14.772 Macro Development - Problem Set 2

Spring 2013

Problem 1: Risk Sharing

Consider H households, with household h consisting of I_h members. There is a single consumption good in this economy. Individuals also care about leisure, thus their per-period utility is $u_{i,h}(c_{i,h}, l_{i,h})$, where i denotes the individual and h denotes the household. This utility function is differentiable in both arguments and satisfies all usual conditions. Individuals discount the future at rate β and maximize expected utility.

Assume that there exist a countable set of payoff relevant states of nature S_t in period t, and we denote a generic state by s_t ($s_t \in S_t$). As usual we denote a history by $s^t = (s_1, ..., s_t)$. The probability of history s^t is $\pi(s^t)$. The sources of individual income are wage income and non labor income. Non-labor income in history s^t is $y_{i,h}(s_t)$ and the hourly wage rate as $w_{i,h}(s_t)$. Note that both income and wages only depend on the current state s_t. Thus, total wage income of individual i in household h and state s_t will be the wage rate $w_{i,h}(s_t)$ times the number of units of time worked. Each individual has a total time endowment of $T_{i,h}$ per period. Finally, there is a transfer schedule amongst households. Let $\tau_h(s)$ denote the (net) transfer received by household h when state s is realized.

Household Level Analysis

We begin by assuming that the risk sharing unit is the household. Assume a unitarian household model in which allocations are decided as a result of an efficient social-planner-like decision rule with weights $\mu_{i,h}$ on individual utility functions.

(1) Write down the program that a household h solves when deciding consumption and labor allocations for its members.

Solution: The household takes the time path of transfers $\{\tau_h(s^t)\}_t$ as given. Hence, total household income after history s^t is given by

$$X_h\left(s^t\right) = \tau_h\left(s^t\right) + \sum_{i=1}^{I_h} y_{i,h}\left(s_t\right).$$

The problem of the family is given by

$$\max_{\left[\left\{c^{i,h}(s^t),l^{i,h}(s^t)\right\}_i^{I_h}\right]_{s^t}} \sum_{i=1}^{I^h} \mu^{i,h} \sum_{t,s^t} \beta^t \pi\left(s^t\right) u^{i,h}\left(l^{i,h}\left(s^t\right), c^{i,h}\left(s^t\right)\right) \tag{1}$$

$$\text{s.t. } \sum_{i=1}^{I^h} c^{i,h}\left(s^t\right) + \sum_{i=1}^{I^h} w^{i,h}\left(s^t\right) l^{i,h}\left(s^t\right) = \sum_{i=1}^{I^h} w^{i,h}\left(s^t\right) T^{i,h} + X^h\left(s^t\right) \text{ for all } s^t.$$

As usual we can solve (1) as a static problem, i.e. for each s^t, $\left\{c^{i,h}\left(s^t\right), l^{i,h}\left(s^t\right)\right\}_i^{I_h}$ solve

$$\max_{\left\{c^{i,h},l^{i,h}\right\}_i^{I_h}} \sum_{i=1}^{I^h} \mu^{i,h} u^{i,h}\left(l^{i,h}, c^{i,h}\right) \tag{2}$$

$$\text{s.t. } \sum_{i=1}^{I^h} c^{i,h} + \sum_{i=1}^{I^h} w^{i,h} l^{i,h} = \sum_{i=1}^{I^h} w^{i,h} T^{i,h} + X^h,$$

where I now suppressed the explicit dependence on s^t.

(2) Characterize the solution to the allocation problem. Please be explicit on which variables $c^{i,h}$ and $l^{i,h}$ depend. Provide a precise intuition why the solution depends on those variables (and why not on some others). Let λ be the multiplier on the constraint. The necessary conditions for this problem are

$$\begin{aligned} \mu^{i,h} u_c^{i,h}\left(l^{i,h}, c^{i,h}\right) &= \lambda \\ \mu^{i,h} u_l^{i,h}\left(l^{i,h}, c^{i,h}\right) &= \lambda w^{i,h}. \end{aligned}$$

