

From Reduced-Form to Structural Evaluation: Expanding Financial Infrastructure and Impact

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What is this lecture about?

- We illustrate the limitations of reduced-form and IV analysis.
 - Highlight the benefit of using reduced-form and structural analysis together
 - Based on Urzua and Townsend (2009) paper.
- Turn to a detailed structural analysis in Keniston et al.(2012) - Using BBL methodology
 - This methodology allows to estimate determinants of costs and demands of a player, i.e., a bank, without having to solve for all strategies of all players (even off equilibrium).
- We then illustrate limitations of this approach when there is a need to know counterfactual strategies (off-equilibrium).

- What is the impact of financial intermediation on productivity?
 - Affects occupational choice as well as allocation of risk
- Consider both static and dynamic structural models, and IV and OLS.
- Goal is to bridge the structural approach and the reduced-form IV approach
- Highlight that under strong assumptions, IV can recover the true LATE, but even then it can be very different from the Average Treatment Effect (ATE), or the Treatment on the Treated (TT) effect.
 - This is driven by the presence of heterogeneity in the population
- Having more margins of decisions, as well as more periods in a dynamic contract increases difficulty to interpret IV.

Standard Model of Occupational Choice

- Individual preferences: $u(c) = c$
- Beginning of period wealth b_i (observed by econometrician)
- Cost of entry into business θ_i^E (private information, with density f_{θ^E})
- Talent as a wage earner θ_i^W (private information, with density f_{θ^W}), independent of θ^E
- End of period wealth:

$$W_i = w + \theta_i^W + b_i \text{ if wage earner}$$

$$W_i = \pi(\theta_i^E, b_i, w) + b_i \text{ if entrepreneur}$$

where profits come from

$$\begin{aligned} \pi(\theta_i^E, b_i, w) &= \max_{\{k, l\}} f(k, l) - wl - k - \theta_i^E \\ \text{s.t.} \quad &: 0 \leq k \leq b_i - \theta_i^E \end{aligned}$$

Standard Econometric Approach for effects of occupational choice

- Decision rule $D = 1$ if person becomes entrepreneur

$$\begin{aligned} D(\theta_i^E, \theta_i^W, b_i, w) &= 1 \text{ if } \pi(\theta_i^E, b_i, w) > w + \theta_i^W \\ &= 0 \text{ else} \end{aligned}$$

- Can reduced form approach identify effects of occupational choice?
- Econometrician observes income (either $\pi + b_i$ or $b_i + w + \theta_i^W$ depending on occupation). End of period income is:

$$Y_i = D_i \left(\pi(\theta_i^E, b_i, w) + b_i \right) + (1 - D_i) \left(w + \theta_i^W + b_i \right)$$

- If assume linear separable model, $\pi = \phi_w w + \phi_\theta \theta_i^E + \phi_b b_i$ then:

$$Y_i = w + b_i + (\phi_b b_i + (\phi_w - 1) w) D_i + \varepsilon_i$$

where $\varepsilon_i = \theta_i^W + (\phi_\theta \theta_i^E - \theta_i^W) D_i$, is correlated with D_i (so simple OLS produces biased estimators).

Standard Econometric Approach for effects of occupational choice II

- Instead, use IV. Instrument: randomly assigned subsidy that increases profits by ψ (only conditional on setting up a firm, cannot be used to finance k).
- New decision rule: $D(\theta_i^E, \theta_i^W, b_i, w) = 1$ if $\pi(\theta_i^E, b_i, w) + \psi_i > w + \theta_i^W$, and $D_i = 0$ else.
- Subsidy is valid instrument:
 - Affects choice of occupation but not potential outcome
 - Satisfies monotonicity assumption: for each individual, an increase in subsidy increases chance of becoming entrepreneur
- If subsidy can take two values, $\bar{\psi}$ and $\bar{\psi}'$ then

$$\Delta^{IV} = \frac{E(Y_i | \psi_i = \bar{\psi}', b_i = b) - E(Y_i | \psi_i = \bar{\psi}, b_i = b)}{E(D_i | \psi_i = \bar{\psi}', b_i = b) - E(D_i | \psi_i = \bar{\psi}, b_i = b)}$$

which is also equal to the local average treatment effect (LATE):

$$\Delta^{LATE} = E \left[\pi(\theta_i^E, b_i, w) - w - \theta_i^W \mid D_i(\bar{\psi}') = 1, D_i(\bar{\psi}) = 0, b_i = b \right]$$

Standard Econometric Approach for effects of occupational choice III

- Treatment on the treated (TT): average benefit of becoming an entrepreneur for those who actually become entrepreneurs

$$\Delta^{TT}(b) = E\left(\pi\left(\theta_i^E, b_i, w\right) - \left(w + \theta_i^W\right) \mid D_i = 1, b_i = b\right)$$

- Average treatment effect (ATE): effect of becoming entrepreneur versus wage earner for the entire population

$$\Delta^{ATE}(b) = E\left(\pi\left(\theta_i^E, b_i, w\right) - \left(w + \theta_i^W\right) \mid b_i = b\right)$$

- If no heterogeneity, or all heterogeneity observed, then $\Delta^{LATE} = \Delta^{ATE} = \Delta^{TT}$. Else, difficult to estimate Δ^{ATE} and Δ^{TT} .

