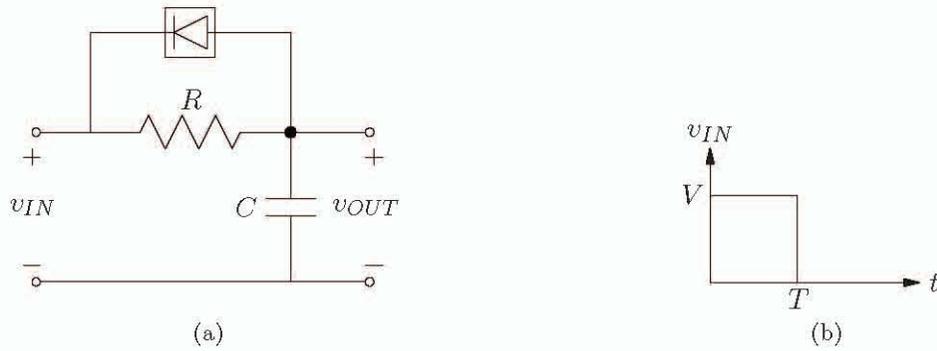


Figure 7.

The network shown in Figure 7a contains an ideal diode. For the network, plot $v_{OUT}(t)$ when $v_{IN}(t)$ is the pulse shown in Figure 7b. The capacitor is initially discharged. Please label all important features of the plot, such as amplitude, time constant, and so forth.

For convenience, we have illustrated the v_D-i_D relationship of the ideal diode in Figure 8b, for the diode terminal variables as defined in Figure 8a.



Figure 8.

Problem 8 (continued):



Problem 9: 4 points

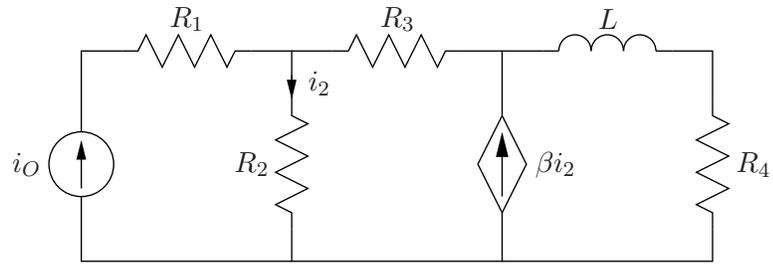


Figure 9.

What is the time constant τ for the circuit depicted in Figure 9?

$\tau =$

Problem 10: 4 points

Inlet Inc. manufactures two types of inverters – the SX model and the EX model – which satisfy the following static disciplines.

Inverter SX satisfies a static discipline with the following voltage levels:

$$\begin{aligned} V_{OH} &= 4V & V_{OL} &= 0.5V \\ V_{IH} &= 2V & V_{IL} &= 1V \end{aligned}$$

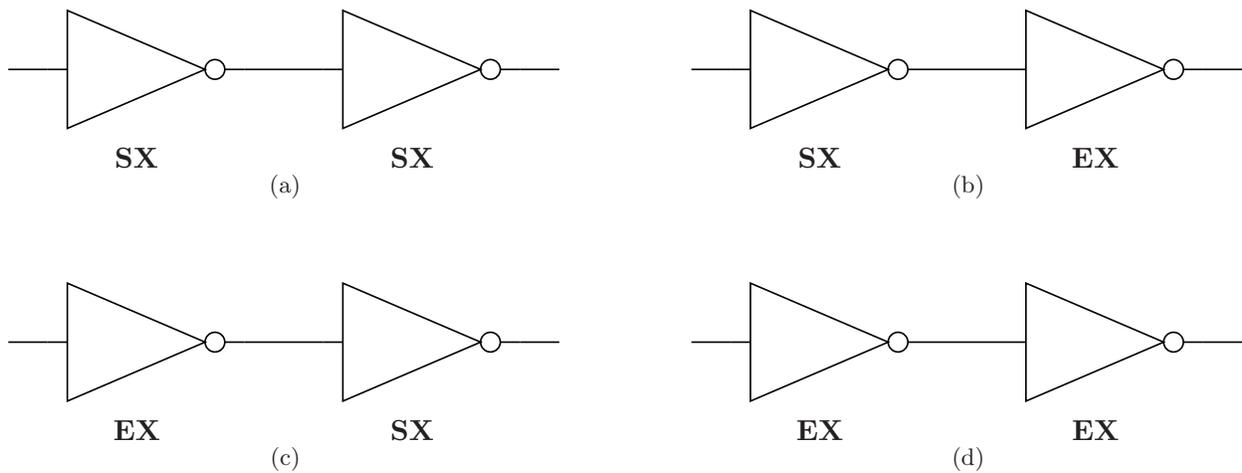
Similarly, Inverter EX satisfies a static discipline with these voltage levels:

$$\begin{aligned} V_{OH} &= 5V & V_{OL} &= 0.7V \\ V_{IH} &= 1.2V & V_{IL} &= 1V \end{aligned}$$

Inlet would like to bid on a contract for buffers issued by QuellCom. The buffers need to operate in a system which follows a static discipline with the following voltage levels:

$$\begin{aligned} V_{OH} &= 4.55V & V_{OL} &= 0.8V \\ V_{IH} &= 1.5V & V_{IL} &= 1V \end{aligned}$$

As an application engineer at Inlet you are tasked with determining whether a pair of inverters connected in series can satisfy QuellCom’s static discipline. For each of the inverter pairs shown below, indicate whether the circuit can serve as a buffer in QuellCom’s systems, or explain in a sentence why not.



Problem 10 (continued):

Is SX-SX a satisfactory buffer?

YES

NO

If not, explanation:

Is SX-EX a satisfactory buffer?

YES

NO

If not, explanation:

Is EX-SX a satisfactory buffer?

YES

NO

If not, explanation:

Is EX-EX a satisfactory buffer?

YES

NO

If not, explanation:

Problem 11: 4 points

Determine an expression for v_O in terms of v_I for the circuit in Figure 10. Assume the MOSFET is operating in its saturation region, with $i_{DS} = (K/2)(v_{GS} - V_T)^2$, and that $v_I < 0$.

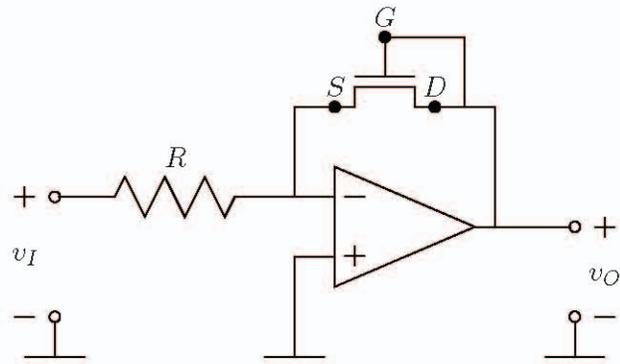


Figure 10.

$v_O =$

Problem 12: 8 points

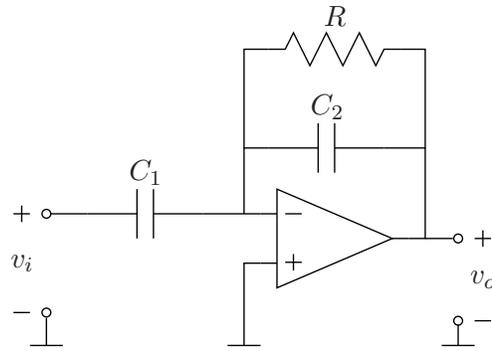


Figure 11.

Q. Rius is an engineer at Inlet Inc. Tasked with building a filter, Rius comes up with the op-amp circuit shown below, and applies the input signal $v_I = A \sin(\omega t)$.

(12A) (4 points) The output $v_o(t)$ is of the form

$$v_o(t) = V_o \sin(\omega t + \theta)$$

Determine the value of V_o and θ in terms of A , R , C_1 , C_2 , and ω .

$$V_o =$$

$$\theta =$$

Problem 12 (continued):

- (12B) (4 points) Experimenting with his circuit, Rius notices that the resistance R is so large compared to the impedances of $C1$ and $C2$ that it has little observable effect on the output even when he doubles or quadruples the value of R . Excited with this observation, Rius removes R from his circuit in an attempt to reduce the number of components. To his dismay, the circuit ceases to work as before. Explain in a sentence or two the reason for the circuit's failure.