Hence, the allocation between consumption and leisure is given by

$$\frac{u_l^{i,h}\left(l^{i,h}, c^{i,h}\right)}{u_c^{i,h}\left(l^{i,h}, c^{i,h}\right)} = w^{i,h} \tag{3}$$

and the consumption allocation across households is given by

$$\mu^{i,h} u_c^{i,h}\left(l^{i,h}, c^{i,h}\right) = \mu^{g,h} u_c^{g,h}\left(l^{g,h}, c^{g,h}\right). \tag{4}$$

From (7) we can express leisure as an individual-specific function of consumption and the wage rate, i.e.

$$l^{i,h} = \phi^{i,h}\left(c^{i,h}, w^{i,h}\right). \tag{5}$$

Hence, we can express (4) as

$$\mu^{i,h} u_c^{i,h}\left(\phi^{i,h}\left(c^{i,h}, w^{i,h}\right), c^{i,h}\right) = \mu^{g,h} u_c^{g,h}\left(\phi^{g,h}\left(c^{g,h}, w^{g,h}\right), c^{g,h}\right). \tag{6}$$

In particular, consider $i = 1$ so that (6) determines $c^{g,h}$ as a function of $c^{1,h}$ and the wage rates $w^{g,h}$ and $w^{i,h}$, i.e.

$$c^{g,h} = \chi^{g,h}\left(c^{1,h}, w^{1,h}, w^{g,h}\right).$$

Hence,

$$\begin{aligned} l^{g,h} &= \phi^{g,h}\left(c^{g,h}, w^{g,h}\right) \\ &= \phi^{g,h}\left(\chi^{g,h}\left(c^{1,h}, w^{1,h}, w^{g,h}\right), w^{g,h}\right) \\ &\equiv \kappa^{g,h}\left(c^{1,h}, w^{1,h}, w^{g,h}\right), \end{aligned}$$

where $\kappa^{g,h}$ is some function specific to individual g.

From the budget constraint of (2) we therefore get that

$$\begin{aligned} \sum_{i=1}^{I^h} w^{i,h}T^{i,h} + X^h &= \sum_{i=1}^{I^h} c^{i,h} + \sum_{i=1}^{I^h} w^{i,h}l^{i,h} \\ &= c^{1,h} + w^{1,h}l^{1,h} + \sum_{g=2}^{I^h} c^{g,h} + \sum_{g=2}^{I^h} w^{g,h}l^{g,h} \\ &= c^{1,h} + w^{1,h}\phi^{1,h}\left(c^{1,h}, w^{1,h}\right) + \\ &\quad \sum_{g=2}^{I^h} \chi^{g,h}\left(c^{1,h}, w^{1,h}, w^{g,h}\right) + \sum_{g=2}^{I^h} \kappa^{g,h}\left(c^{1,h}, w^{1,h}, w^{g,h}\right). \end{aligned}$$

This is an equation which determines $c^{1,h}$ as a function of a bunch of things, in particular

$$c^{1,h} = f^{1,h}\left(w^{1,h}, w^{2,h}, ..., w^{3,h}, \sum_{i=1}^{I^h} w^{i,h}T^{i,h} + X^h\right),$$

i.e. consumption depends on *all* wage rates $\left[w^{1,h}, ..., w^{I_h,h}\right]$ and total income $\sum_{i=1}^{I^h} w^{i,h}T^{i,h} + X^h$. That consumption only depends on total income and not on its individual components is the usual result. However, now consumption depends on all wage rates of household members. This is due to the leisure-labor choice encapsulated in (7).

(3) With your answer to the previous question in mind: what do you think of the usual risk sharing regressions? Why might a significant effect of individual income in the consumption regression not be informative about the absence of risk-sharing?

Solution: This model suggests, that individual income depends on total income and all wage rates. In particular: if consumption and leisure are substitutes (i.e. the marginal utility of consumption is high if leisure is low), individual consumption will be positively related to the wage rate - if wages are high, the individual should work and hence receive consumption due to the complementarity. If the wage rate is positively correlated with personal income, we will find in a regression that personal income is correlated with consumption conditional on aggregate income. This however, is an implication of optimal risk sharing.

(4) Now suppose that consumption and leisure are separable in individuals' preferences. Formally, suppose that $u^{i,h}(c,l) = v^{i,h}(c) + q^{i,h}(l)$. Which variables determine individual leisure and consumption now?