Parametric estimation for effects of occupational choice (ATE and TT)

- Can find ATE and TT if make additional parametric assumptions on functional forms for profits (f quadratic) and distribution functions of θ s (normally distributed)
- Probability of being entrepreneur:

$$\begin{aligned} & \Pr \left(\pi \left(\theta_i^E, b_i, w \right) + \psi_i > w + \theta_i^W \right) \\ &= \Pr \left(\phi_w w + \phi_\theta \theta_i^E + \phi_b b_i + \psi_i > w + \theta_i^W \right) \\ &= \Phi \left(\frac{(\phi_w - 1) w + \phi_b b_i + \psi_i}{\sqrt{\sigma_W^2 + \phi_\theta^2 \sigma_E^2}} \right) \end{aligned}$$

where σ_W^2 and σ_E^2 are the variances of θ^W and θ^E , respectively.

Parametric estimation for effects of occupational choice (ATE and TT) II

- Expected profits conditional on being an entrepreneur:

$$\begin{aligned} & E \left(\pi \left(\theta_i^E, b_i, w \right) \mid D_i = 1, b_i, \psi_i \right) \\ &= \phi_w w + \phi_b b_i - \frac{\phi_\theta^2 \sigma_E^2}{\sqrt{\sigma_W^2 + \phi_\theta^2 \sigma_E^2}} \lambda \left(\frac{(\phi_w - 1) w + \phi_b b_i + \psi_i}{\sqrt{\sigma_W^2 + \phi_\theta^2 \sigma_E^2}} \right) \end{aligned} \quad (1)$$

where $\lambda (\cdot)$ is a function (the Mills' ratio).

- Hence, correct regression is of profits/earnings onto the wage, b_i , and λ - note that ϕ_θ and σ_E^2 cannot be separately identified.

Parametric estimation for effects of occupational choice (ATE and TT) III

- Average wages among entrepreneurs (unobserved of course):

$$\begin{aligned} & E\left(w + \theta_i^E \mid D_i = 1, b_i, \psi_i\right) \\ &= w + \frac{\sigma_W^2}{\sqrt{\sigma_W^2 + \phi_\theta^2 \sigma_E^2}} \lambda \left(\frac{(\phi_w - 1)w + \phi_b b_i + \psi_i}{\sqrt{\sigma_W^2 + \phi_\theta^2 \sigma_E^2}} \right) \end{aligned} \quad (2)$$

which depends only on identified parameters (from the probit), so can be constructed for all b_i and ψ_i values.

Parametric estimation for effects of occupational choice (ATE and TT) IV

- Hence, can compute:

$$\begin{aligned} & \Delta^{TT}(b, \psi) \\ = & \underbrace{E\left(\pi\left(\theta_i^E, b_i, w\right) \mid D_i = 1, b_i = b, \psi_i = \psi\right)}_{\text{identified from (1)}} \\ & - \underbrace{E\left(w + \theta_i^E \mid D_i = 1, b_i = b, \psi_i = \psi\right)}_{\text{identified from (2)}} \end{aligned}$$

$$\Delta^{ATE}(b) = E\left(\pi\left(\theta_i^E, b_i, w\right) - \left(w + \theta_i^E\right) \mid b_i = b\right) = (\phi_w - 1)w + \phi_b b$$

- To get unconditional version, just integrate over b and ψ over appropriate region.

Further Note on Heterogeneous Treatment effects I

- Method by Heckman and Vytlacil (2001)
- Compute the Local IV estimator, Δ^{LIV} :

$$\Delta^{LIV}(p, b) = \frac{\partial E(Y_i | p_i, b_i = b)}{\partial p_i} \Big|_{p_i = p}$$

where p_i is the propensity score, here $p_i = \theta_i^W - \phi_\theta \theta_i^E$.

- This can identify the treatment parameter

$$\Delta^{MTE}(p, b) = E\left(\pi(\theta_i^E, b_i, w) - (w + \theta_i^W) \mid b_i = b, \theta_i^W - \phi_\theta \theta_i^E = p\right)$$

(treatment effect for those individuals indifferent between occupations, given p and b).

Further Note on Heterogeneous Treatment effects II

- Can then obtain Δ^{ATE} and Δ^{TT} as weighted averages of Δ^{MTE} :

$$\Delta^{TT}(b) = \int \Delta^{MTE}(u, b) \omega^{TT}(u, b) du$$

$$\Delta^{ATE}(b) = \int \Delta^{MTE}(u, b) \omega^{ATE}(u) du$$

where $\omega^{ATE}(u) = 1$,

$\omega^{TT}(u, b) = \Pr(p(w, b, \psi) > u) / \int \Pr(p(w, b, \psi) > u) du$

- To compute $\Delta^{LIV}(p, b)$, can approximate it by a polynomial on p_i .

Measuring Impact of Occupations on Income I

- Directly simulate data from model to compare different estimates. Parameterize model (see table 1 in paper for all details).
- How do we do this?
- We fix some parameters for the full model ('calibrate' it), randomly assign a subsidy to some agents.
- Model then tells us what occupation each agent chooses and what his realized income is. We also know what his counterfactual would have been without the subsidy.
- Directly estimate the effects of occupation by directly looking at income before and after the subsidy for the same individual.
- Then try to directly run the IV regression on the model-generated data (see the next slide).