Explanation:

Problem 13: 4 points

Consider the circuit shown in Figure 12. The parameters characterizing the behavior of the MOSFET are $K = 2 \text{ mA/V}^2$, and $V_T = 1 \text{ V}$, and $i_{DS} = (K/2)(v_{GS} - V_T)^2$.

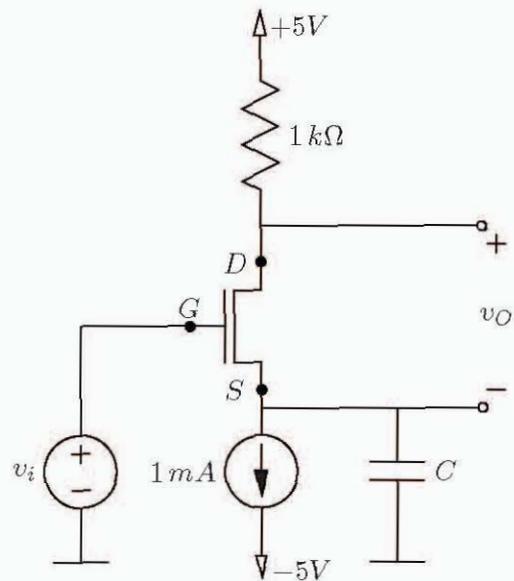


Figure 12.

Determine the value of V_O , the DC component of the output voltage. Assume that $v_i = 0$.

$V_O =$

Problem 14: 4 points

Consider the circuit shown in Figure 13. The parameters characterizing the behavior of the MOSFET are $K = 2 \text{ mA/V}^2$, and $V_T = 1 \text{ V}$, and $i_{DS} = (K/2)(v_{GS} - V_T)^2$.

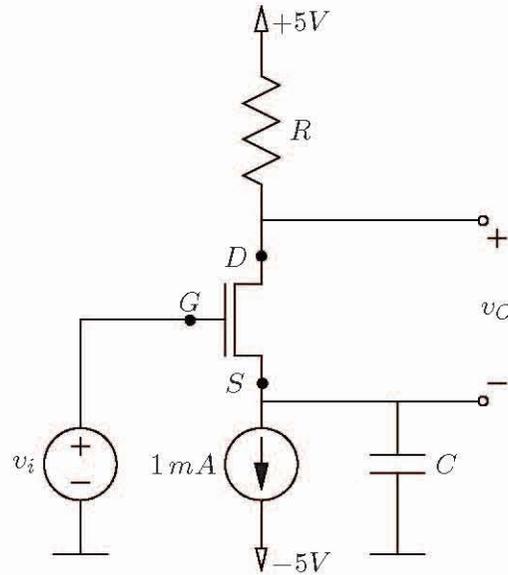


Figure 13.

Assume that the input for this circuit is $v_i(t) = V_i \cos(\omega t)$, where V_i is a small-signal amplitude, and assume that the MOSFET can be characterized by a transconductance g_m . The small-signal output $v_o(t)$ can be expressed in the following form:

$$v_o(t) = V_o \cos(\omega t + \theta)$$

Determine the values of V_o and θ , expressing your answers in terms of V_i , ω , R , C , and g_m .

Problem 14 (continued):