Solution: If preferences are separable, (7) simplifies to

$$\frac{u_l^{i,h}\left(l^{i,h},c^{i,h}\right)}{u_c^{i,h}\left(l^{i,h},c^{i,h}\right)}=\frac{q_l^{i,h}\left(l^{i,h}\right)}{v_c^{i,h}\left(c^{i,h}\right)}=w^{i,h}. \tag{7}$$

This shows that (5) still applies, i.e. $l^{i,h}=\phi^{i,h}\left(c^{i,h},w^{i,h}\right)$. All the other steps to derive the consumption allocation rule did not use the separability between consumption and leisure. Hence, individual consumption allocation depend on the same variables as above despite the separability of preferences. *But:* there is another relation we can use! By looking at (4) right away we get that

$$\mu^{i,h}v_c^{i,h}\left(c^{i,h}\right)=\mu^{g,h}v_c^{g,h}\left(c^{g,h}\right),$$

from which we get that for all $i=2,...,H$

$$c^{i,h}=\chi^{i,h}\left(c^{1,h}\right).$$

Hence, $C^h\equiv\sum c^{i,h}=\sum\chi^{i,h}\left(c^{1,h}\right)$ so that $c^{1,h}=h\left(C^h\right)$ so that we can still write an equation of the form

$$c^{i,h}=f^{i,h}\left(C^h\right),$$

i.e. individual consumption should only depend on aggregate consumption (*not* aggregate income).

(5) How could this allocation be decentralized, (assuming that each agent is free to decide how much to work)?

Solution: Under the usual assumptions on preferences the second welfare theorem applies. Hence, there is an equilibrium with transfers, where individual household members receive endowments $e^{i,h}$ and then either trade in Arrow-Debreu markets for state-contingent commodities or simply have a full set of state-contingent one-period assets and markets for labor and the consumption good are open in each state s^t.

(6) Does individual labor supply of agent i in household h depend on wages and incomes of individuals in the household? Why or why not?

Solution: As clearly seen from above: individual labor supply is given by

$$T^{i,h}-l^{i,h}=T^{i,h}-\zeta^{1,h}\left(w^{1,h},w^{2,h},...,w^{3,h},\sum_{i=1}^{I^h}w^{i,h}T^{i,h}+X^h\right),$$

i.e. labor supply depends on wages of other people in the household but not on their non-labor income (conditional on aggregate income $\sum_{i=1}^{I^h}w^{i,h}T^{i,h}+X^h$). The intuition that the wages of other individuals determine labor supply is the same as for consumption: with non-separable preferences pareto-optimality requires that marginal utilities of consumption are equalized. As this marginal utility depends on individual leisure, the labor allocations depend on the distribution of wages in the economy.

(7) Suppose only for this question that the utility function was CARA only in consumption (i.e.,$u^{i,h}(c) = -e^{-\sigma_{ih}c}$ and you would like to identify the risk aversion of agents in a household. Could you identify the parameter σ_{ih} using a regression? Why or why not? If leisure does not enter the utility function, we clearly have $l^{i,h} = 0$, as leisure has the price $w^{i,h}$. From (6) we have

$$\mu^{1,h} e^{-\sigma^{1,h} c^{1,h}} \sigma^{1,h} = \mu^{g,h} e^{-\sigma^{g,h} c^{g,h}} \sigma^{g,h}.$$

Hence,

$$c^{g,h} = \frac{1}{\sigma^{g,h}} ln \left(\frac{\mu^{g,h}}{\mu^{1,h}} \frac{\sigma^{g,h}}{\sigma^{1,h}} \right) + \frac{\sigma^{1,h}}{\sigma^{g,h}} c^{1,h}.$$

