

## SP 713 Slide Rule Practice for our MIT Museum visit

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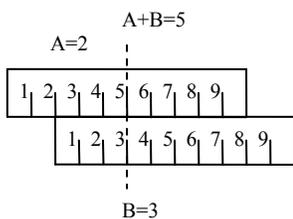
On your slide rule, look for the rows of short vertical marks, with numbers just above them. Notice how the spacing goes between one numbered marking and the next. Each horizontal row of markings is a “scale”. At the leftmost end of each scale, is the name of that scale. Notice the feel of the instrument (especially with the wood ones) with its slide, and the sense of scale and spacing that develop as you begin using it.

On the **plastic slide rules** from the museum, these labels are: Top: A; Slide B, CI, C [Reverse of slide: S,L, T – if SLT is on the front of your rule, flip it around to bring B, CI, C to the front]; Bottom: D, K.

The **wood slide rules** from the museum have many more scales, and these scales are arranged differently on different rules. Look on both sides of the rule to find these scales: C, D, A, B, CI, DI, CF, DF.

In *Powers of Ten*, when the scene went from one scale to the next, such as  $10^1$  (the man on the blanket) to  $10^2$  (the ball field), the exponent increased by the addition of 1, while the number increased by a factor of 10. When going down in scale, such as from  $10^0$  (the hand) to  $10^{-1}$  (part of skin), the exponent changed by the subtraction of 1, while the number decreased by a factor of  $1/10$ . These exponents are logarithms of the original number (in this case, in base 10).

You can use two rulers side by side to add numbers. Suppose you want to compute  $A+B$ . If you place the 0 mark of the second ruler at  $A$  on the first ruler, and then find  $B$  on the second ruler, then the number on the first ruler adjacent to  $B$  on the second ruler is  $A+B$ .



The exercises you did with Galileo’s compass used a linear scale like this pair of rulers. Galileo put other scales on his instrument where the spacing between one numbered mark and the next higher number was not the same; these scales represented squares, cubes, and so on.

Suppose that the sequence of numbers on these marks, instead of increasing by one, changes in some other way. Then what you do in applying the above operation changes. Suppose, e.g., that you have a ruler where the labels on equidistant spaced marks are 1, 2, 4, 8, 16, ... and another ruler that is the same. Then if  $A$  is a place on the first ruler, and the 1 on the second ruler is aligned with that, and  $B$  is a place on the second ruler, then the place that  $B$  aligns with on the first ruler is  $A$  times  $B$ .

The reason that this happens is that the actual distance between 1 and each label is the logarithm (to the base 2) of the label. For example, 8 is 3 units from 1 because  $2^3=8$ . So, “adding” two numbers is actually adding powers of two, which results in multiplying the two numbers. If  $A$  and  $B$  are the two numbers on your scale, you are actually adding  $\log_2(A)$  to  $\log_2(B)$  to get a

number  $\log_2(C)$ , which is depicted as C, which is A times B. [This is like in the *Powers of Ten* where instead of multiplying two numbers to get their product ( $10^1 \times 10^2 = 10^3$ ), the product is found by addition ( $1+2=3$ ).]

The way a slide rule works is similar, except that the logarithms are in base 10 rather than base 2. You can observe, however, that the distances between 1 and 2, between 2 and 4, between 4 and 8, are still all equal on the slide rule's scales. Why? By the base conversion formula,  $\log_{10}(A) = \log_2(A) / \log_2(10)$ , so that all we are doing by changing the base of the logarithm is to change the distance between ticks. If you want a base-2 slide rule, consider the markings between 1 and 2 on the C and D scales.

What if you want to multiply more than 2 numbers? Think about what you would do to add more than two numbers on two rulers, as above. You would add the two numbers, put your finger on the sum, and then reposition the other ruler to align with your finger. This is what the cursor does on the slide rule: it serves as a virtual finger!

