14.772 Pset 3 - Hsieh and Klenow (2009)

[Part I]

(a) Given the production function given in formula (3) of the paper, solve the cost minimization problem

$$P_s Y_s = \min \sum_{i=1}^{M_s} P_{si} Y_{si}$$

subject to

$$Y_s = \left(\sum_{i=1}^{M_s} Y_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma-1}{\sigma}}. \tag{2}$$

Determine P_s.

(b) Let λ_s be the multiplier on the constraint (2). Show that the profit maximization of firm i in industry s is

$$\max_{Ysi,Lsi,Ksi} (1-\tau_{Y_{si}}) \lambda_s \frac{\sigma-1}{\sigma} (Y_{si})^{\frac{\sigma-1}{\sigma}} - wL_{si} - (1+\tau_{Ksi}) RK_{si}$$

subject to $Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}$.

(c) Use the solution to the firm maximization problem and the expression of P_s to derive the formula (15). *NOTE: In their original QJE paper, there are a couple of typos! In particular, (12) and (13) are not correct if $\overline{MRPL_s}$ and $\overline{MRPK_s}$ are defined as in their paper following (12) and (13). As a hint: define*

$$\frac{1}{\overline{MRPL_s}} = \sum_{i=1}^{M_s} \left(\frac{1}{\overline{MRPL_{si}}} \frac{P_{si} Y_{si}}{P_s Y_s} \right)$$

and $\overline{MRPK_s}$ similarly.

(d) There is a large literature trying to link the distortions $(\tau_{Y_{si}}, \tau_{K_{si}})$ to financial frictions individual firms face. To see the relation between these exogenous taxes and credit constraints, suppose that there are no taxes (i.e. $\tau_{Y_{si}} = \tau_{K_{si}} = 0$) but firm i faces a credit constraint of the form

$$wL_{si} + \zeta RK_{si} \leq W(z_{si}, \eta),$$

where z_{si} is a firm characteristic (e.g. wealth), η parametrizes the financial system and ζ parametrizes how much of capital expenses can be pledged. Suppose that W is increasing in both argument, i.e. wealthy firms are less constrained and better financial system are associated with higher values of η. Derive the firm's factor demands taking prices factor prices as given. What are the firm-specific "taxes" in this framework? Suppose that $A_{si} = A$, i.e. all firms have the same productivity. Which firms face high "output-taxes $\tau_{Y_{si}}$"? Under what conditions would a researcher conclude that $\tau_{K_{si}} = 0$?

[Part II]

This part concerns the analysis of equations in Appendix I in the paper.

(a) Show that $TFP = \overline{TFPR} \frac{w}{P\gamma}$ in which $\overline{TFPR} = \sum_{i=1}^{M} \frac{L_i}{L} TFPR_i$

(b) Suppose $(1-\tau_i) = a\frac{1}{A_i}$. Using the labor market clearing condition, show that

$$TFP = \frac{1}{M} \frac{\sum_{i=1}^{M} A_i}{L^{1-\gamma}}$$

independent of a. Give a concise interpretation why aggregate TFP is independent of a. What is the crucial assumption for this result?

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