Measuring Impact of Occupations on Income II

- Subsidy = $\{0, 1\}$.
- Suppose researcher tries to estimate effect of occupational choice from $\kappa_3 + \kappa_2 b_i$ below:

$$Y_i = \kappa_0 + \kappa_1 b_i + \kappa_2 b_i D_i + \kappa_3 D_i + \varepsilon_i$$

- Can use subsidy as IV for D_i .
- OLS and IV are very different (see next slide): IV shows negative impact, OLS positive - because occupational choice is related to unobserved talent, hence endogenous. IV is 'correct': individuals who switch occupation as result of subsidy are those with lower profits and higher wages (than those who already are entrepreneurs).

Using Model structure to generate counterfactual outcomes and causal effects of occupation

- Since we know structure of model - can generate counterfactuals.
- Provide individuals' who originally did not get subsidy with the subsidy and compute LATE generated from model (directly) - finding: LATE very similar to IV (negative again)
- TT and ATE computed as positive numbers (overall, there are positive benefits from being an entrepreneur).
- Conclusion: the econ model delivered a valid instrument which does correctly identify the causal effect, and the causal effect can differ from ATE or TT.

OLS and IV Estimates

Model of occupational choice-estimates from cross-sectional data

Paramter	Estimates	
	Δ^{OLS}	Δ^{IV}
k_0	0.606**	1.189**
k_1	1.155**	1.142**
k_2	-0.136**	-0.082
k_3	0.457**	-0.356*
Average Effect ($k_2\bar{b} + k_3$)	0.303**	-0.450

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Occupational Choice under Financial Intermediation

- Incorporate an intermediated sector in model above.
- Individual specific cost of using financial sector, Q_i (travel time, effectiveness of bank in the village, etc..)
- Maximization of entrepreneur in intermediated sector (neoclassical separation between production and household wealth):

$$\max_{k,l} f(k, l, \theta_i^E) - wl - (1+r)(k + \theta_i^E)$$

- Occupation choice for agents in intermediated sector:

$$\begin{aligned} D(\theta_i^E, \theta_i^W, w, r) &= 1 \text{ if } \pi(\theta_i^E, w, r) + b_i(1+r) - Q_i + \psi_i \\ &> w + \theta_i^W + b_i(1+r) - Q_i \\ &= 0 \text{ else} \end{aligned}$$

Occupational Choice under Financial Intermediation II

- Outcome observed under intermediation:

$$\begin{aligned} & Y_I \left(\theta_i^E, \theta_i^W, b_i, w, r \right) \\ &= D \left(\theta_i^E, \theta_i^W, w, r \right) \left(\pi \left(\theta_i^E, w, r \right) + b_i (1 + r) \right) \\ &\quad + \left(1 - D \left(\theta_i^E, \theta_i^W, w, r \right) \right) \left(w + \theta_i^W + b_i (1 + r) \right) \end{aligned}$$

(not counting subsidy and intermediation costs)

- Without intermediation, occupational choice as before:

$D_i \left(\theta_i^E, \theta_i^W, b_i, w \right) = 1$ if $\pi \left(\theta_i^E, b_i, w \right) + \psi_i > w + \theta_i^W$ and $D_i = 0$ else.

Hence observed outcome under autarky (A) is (not counting subsidy)

$$\begin{aligned} Y_A \left(\theta_i^E, \theta_i^W, b_i, w \right) &= D_i \left(\theta_i^E, \theta_i^W, b_i, w \right) \left(\pi \left(\theta_i^E, b_i, w \right) + b_i \right) \\ &\quad + \left(1 - D_i \left(\theta_i^E, \theta_i^W, b_i, w \right) \right) \left(w + \theta_i^W + b_i \right) \end{aligned}$$

Occupational Choice under Financial Intermediation III

- Choice of sector (intermediated vs. not): $Y_i = 1$ if in intermediated sector, 0 else.

$$Y_i \left(\theta_i^E, \theta_i^W, b_i, w, r, \psi_i, Q_i \right) = 1$$
$$\text{if } \left(+ \left[D \left(\theta_i^E, \theta_i^W, w, r \right)^{[Y_I - Y_A]} - D \left(\theta_i^E, \theta_i^W, b_i, w \right) \right] \psi_i - Q_i \right) \geq 0$$
$$Y_i = 0 \text{ else}$$

- Effect of financial intermediation at individual level is:

$$\Delta_i^Y = Y_I - Y_A$$

Identifying the Effects of Financial Intermediation

- ATE and TT:

$$ATE = E(\Delta_i^Y)$$

$$TT = E(\Delta_i^Y | Y_i = 1)$$

$$= E\left(Y_I(\theta_i^E, \theta_i^W, b_i, w, r) - Y_A(\theta_i^E, \theta_i^W, b_i, w) | Y_i = 1\right)$$

- Shortcut: denote by $D_i = D(\theta_i^E, \theta_i^W, w, b_i)$ the occupation choice under autarky and $D_i(r) = D(\theta_i^E, \theta_i^W, w, r)$ the occupation choice under intermediation.
- Observed outcome:

$$\tilde{\zeta}_i = Y_i \times Y_I + (1 - Y_i) \times Y_A$$

Identifying the Effects of Financial Intermediation II

- Observed outcome depends on all choices and outcomes, even if just interested in effect of financial intermediation:

$$\zeta_i = Y_i \times \left[\begin{array}{c} D_i(r) \left(\pi \left(\theta_i^E, w, r \right) + (1+r) b_i \right) \\ + \\ (1 - D_i(r)) \left(w + \theta_i^W + b_i (1+r) \right) \end{array} \right] \\ + (1 - Y_i) \left[\begin{array}{c} D_i \left(\pi \left(\theta_i^E, b_i, w \right) \right) + b_i \\ + \\ (1 - D_i) \left(w + \theta_i^W + b_i \right) \end{array} \right]$$