$$V_o =$$

$$\theta =$$

Problem 15: 4 points

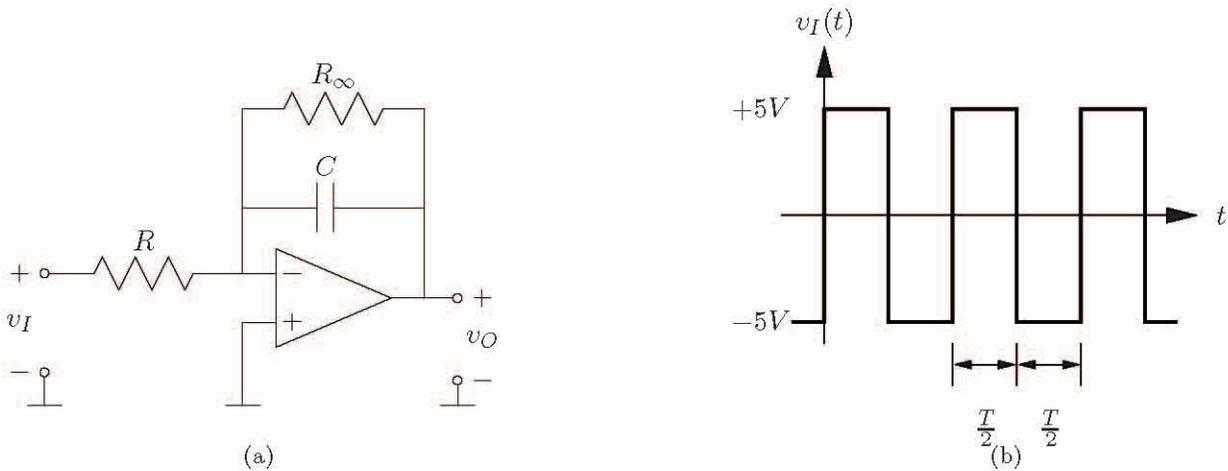


Figure 14.

The circuit depicted in Figure 14a is driven by a signal $v_I(t)$ which is a square wave oscillating between $+5V$ and $-5V$ with a period of T . The initial capacitor voltage $v_C(t = 0) = 0$. The resistor R_∞ is included to insure that the op-amp is operating in a stable range, but it is large enough to be ignored in your analysis. Sketch the response $v_O(t)$ on the axes provided, labelling all of the important features of the waveform.



Problem 16: 4 points

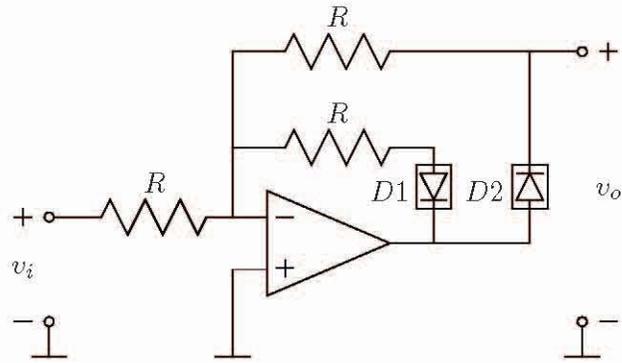


Figure 15.

The diodes $D1$ and $D2$ in the circuit of Figure 15 are ideal diodes. If the circuit is driven by a signal $v_i(t) = V_i \cos(\omega t)$, sketch the response $v_o(t)$ on the axes provided, labelling all of the important features of the waveform. (The characteristics of the ideal diode were given in Problem 8, in Figure 8.)



Problem 17: 4 points

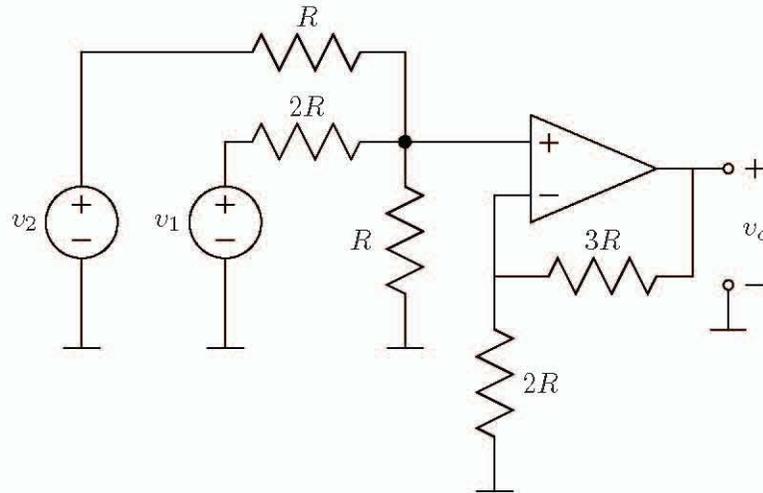


Figure 16.

Derive an expression for $v_o(t)$ in terms of $v_1(t)$ and $v_2(t)$.