Substituting this in the budget constraint yields

$$\begin{aligned}
\sum_{i=1}^{I^h} w^{i,h}\left(s^t\right) T^{i,h} + X^h\left(s^t\right) &= \sum_{i=1}^{I^h} c^{i,h}\left(s^t\right) + \sum_{i=1}^{I^h} w^{i,h}\left(s^t\right) l^{i,h}\left(s^t\right) \\
&= \sum_{i=1}^{I^h} c^{i,h}\left(s^t\right) \\
&= c^{1,h} + \sum_{g=2}^{I^h} \frac{1}{\sigma^{g,h}} ln \left(\frac{\mu^{g,h}}{\mu^{1,h}} \frac{\sigma^{g,h}}{\sigma^{1,h}} \right) + c^{1,h}\sigma^{1,h} \sum_{g=2}^{I_h} \frac{1}{\sigma^{g,h}} \\
&= c^{1,h}\left(1 + \sigma^{1,h} \sum_{g=2}^{I_h} \frac{1}{\sigma^{g,h}} \right) + \sum_{g=2}^{I^h} \frac{1}{\sigma^{g,h}} ln \left(\frac{\mu^{g,h}}{\mu^{1,h}} \frac{\sigma^{g,h}}{\sigma^{1,h}} \right) \\
&= c^{1,h}\sigma^{1,h}\left(\frac{1}{\sigma^{1,h}} + \sum_{g=2}^{I_h} \frac{1}{\sigma^{g,h}} \right) + \sum_{g=2}^{I^h} \frac{1}{\sigma^{g,h}} ln \left(\frac{\mu^{g,h}}{\mu^{1,h}} \frac{\sigma^{g,h}}{\sigma^{1,h}} \right) \\
&= c^{1,h}\sigma^{1,h}\left(\sum_{g=1}^{I_h} \frac{1}{\sigma^{g,h}} \right) + \sum_{g=2}^{I^h} \frac{1}{\sigma^{g,h}} ln \left(\frac{\mu^{g,h}}{\mu^{1,h}} \frac{\sigma^{g,h}}{\sigma^{1,h}} \right).
\end{aligned}$$

Solving for consumption yields

$$\begin{aligned}
c^{1,h} &= \frac{\frac{1}{\sigma^{1,h}}}{\sum_{g=1}^{I_h} \frac{1}{\sigma^{g,h}}} \left[-\sum_{g=2}^{I^h} \frac{1}{\sigma^{g,h}} ln \left(\frac{\mu^{g,h}}{\mu^{1,h}} \frac{\sigma^{g,h}}{\sigma^{1,h}} \right) + \left(\sum_{i=1}^{I^h} w^{i,h}\left(s^t\right) T^{i,h} + X^h\left(s^t\right) \right) \right] \\
&= -\frac{\frac{1}{\sigma^{1,h}} \sum_{g=2}^{I^h} \frac{1}{\sigma^{g,h}} ln \left(\frac{\mu^{g,h}}{\mu^{1,h}} \frac{\sigma^{g,h}}{\sigma^{1,h}} \right)}{\sum_{g=1}^{I_h} \frac{1}{\sigma^{g,h}}} + \frac{\frac{1}{\sigma^{1,h}}}{\sum_{g=1}^{I_h} \frac{1}{\sigma^{g,h}}} \left(\sum_{i=1}^{I^h} w^{i,h}\left(s^t\right) T^{i,h} + X^h\left(s^t\right) \right) \\
&\equiv \alpha_1 + \frac{\frac{1}{\sigma^{1,h}}}{\sum_{g=1}^{I_h} \frac{1}{\sigma^{g,h}}} Y^h,
\end{aligned}$$

where Y^h is aggregate household income and α_1 is a individual-specific fixed effect. This shows that $\sigma^{i,h}$ is *not* identified - it is only the *relative* risk aversion (relative to the other household members), which is identified from a regression. In particular, relatively risk-neutral households get a big share of aggregate resources and are therefore subject to a larger part of the aggregate risk in this economy.

(8) Express the indirect utility function of household h, ω^h, implicitly. What are the arguments of the indirect utility function?