Suppose, for example, that you want to multiply  $2 \cdot 2 \cdot 2$ . You might start by aligning 2 on the D scale with 1 on the C scale, after which 2 on the C scale corresponds to  $4 = 2 \cdot 2$  on the D scale. Put the cursor there to remember that. Then slide the 1 on the C scale to that position, and then look where 2 on the D scale falls. This is at  $2 \cdot 2 \cdot 2 = 8$ .

One thing you might notice is that when a product is greater than 10, the C and D scales have to be "reversed", i.e., you must place the 1.0 on the right hand side of the C scale onto the appropriate number on the D scale. Suppose, e.g., that you want to multiply  $4 \cdot 7$ . If you start by putting the left hand 1 on the C scale next to the 4 on the D scale, you will see (to your disgust) that you can't read what is next to 7 on the C scale. But you can do something else. Align the 1.0 on the *right hand side* of the C scale to 4, and then observe that next to the 7 on the C scale, you can read 2.8. Thus you did multiply  $4 \cdot 7$ , but the slide rule does not account for an extra power of ten that you need to remember. You can always multiply any two numbers by either aligning the left-hand or right-hand 1 on the C scale with the first, but if you use the right-hand side, your result must be multiplied by 10.

If this is a little confusing, try multiplying on the A and B scales instead. The A scale is two smaller D scales end-to-end, while the B scale is two C scales end-to-end. These can be used to multiply, just like the C and D scales. Consider, e.g., computing 8 times 7. Align 1 on the B scale with 8 on the first half of the A scale. Then find 7 on the first half of the B scale. This is next to 5.6 on the *second half* of the A scale. But since we are using the second half of the scale, the result is 56, and not 5.6.

An expert slide-rule user will use the A/B or C/D scales interchangeably for multiplying and dividing, whichever is more convenient.

Looking at the slide rule, you will notice that the ticks on the A scale are half as far from each other as on the D scale. Thus the numbers on the A scale are the *squares* of the numbers on the D scale, because  $2\log(X) = \log(X^2)$ . To convince yourself of this, align your cursor over 5 on the D scale and read 2.5 on the A scale. You must remember the appropriate power of ten to infer that

$5 \cdot 5 = 25$ . Likewise, the numbers on the B scale are the squares of the numbers on the C scale. In like manner, the K scale is made of three D scales, so that the numbers on the K scale are the *cubes* of the D scale numbers! The reason for square and cube scales is to compute areas (with squares) and volumes (with cubes).

Suppose, for example, that you want to compute the area of a circle (which comes up a lot). We know the formula is  $\pi r^2$ , where  $r$  is the radius and  $\pi = 3.14159$ . There are several ways to do this. One way is to enter  $r$  on the C scale, read  $r^2$  on the A scale, and *staying on the A scale*, align 1 on the B scale with  $r^2$ . Then the position of  $\pi$  on the B scale corresponds with  $\pi r^2$  on the A scale. This is such a common operation to do that  $\pi$  is usually explicitly marked on the A, B, C, and D scales.

Square roots are as easy to find as squares. If you start with a number on the A scale, its square root is found at the same position on the C scale. Likewise, the square root of any number on the B scale is at the same position on the D scale. Use the cursor to find them.

A second thing to notice is that the CI scale is the reverse of the C scale, exactly backwards! Given  $X$  on the C scale, the corresponding number on the CI scale is  $1/X$ . What is division? On a slide rule, it is subtraction! If you want to divide  $A$  by  $B$ , you need to go backwards by  $B$  units. All that CI does, is to do this division for the C scale (DI does this for the D scale). On the wood slide rules, scales that read backwards have numerical labels in red.

Large slide rules sometimes have CF and DF scales, which are the C and D scales multiplied by  $\pi$ . They are called CF and DF because multiplication is the same as “folding” the scale so that  $\pi$  on CF, DF corresponds to 1 on C, D respectively.