$v_o(t) =$

Problem 18: 4 points

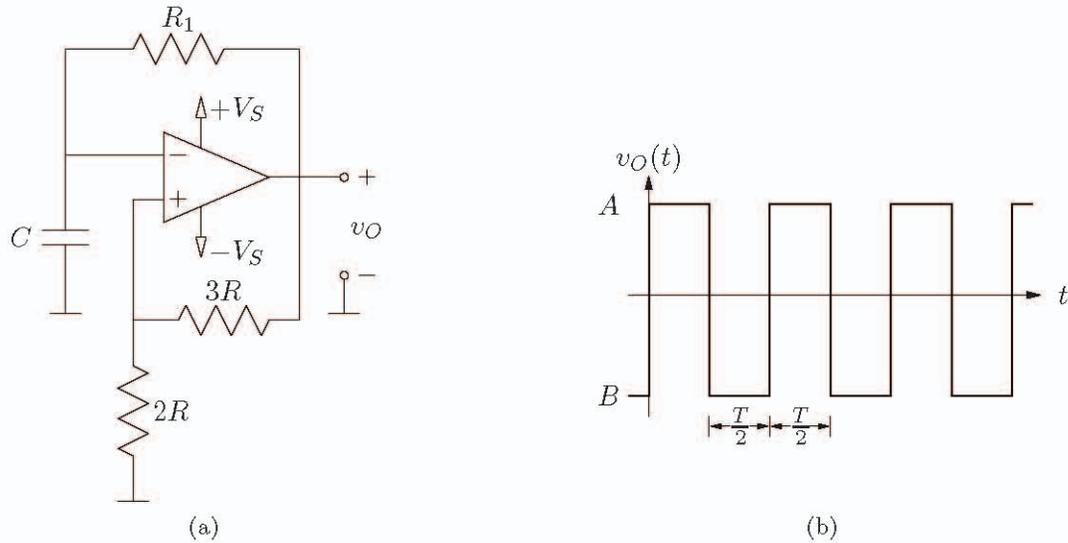


Figure 17.

For the circuit in Figure 17a, the output waveform $v_O(t)$ is illustrated in Figure 17b. The important features of the waveform are denoted A , B , and T ; please find expressions for these variables in terms of R , R_1 , C , and the power supply rails of the op-amp, which are $+V_S$ and $-V_S$.

$$A =$$

$$B =$$

$$T =$$

Problem 19: 20 points

A car travels at a constant speed along a bumpy road. As a result of the bumps, the velocity of the car's axle perpendicular to the road is $V(t)$. Taking into account the dynamics of the car's suspension system, the equation of motion for the body of the car (the chassis) can be formulated as follows:

$$M \frac{du}{dt} + Bu(t) + K \int_{-\infty}^t u dt = -M \frac{dV}{dt} \quad (1)$$

where $u(t)$ is the velocity of the body of the car (relative to the axle), M is the mass of the car, K is the spring constant of the springs connecting the body to the axle, and B is the coefficient of viscous damping for the shock absorbers.

In this problem, you will be asked to make a circuit model corresponding to the equation of mechanical dynamics from Equation 1, and then you will use the model to investigate the car's motion due to the bumpy road.

Note: Each of the parts of this problem can be worked independently of the others.

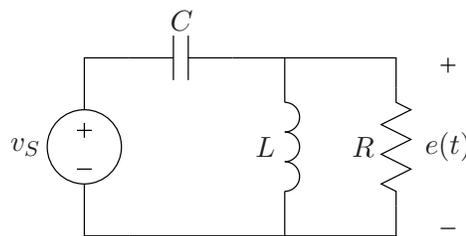


Figure 18.

- (19A) (5 points) The circuit in Figure 18 is proposed to model Equation 1, where the node voltage $e(t)$ is identified with the car body's velocity $u(t)$. Determine the values of v_S , C , L , and R in terms of the parameters in Equation 1 so that the solution for $e(t)$ will be identical to the solution of Equation 1 for $u(t)$.

Problem 19 (continued):

$$v_s =$$

$$C =$$

$$L =$$

$$R =$$

Problem 19 (continued):

(19B) (5 points) Referring to the circuit model, assume that the response is oscillatory.

- What is the frequency of oscillation ω_{osc} ?
- How long must one wait for the amplitude of the transient oscillation to decay to $1/e$ of its initial amplitude?

$$\omega_{osc} =$$

$$t_{1/e} =$$

Problem 19 (continued):

- (19C) (5 points) Now assume that it is desirable to avoid oscillatory transient behavior. What is the maximum value of R in this case?

$$R_{max} =$$

Problem 19 (continued):

- (19D) (5 points) Once again referring to the circuit, suppose that the source $v_S(t)$ is sinusoidal, i.e. $v_S(t) = A \sin(\omega t)$. Find an expression for the steady-state voltage $e(t)$.

$e(t) =$