- Assume linear profit functions under both autarky and intermediation.

$$\begin{aligned} \pi \left(\theta_i^E, b_i, w \right) &= \gamma_w w + \gamma_b b_i + \gamma_\theta \theta_i^E \\ \pi \left(\theta_i^E, w, r \right) &= \delta_w w + \delta_r r + \delta_\theta \theta_i^E \end{aligned}$$

Identifying the Effects of Financial Intermediation II

- Hence observed effect $\tilde{\zeta}_i$ can be rewritten, using the functional form assumptions as:

$$\begin{aligned}\tilde{\zeta}_i &= w + b_i + rY_i b_i \\ &\quad + (\gamma_w - 1) w D_i (1 - Y_i) + \gamma_b b_i D_i (1 - Y_i) \\ &\quad + ((\delta_w - 1) w + \delta_r r) D_i (r) Y_i + \delta_b b_i Y_i D_i (r) \\ &\quad + \eta_i (\theta_i^E, \theta_i^W, r, Q)\end{aligned}$$

where $\eta_i =$

$(\delta_\theta \theta_i^E - \theta_i^W) Y_i D_i (r) - (\gamma_\theta \theta_i^E - \theta_i^W) Y_i D_i + (\gamma_\theta \theta_i^E - \theta_i^W) D_i + \theta_i^W$ so it depends on unobserved talents.

Identifying the Effects of Financial Intermediation III

- Effect of financial intermediation is:

$$\begin{aligned}\Delta_i^Y &= \frac{\Delta \tilde{\zeta}_i}{\Delta Y_i} \\ &= rb_i + (((\delta_w - 1) w + \delta_r r) D_i (r) - (\gamma_w - 1) w - \gamma_b b_i) D_i \\ &\quad + \frac{\Delta \eta_i}{\Delta Y_i}\end{aligned}$$

which depends on occupation of individual under each regime and unobserved talents.

- Cannot be estimated by simple OLS since unobserved talent enters error term η_i .

Identifying the Effects of Financial Intermediation IV

- Is Q_i a good instrument for Y_i ? It affects choice of intermediation but not potential outcomes. Can estimate IV effect if have two values of entry costs, \bar{Q} and \bar{Q}' :

$$\Delta^{IV(Q)} = \frac{E(\xi_i | Q_i = \bar{Q}', b_i = b) - E(\xi_i | Q_i = \bar{Q}, b_i = b)}{E(Y_i | Q_i = \bar{Q}', b_i = b) - E(Y_i | Q_i = \bar{Q}, b_i = b)}$$

to identify local treatment effect of financial intermediation on income

$$\Delta^{LATE(Q)} = E(Y_I - Y_A | b_i = b, Y_i(\bar{Q}') = 1, Y_i(\bar{Q}) = 0)$$

- What does this measure? Gains in outcomes (profits and wages) for those induced to join intermediation sector as consequence of reduction in intermediation costs (all margins adjusting together).
- It does NOT measure effects of financial intermediation on profits for entrepreneurs or wages for wage earners: change in Q also induces endogenous changes in occupation (i.e., NOT holding occupation constant)!

Identifying the Effects of Financial Intermediation V

- How about computing an Δ^{IV} separately for wage earners and entrepreneurs? Would that capture the local causal effects of financial intermediation?
- No: responses in occupational choice are not uniform. If restrict to entrepreneurs, we lose gains from those initial entrepreneurs who became wage earners in response to change in intermediation cost.
- What does it identify if we compute it by group? Identifies effect of financial intermediation on entrepreneurs (resp., wage earners) who would not have switched occupations as a result of the change in the instrument.

Identifying the Effects of Financial Intermediation VI

- Can also compute IV estimator to identify the LATE of effect of occupation:

$$\Delta^{IV}(\bar{\psi}, \bar{\psi}', b) = \frac{E(\xi_i | \psi_i = \bar{\psi}', b_i = b) - E(\xi_i | \psi_i = \bar{\psi}, b_i = b)}{E(\tilde{D}_i | \psi_i = \bar{\psi}', b_i = b) - E(\tilde{D}_i | \psi_i = \bar{\psi}, b_i = b)}$$

where $\tilde{D}_i = D_i(r)Y_i + D_i(1 - Y_i)$.

- Under uniform effect of ψ on \tilde{D} , Δ^{IV} identifies the LATE of occupation on income.
- Again, caution: Δ^{IV} cannot measure effects for those induced to enter entrepreneurship as a result of the subsidy: since produces intermediation choices which are non uniform and endogenous.
- We can use Δ^{IV} to identify the effects of entrepreneurship if there was a subpopulation for which the subsidy changed but intermediation would not change (e.g., they have too high Q and would never enter intermediation in any case).

How does econometric estimation perform? Simulations

- Again, parameterize and simulate the model.
- What would happen if an econometrician estimates:

$$Y_i = \kappa_0 + \kappa_1 b_i + \kappa_2 b_i Y_i + \kappa_3 Y_i + \varepsilon_i$$

- OLS and IV both positive, but OLS is double effect of IV (because of selection).
- Counterfactual analysis (simulations to uncover true causal effects): since we know all parameters of model, we can simulate outcomes, also for various subgroups and see the true effects of intermediation and occupational choice. Let's compare these to the OLS and IV findings. For example, can see effects on individuals switching from "wage-earner under autarky" to "entrepreneur with financial system access" - which is impossible without a structural model.