At this point, however, it will become clear that a lot is not written on the slide rule. For example, consider how to compute 4 times 4. You line up 1 with 4 and read across from the other 4, getting  $4 \cdot 4 = 1.6$ . You know this is close, because  $4 \cdot 4 = 16$ . What went wrong? On a slide rule, all you ever get is the *first digit* of a multiplication (and maybe the second). You have to decide where that digit goes in the result. So it is always necessary to keep in mind “how many 0’s to add” to the number. The slide rule doesn’t do the whole multiplication, but instead, acts in synergy with you as a human being and helps you in certain ways and not others.

Consider, e.g., the problem of computing  $234 \cdot 31$ . On a calculator, you would type in 234 and 31. On a slide rule, you set the scales to 2.34 and 3.1, and read the result as (about) 7.25. You must then do a bit of your own calculation: if  $2.34 \cdot 3.1 = 7.25$ , then  $(2.34 \cdot 10 \cdot 10) \cdot (3.1 \cdot 10) = 7.25 \cdot (10 \cdot 10 \cdot 10) = 7250$ . To get the answer, you must remember the factors of 10 and add them to get a power of ten by which to adjust the final result. Why does this work? Multiplication is *commutative*, i.e.,  $2.34 \cdot 10 \cdot 10 \cdot 3.1 \cdot 10$  is always  $2.34 \cdot 3.1 \cdot 10 \cdot 10 \cdot 10$ .

At this point, you might have noticed that a slide rule has only three digits or so of precision. For example, if you multiply 3452 and 5684, you will actually enter 3.45 and 5.68 into the slide rule, and obtain 1.96. Accounting for 10’s,  $3452 \cdot 5684 \approx 3.45 \cdot 5.68 \cdot 10^6 = 1.96 \cdot 10 \cdot 10^6 = 19,600,000$  (note that I added a factor of 10 because the product was greater than 10).

Why only three digits? Any engineer would tell you that most *measurements* aren't that precise. If one only has three digits of precision in a measurement, then the product cannot have more than three digits of precision either. Modern calculators provide extra precision that is mostly an illusion, because to use a 16-digit precision one would have to be able to *measure something to 16 digits of accuracy*, which we still cannot do in the present day!

Obviously, it takes some skill and experience to make calculations "easier" on a slide rule. Consider calculating the volume of a sphere, which is  $(4/3)\pi r^3$ . The "normal" way to do this is to multiply  $r \cdot r \cdot r \cdot (4/3) \cdot \pi$ , which takes a lot of setting and resetting of the cursor. If you are in a hurry, you can set  $r$  on the cursor on the D scale, read  $r^3$  on the K scale, align 1 on the C scale with that number on the D scale, and then multiply it by  $4/3 \approx 1.33$  and  $\pi \approx 3.14$ , thus saving a couple of multiplications (but losing a bit of accuracy). If you do this enough, however, you will learn instead to multiply  $r^3$  by  $4.2 \approx (4\pi/3)$ .

One power of Galileo's instrument was in doing proportion problems (for example, Operation IV and V in his book). The typical proportion problem is that A is to B as C is to D. For example, if you are baking a cake, you know that the ratio of flour to eggs should be a constant. To do this problem on a slide rule, you use the rule in a surprising way. So far, we have aligned 1 with the multiplicand. Here we will align the two numbers in question on the C and D scales.

Suppose, for example, that you know that you need two cups of flour for every three eggs and (oops) you just put in four eggs. Since it is rather difficult to put an egg back in its shell, how much flour do you need now? Align 2 on the C scale with 3 on the D scale. Now all pairs of C/D scale values *have the same ratio*. Thus look at where 4 is on the D scale, and that will tell you that you need about 2.68 cups of flour instead of 2. (Converting .68 cups of flour into tablespoons requires multiplying by 16!)

The key idea in the above, and in all slide rule use, is that when C and D are in a fixed orientation, *all pairs of numbers on C and D have the same ratio*. This is also the principle behind Galileo's compass, but there, C is analogous to length along a caliper and D is analogous to distance between calipers. Multiplication is a matter of solving the proportion problem

"1 (on C) is to X (on D) as Y (on C) is to X·Y (on D)."