How does econometric estimation perform? Simulations II

- Findings from the simulations: Overall LATE across population is very close to the IV coefficients.
- Notice that changes in Q can make people move away from entrepreneurship towards wage work: illustrates non uniform changes, as some people now find it better to just put their money in the bank and work as wage earners (if have low talent for entrepreneurship for example).
- Similarly, changes in a subsidy cause people to non-uniformly change to intermediated sector.

Further Results of Effects of Occupation on Income

- Suppose econometrician tries to estimate the following model of the effects of occupation on income:

$$Y_i = \tau_0 + \tau_1 b_i + \tau_2 b_i D_i + \tau_3 D_i + \varepsilon_i$$

where again, $D_i = 1$ if individual i is an entrepreneur and 0 otherwise.
Results on the next slide.

- Again, as for the effects of intermediation, OLS delivers a positive effect whereas IV suggests negative effect of occupation (entrepreneur).

Model Generated Local Average treatment Effects

Model of occupational choice and financial intermediation

Parameter	Value	Number of Movers	Direction
$\Delta^{\text{LATE}(\Psi)}(1,0)$	-0.466	2,219	From Wage Earner to Entrepreneur
	-0.444	1,548	From wage worker under autarky to entrepreneur under autarky
	-0.278	278	From wage worker under autarky to entrepreneur under financial intermediation
	-0.724	322	From wage worker under financial intermediation to entrepreneur under autarky
	-0.519	71	From wage worker under financial intermediation to entrepreneur under financial intermediation
$\Delta^{\text{LATE}(Q)}(0.25,1)$	0.388	3,757	From Autarky to Financial Intermediation
	0.355	911	From wage worker under autarky to wage worker under financial intermediation
	-0.203	176	From wage worker under autarky to entrepreneur under financial intermediation
	0.752	75	From entrepreneur under autarky to wage worker under financial intermediation
	0.430	2,595	From entrepreneur under autarky to entrepreneur under financial intermediation

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- Consider now a dynamic model with household discounted expected utility

$$E_0 \left(\sum_{t=0}^{\infty} \beta_i^t u(c_{it}) \right)$$

- Individuals differ in their discount factors, $\beta_i = \bar{\beta} + \theta_i$ where $\bar{\beta}$ is common knowledge but θ_i is private.
- Let s_{it} be savings, as fraction of wealth k_{it} . ψ_t^E is proportion of savings invested in risky enterprise sector, ψ_t^W is fraction invested in wage sector activities.
- Investment in enterprise yields $\delta_t^E + \varepsilon_{it}^E$ where ε_{it}^E is a random shock and investment in wage activities yields $\delta_t^W + \varepsilon_{it}^W$.

- Law of motion of wealth in autarky:

$$k_{it+1} = s_{it} \times \left[\psi_t^E \times \left(\delta_t^E + \varepsilon_{it}^E \right) + \psi_t^W \times \left(\delta_t^W + \varepsilon_{it}^W \right) \right] \times k_{it} \quad (3)$$

- Consumption in autarky is $c_{it}^A = (1 - s_{it}) k_{it}$.
- Welfare under autarky satisfies Bellman equation:

$$W_0(k_{it}, \theta_i) = \max_{\psi_t^E, \psi_t^W, c_{it}, s_{it}} u(c_{it}) + \beta_i E(W_0(k_{it+1}, \theta_i))$$

subject to (3).

- With CRRA preferences, risky assets and wage sector investments are constant fractions of available resources:

$$c_{it}^A = \tilde{\alpha}_i^A k_{it} = \tilde{\alpha}_{it} \left(y_{it}^E + y_{it}^W \right)$$

with $\tilde{\alpha}_i^A = (1 - \beta_i)$, y_{it}^E is the income from enterprise, and y_{it}^W the income from labor

$$y_{it}^E = \psi_{t-1}^E \left(\delta_{t-1}^E + \varepsilon_{it-1}^E \right) k_{it-1} s_{it-1}$$

$$y_{it}^W = \psi_{t-1}^W \left(\delta_{t-1}^W + \varepsilon_{it-1}^W \right) k_{it-1} s_{it-1}$$

- Hence, consumption in autarky is: $c_{it}^A = (1 - \bar{\beta} - \theta_i) y_{it} = \alpha^A y_{iy} + \varepsilon_{it}^A$ where $y_{it} = y_{it}^E + y_{it}^W$ is total income, $\alpha^A = 1 - \bar{\beta}$ and $\varepsilon_{it}^A = -\theta_i y_{it}$.

- In intermediated sector, households share all idiosyncratic shocks. Law of motion of wealth:

$$k_{it+1} = s_{it} k_{it} \max \left\{ \delta_t^W, \delta_t^E \right\} (1 - \tau) \quad (4)$$

where τ is marginal intermediation transaction cost.

- Value function in the intermediation sector satisfies Bellman Equation:

$$V_I(k_{it}, \theta_i) = \max_{c_{it}, s_{it}} [u(c_{it}) + \beta_i E(V_I(k_{it+1}, \theta_i))]$$

subject to (4).

- Under CRRA preferences, we have again: $c_{it}^I = \tilde{\alpha}_i^I A_t$ where $A_t = \max \left\{ \delta_{t-1}^W, \delta_{t-1}^E \right\} (1 - \tau)$ and $\alpha_i^I = 1 - \bar{\beta} - \theta_i$
- $c_{it}^I = \alpha^I A_t + \varepsilon_{it}^I$ with $\alpha^I = 1 - \bar{\beta}$ and $\varepsilon_{it}^I = -\theta_i A_t$ is the unobserved component.

Once-and-for-all participation decision

- At $t = 0$, household decides whether to enter intermediated sector once and for all.
- Z_i are individual-specific participation costs.
- Participation decision is I_{i0} with: $I_{i0} = 1 \Leftrightarrow V_I(k_{i0} - Z_i, \theta_i) \geq W_0(k_{i0}, \theta_i)$
- Observed consumption is then

$$\begin{aligned}c_{it} &= c_{it}^A (1 - I_{i0}) + c_{it}^I I_{i0} \\c_{it} &= \alpha^A y_{it} + \left(\alpha^I A_t - \alpha^A y_{it} \right) I_{i0} + v_{it}\end{aligned}$$

with $v_{it} = \varepsilon_{it}^A + I_{i0} \left(\varepsilon_{it}^I - \varepsilon_{it}^A \right)$. Note that error v_{it} depends on decision I_{i0} and hence c_{it} 'regression' is endogenous. Need IV strategy.

Once-and-for-all participation decision

- Potential instrument: Z_i : only affects decision at time 0 but not potential outcomes (i.e., consumption c_{it}^A or c_{it}^I) for $t > 0$
- Will identify LATE
- ATE and TT difficult as before because of heterogeneous treatment effects - only if no selection on unobserved gains (unlikely) would ATE, TT and LATE coincide.

Sequential participation decision

- Suppose instead that participation decision is made each period. Then, for those not yet in the intermediated sector at t , value function satisfies:

$$W_0(k_{it}, \theta_i) = \max_{\psi_t^E, \psi_t^W, c_{it}, s_{it}} \left\{ U(c_{it}) + \beta_i E \max \{ W_0(k_{it+1}, \theta_i), V_1(k_{it+1} - Z_i, \theta_i) \} \right\}$$

subject to

$$k_{it+1} = s_{it} \times \left[\psi_t^E \times (\delta_t^E + \varepsilon_{it}^E) + \psi_t^W \times (\delta_t^W + \varepsilon_{it}^W) \right] \times k_{it}$$

- Threshold value $k^*(Z_i, \theta_i)$ defines participation.
- Savings s_t and investments ψ_t^E, ψ_t^W will now depend on wealth k_{it} even with CRRA: hence variation in Z_i affects not just decision to participate (k^*), but also pre-participation outcomes.

Identification Power of Policies

- Unanticipated policies can help identify effect of financial intermediation.
- An unanticipated once-for-all change in Z_i at time t^* effectively transforms t^* into 'period 0' of the previous example, in which Z_i was a valid instrument - we can analyze the agent's decision as if it was a once-for-all decision.
- This policy is a valid instrument, because, as in the once-for-all choice of intermediation example, Z_i affects participation, but not potential outcomes.

- Cautious when using reduced-form IV.
- IV is not always wrong: the lesson is to use it carefully and through the lens of a model.

- Observation: banks and cajas in Spain locate around their home provinces. How can we estimate costs of bank expansion?
- Use Method of Simulated Maximum Likelihood, similar to Bajari, Benkard, and Levin (2007, hereafter, BBL).
- Important distinction to before: intermediation cost Q was random before. Here, banks are choosing where to locate - different setup!

Before we start: main messages

- First, the 'reactions' of other banks are estimated from the data. No need to solve their behavior fully - great simplification.
- Second, we only need to 'simulate forward' once - and we get the value functions, without all counterfactual, alternative strategies specifically considered. This is the key point of BBL. Details below!

Model of bank profits and entry

- Banks operate chains of branches, making loans and collecting deposits. Earn profit π_{ipt} from bank i in province p in year t .
- Vector of state variables for each bank/province/year is s_{ipt} (e.g., GDP of province, number of own branches and rival branches, distance from original province, etc..)
- Vector of state variables for all provinces for a given bank/year is s_{it} .
- Entry indicator variables l_{ipt} and let η_{ipt} be number of new branches.

Maximization problem of the bank

$$V(s_{i\tau}) = \max_{\eta_i, l_i} \left\{ \sum_{t=\tau}^{\infty} \beta^{t-\tau} \left(\sum_{p=1}^P \pi(s_{ipt}) + C^P(l_{ipt}|s_{it}) + C^B(\eta_{ipt}|s_{ipt}) \right) \mid s_{i\tau} \right\}$$

where $C^P(l_{ipt}|s_{it})$ is the cost the bank pays if it enters the province in year t and $C^B(\eta_{ipt}|s_{ipt})$ is the cost incurred to open η_{ipt} branches in that province

Hypothesis tested: C_{ipt}^P is a function of distance of province to the bank's existing network of branches in other provinces.