Division is a matter of solving the related proportional problem

"X (on C) is to Y (on D) as X/Y (on C) is to 1 (on D)."

A skilled slide rule user realizes that there are two more usable laws that are inverses of the above:

"1 (on D) is to X (on C) as Y (on D) is to X·Y (on C)."

"X (on D) is to Y (on C) as X/Y (on D) is to 1 (on C)."

and (somewhat counter-intuitively):

"1 (on CI) is to X (on D) as Y (on CI) is to X/Y (on D)."

The reason the CF and DF scales are useful is that:

"1 (on C) is to  $\pi$  (on CF) as X (on C) is to  $\pi X$  (on CF)."

"1 (on C) is to  $\pi$  (on CF) as  $X/\pi$  (on C) is to X (on CF)."

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<sup>1</sup> Engineers often point out that while a high-school math teacher estimates  $\pi$  as 22/7, and mathematicians calculate  $\pi$  to thousands of digits, for engineers, an estimate of "3" is sufficient, because if you need a more accurate estimate, you are not doing safe engineering!

Specialized slide rules intended for specific calculations often include a custom folded scale for multiplying by some common constant, e.g.,  $(4\pi/3)$ .

There are many kinds of so-called “calculators” based upon the slide rule principle, but specialized for one kind of operation. Boaters use a “dead reckoning” circular slide rule that computes distance traveled from product of speed and time. Looking carefully, both speed and time are log scales! Photographic “exposure calculators” use the slide rule’s law of proportion: putting an F/stop over a shutter speed results in all pairs of F/stops and shutter speeds that give the same exposure (a ratio) (the photographic slide rule is based upon base-2 logarithms, as in the example at top).

You might want to think about the laws of proportion that hold for the A, B, and K scales.

Some problems to think about:

1. When you put 1 on the C scale over 4 on the D scale, where is 1 on the D scale over?  
Hint: try it on A and B scales too.
2. Put the cursor over 4 on the B scale. What number is the cursor over on the CI scale?  
What is the relationship between this number and  $\frac{1}{4}$ ?
3. If you put 1 on the CI scale adjacent to 6 on the D scale, then what is the number on the D scale adjacent to 2 on the CI scale?
4. Suppose you put X on the A scale next to  $\pi$  on the B scale, put the cursor on the 1 on the B scale, and then read the D scale. What is the geometric interpretation of this number?
5. If X on the C scale is adjacent to Y on the D scale, then what is 1 on the C scale always adjacent to on the D scale?
6. Suppose that 3 on the C scale is over 4 on the D scale. What is the ratio between the A and B scales?
7. Using the slide rule, “prove” that if you divide a number X by Y and then multiply it by the same Y, you get the same number X as you had initially. Hint: use the laws of proportion.
8. You drop a round ball into water and observe that it displaces 3 cubic inches of water. What is its radius in inches, assuming that its volume is  $(4/3)\pi r^3$ ? Describe the easiest way to compute this on the slide rule.

### **Slide Rule resources on the SP713 website-**

Under [Slide Rule](#)

#### **Buying a Slide Rule**

For an overview of the historical experience of an engineering student getting introduced to a slide rule, page through the cartoons and descriptions of various scales and instruments.

Under [Slide Rule](#)

#### **Slide Rule Instructions**

This site provides a powerpoint set of instructions for using a slide rule

The following pages in the powerpoint cannot be done with the plastic rules due to missing scales: 16, 17, 20-26, 36-52.

**The wood slide rules** that the Museum loaned have nearly all the scales described in the exercises; the placement of these scales may be different than on the yellow demonstration one. For the exercise on page 26, what they call the ST scale is labeled SRT scale on your rules. When they tell you to look for the answer on the C scale, on your rules it will appear on the D scale (or flip the rule over and you will find it on both C and D scales). You cannot do the exercise on page 39; your rules do not have an LL0 scale.

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