- 1 Banks chose interest rates at national level (nash eqm)
- 2 Make loans and take deposits in provinces in which already operating - get profits
- 3 Privately observed cost of entry and opening new branches realized
- 4 Decision to enter provinces and open new branches in existing ones - incur start up costs
- 5 GDP in province evolves exogenously

Static Actions

- Deposit rate $r_{dep,t}$, borrowing rate in interbank loan market, ρ
- Within period profit of bank i in period t if active in province p :

$$\pi(s_{ipt}) = \sum_{z=1}^4 l_{iptz} (r_{zt} - \rho_t) + d_{ipt} (\rho_t - r_{dep,t}) - AC_t \times n_{it}$$

where z indexes four sectors.

- l_{iptz} = quantity of loans in sector z , d_{ipt} is total province deposit for the bank in year t , AC_t is the cost of operating a bank branch.

Static Actions II

- Number of branches of competitors in province p for bank i in year t :
 $n_{-ipt} = \sum_{b=1}^B n_{bpt}$
- Province/bank/sector level fixed effects δ_{ipz}
- Demand for loans:

$$l_{iptz} = \beta_{1z} n_{ipt} + \beta_{2z} n_{ipt}^2 + \beta_{3z} n_{-ipt} + \beta_{4z} n_{-ipt}^2 \\ + \beta_{5z} r_{zit} + \beta_{6z} GDP_{pt} + \beta_{7z} \frac{\sum_{b=1}^B n_{bpt} r_{zbt}}{n_{-ipt}} + \delta_{ipz} + \varepsilon_{iptz}$$

- Demand for deposits is identical.

- Interest rate determined according to Nash Equilibrium. For bank operating in set P_i of provinces, optimization problem is:

$$\max_{r_{zit}} \sum_{P_i} l_{iptz} (r_{zt} - \rho_t) + d_{ipt} (\rho_t - r_{dep,t}) - AC_t \times n_{it}$$

chosen simulatenously with other banks.

Dynamic Actions I

- Each period, decide on entry and number of branches
- Cost of entry into province p :

$$C^P \left(l_{pit} | s_{ipt}, v^P; \gamma \right) = l_{ipt} \left(\gamma_0 + \gamma_1 dist_{ipt} + \gamma_2 v_{pt}^P \right)$$

where v_{pt}^P is iid $N(0, 1)$.

- Distance modeled as:

$$dist_{ipt} = \sum_{m \in P_i, m \neq p} \frac{n_{imt}}{kms(p, m)}$$

(where $kms(p, m)$ is distance in km between provinces p and m).

- Cost of opening branches:

$$C^B \left(\eta | s_{ipt}, v_{ipt}^B; \alpha \right) = I(\eta > 0) \left(\alpha_0^o \eta + \alpha_1^o \eta v_{ipt}^B \right) + I(\eta < 0) \left(\alpha_0^c \eta + \alpha_1^c \eta v_{ipt}^B \right)$$

Shocks enters both cost of opening new branches and liquidating (closing) existing ones.

- Vector to be estimated, $\theta = (\beta_{1z}, \dots, \beta_{7z}, \gamma_0, \gamma_1, \gamma_2, \alpha_0^o, \alpha_1^o, \alpha_0^c, \alpha_1^c)$ includes β (coefficients on loan demand/supply functions), γ (vector of coefficients of entry costs), α (vector of coefficients on cost of opening/closing branches).
- Estimated reduced form Markov process for province GDP and number of other banks' branches (polynomials)
 - For GDP, GDP_t is only function of GDP_{t-1}
 - For the number of competitor's branches, predict n_{-ipt} from OLS regression of n_{-ipt} on polynomial terms of $n_{ip(t-1)}$, $\log(GDP_{t-1})$ and $n_{-ip(t-1)}$.
- Estimate demand parameters β using static methods, with IV = lagged values of number of own and competitors' branches as instruments for current levels.
- Decisions to enter provinces and construct branches: complex functions of states: unfeasible to solve. Instead, estimate them based on observed actions.

Estimation II

- Semi-parametric estimation of decision to enter new province as function of states:

$$\Pr \left(l_{ipt}^P = 1 | s_{it} \right) = F \left(n_{-ipt}, GDP_{ipt}, kms_{ipt} \right)$$

with $F ()$ being a flexible functional form (e.g., logit on 3rd order polynomial of states).

- Choice to open new branches:

$$E \left(\eta_{ipt} | s_{ipt} \right) = \left(n_{ipt}, n_{-ipt}, GDP_{ipt} \right)$$

where $H ()$ estimated via ordered probit on third order polynomial of states.

- Potential concern in the estimation of these policy functions lies in the number of state variables to include in these regressions. Because banks consider their full forward expansion paths when deciding to enter a province, the characteristics of all surrounding provinces may also be included among the state variables, thus potentially increasing them to an unfeasibly large dimension.

Estimation III

- What about fixed costs parameters α and γ ? - use BBL technique.
- $V_i(s_{it}|\sigma_{it}; \theta)$ (resp., $V_i(s_{it}|\sigma'_{it}; \theta)$) is expected current and future profit under actual strategy σ_{it} (resp., strategy σ'_{it}).
- Given entry shock received at true parameter, it must be:

$$V_i(s_{it}|\sigma_{it}; \theta^0) \geq V_i(s_{it}|\sigma'_{it}; \theta^0)$$

- Strategy: generate estimates of actual and counterfactual value functions using forward simulation, then find θ that maximizes prob that inequality above holds at all entry decisions.

- Simulate path of actions taken by bank (given that we know static demands, state transitions and policy functions):
 - start from state after entry $s_{i(t+1)}$ and draw shocks $v_{\rho(t+1)}^B$ and $v_{\rho(t+1)}^P$
 - for each new province, predict if entry by testing if $\Phi\left(v_{ip(t+1)}^P\right) > \hat{F}\left(n_{-ip(t+1)}, GDP_{ip(t+1)}, dist_{ip(t+1)}\right)$ (if yes, $\hat{l}_{ip(t+1)} = 1$)
 - predict new branches closed/opened by evaluating:
$$\hat{\eta}_{ipt+1} = \hat{H}\left(n_{ip(t+1)}, n_{-ip(t+1)}, GDP_{ip(t+1)}, v_{\rho(t+1)}^B\right)$$
 - update GDP, interest rates, number of other banks' branches according to transition functions
 - start from new state generated, $s_{i(t+2)}$ and iterate

Dynamic Parameter Estimation

- Suppose (to illustrate) that the bank decides not to enter province p . Then the following inequalities hold (second line substitutes the parameterizations assumed):

$$\left[\begin{array}{l} C^P \left(l_{ipt} = 1 | s_{it}, v_{ipt}^P; \gamma \right) \\ + C^B \left(\eta_{ipt} | s_{ipt}, v_{ipt}^B; \alpha \right) \\ + \beta E \left(V_i \left(s_{i(t+1)} | \sigma'_{i(t+1)}; \theta \right) \right) \end{array} \right] \leq \beta E \left(V_i \left(s_{i(t+1)} | \sigma_{i(t+1)}; \theta \right) \right)$$

$$\left[\begin{array}{l} \gamma_0 + \gamma_1 dist_{ipt} + \gamma_2 v_{pt}^P + \alpha_0^o \eta_{ipt} \\ + \alpha_1^o \eta_{ipt} v_{ipt}^B \\ + \beta E \left(V_i \left(s_{i(t+1)} | \sigma'_{i(t+1)}; \theta \right) \right) \end{array} \right] \leq \beta E \left(V_i \left(s_{i(t+1)} | \sigma_{i(t+1)}; \theta \right) \right)$$

Dynamic Parameter Estimation

- Rearranging:

$$\begin{aligned} & Pr (l_{ipt} = 1 | s_{it}; \theta) \\ = & Pr \left(\gamma_2 v_{ipt}^P \geq \left(\begin{array}{c} -\gamma_0 - \gamma_1 dist_{ipt} - \alpha_0 \eta_{ipt} - \alpha_1^0 \eta_{ipt} v_{ipt}^B \\ +\beta E \left(\begin{array}{c} V_i (s_{i(t+1)} | \sigma_{i(t+1)}; \theta) \\ -V_i (s_{i(t+1)} | \sigma'_{i(t+1)}; \theta) \end{array} \right) \end{array} \right) \right) \end{aligned}$$

- Complication: two sources of uncertainty, future profits from entry and current value of shocks.

Dynamic Parameter Estimation II

- Expected profits from entry = generated by difference in two forward simulations (one assuming entry in the province this period, the other assuming not).
- Integrate these differences over current period shocks to cost of opening branches: creates joint distribution of shocks and branch openings $g(\eta_{ipt}, v_{ipt}^B)$.
- Hence entry probability is (using first period shock draws to integrate over combinations of branches/shocks):

$$\Pr(l_{ipt} = 1 | s_{it}; \theta) = 1 - \frac{1}{M} \sum_{m=1}^M \Phi \left(\left(\begin{array}{c} -\gamma_0 - \gamma_1 dist_{ipt} - \alpha_0^o \eta_{ipt,m} \\ -\alpha_1^o \eta_{ipt,m} v_{ipt,m}^B \\ + \frac{1}{M} \sum_{m=1}^M (W_{i,m}(s_{it} | \sigma_{it}; \theta) - W_{i,m}(s_{it} | \sigma'_{it}; \theta)) \end{array} \right) \right)$$

- Likelihood Function:

$$\max_{\theta} \prod_t \prod_p \Pr (l_{ipt} = 1 | s_{it}; \theta)^{l_{ipt}} (1 - \Pr (l_{ipt} = 1 | s_{it}; \theta))^{(1-l_{ipt})}$$

- Important simplification: counterfactual strategies are only made of 'single-province' deviations. Rules out strategies like entering several provinces simultaneously, but not individually.
- What is the great advantage of this approach? We are not solving for ALL strategies of all players backwards - much simpler.

- Keniston et al. paper deduces the behavior of the 'market' from the data (taking as given the observed Markov structure from the data), and optimizes only for one bank at a time, then repeats for other banks.
- They do not have to compute the Nash equilibrium, with all players, in order to estimate all parameters simulatenously.
 - This is a big computational simplification.
 - This is essence of BBL

- What is this missing?
- On equilibrium path, we know how all competitors will react. Then, we optimize a given bank's problem on that equilibrium path.
 - Note that we did not compute the strategies of all other players: we just observed the equilibrium in the data
- But what if the bank tries out a counterfactual strategy? (which it must do since it is choosing the optimum).
- Then generates an off-equilibrium situation and other players will adapt - will also play off-equilibrium strategies, which we do not know.
- We cannot see the off-equilibrium strategies in the data - only the equilibrium.

- Bottomline is that all the assumptions needed from BBL are hard to maintain when we switch to more complex bank problems.
- For example, when we switch to entry problems with endogenous markets, rather than exogenous provinces, and evolving state variables.

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