Solution: The indirect utility function of the household is simply the solution to (8), i.e. it is defined by

$$\begin{aligned}
\omega^h\left(\left\{X^h\left(s^t\right)\right\}_t, \left\{w^{i,h}\left(s^t\right)\right\}_{i,t}\right) &= \max_{\left[\left\{c^{i,h}(s^t), l^{i,h}(s^t)\right\}_i^{I_h}\right]_{s^t}} \sum_{i=1}^{I^h} \mu^{i,h} \sum_{t,s^t} \beta^t \pi\left(s^t\right) u^{i,h}\left(l^{i,h}\left(s^t\right), c^{i,h}\left(s^t\right)\right) \qquad (8) \\
& \text{s.t. } \sum_{i=1}^{I^h} c^{i,h}\left(s^t\right) + \sum_{i=1}^{I^h} w^{i,h}\left(s^t\right) l^{i,h}\left(s^t\right) = \sum_{i=1}^{I^h} w^{i,h}\left(s^t\right) T^{i,h} + X^h\left(s^t\right) \text{ for all } s^t.
\end{aligned}$$

The indirect utility function depends on the entire sequence of transfers and the entire sequence of wages for all family members. Clearly both of these "objects" have to be known to solve (8) and hence to define the indirect utility function ω^h.

Village Level Analysis

We now assume that the village is the risk sharing unit.

(9) Set up the planning problem for the village and prove that this problem can be solved by the determination of state contingent transfers τ_{ih} to maximize the weighted sum of household indirect utilities.

Solution: See my risk sharing notes.

(10) Characterize the allocation. How would the allocation rule differ in two different states, s and s', that satisfy the following property $\sum_{h=1}^{H} \sum_{i=1}^{I_h} y^{i,h}(s) = \sum_{h=1}^{H} \sum_{i=1}^{I_h} y^{i,h}(s')$?

Solution: See my risk sharing notes.

(11) What regression would you run to test for risk-sharing within the family and across families within the village?

Solution: Optimal household risk-sharing requires that individual consumption should not depend on individual income once household income and the wages of the household are controlled for. This restriction can be tested using individual panel data. Risk sharing in the village implies that household transfers, i.e. $\tau^h\left(s^t\right) = \sum_{i=1}^{I_h}\left(c^{i,h}\left(s^t\right) - \left[y^{i,h}\left(s^t\right) + w^{i,h}\left(s^t\right)\left(T^{i,h} - l^{i,h}\left(s^t\right)\right)\right]\right)$, should only depend on aggregate income once the distribution of wages is controlled for.

Policy Experiment

Suppose that a new government comes to power and considers that some of the wages paid in the village are extremely low. As a result, the government implements an employment income guarantee scheme in this village. Effectively, this introduces an outside option of w for all individuals in the village. Thus all the wages $w^{i,h}(s^t)$ below w become w.

(12) Construct an example for which you can characterize the consumption and leisure allocations across individuals in closed form (or at least in a way that allows you to study how these allocation depend on individual wages). How does consumption and leisure of the "affected" individuals react to the minimum wage? How are the other individuals affected? What drives those results in your example? Do you think those results generalize? [HINT: As I did not tell you what example to pick, there is no "right" or "wrong" in this exercise. Just try to find an example, which works out nicely and discuss your findings.]

Solution: Suppose that $u^{i,h}(c,l) = ln(c) + \xi ln(l)$. Then (3) implies that

$$\frac{\xi\left(l^{i,h}\right)^{-1}}{\left(c^{i,h}\right)^{-1}} = \frac{\xi c^{i,h}}{l^{i,h}} = w^{i,h} \Rightarrow c^{i,h}\xi = w^{i,h}l^{i,h}.$$

Similarly, (4) implies that

$$\mu^{i,h}\frac{1}{c^{i,h}} = \mu^{g,h}\frac{1}{c^{g,h}} \Rightarrow c^{g,h} = \frac{\mu^{g,h}}{\mu^{i,h}}c^{i,h}.$$

Hence,

$$\begin{aligned}
\sum_{i=1}^{I^h} w^{i,h}T^{i,h} + X^h &= \sum_{i=1}^{I^h} c^{i,h} + \sum_{i=1}^{I^h} w^{i,h}l^{i,h} \\
&= c^{1,h} + w^{1,h}l^{1,h} + \sum_{g=2}^{I^h} c^{g,h} + \sum_{g=2}^{I^h} w^{g,h}l^{g,h} \\
&= c^{1,h} + \xi c^{1,h} + \sum_{g=2}^{I^h} c^{g,h} + \xi\sum_{g=2}^{I^h} c^{g,h} \\
&= (1+\xi)\left[c^{1,h} + \sum_{g=2}^{I^h} c^{g,h}\right] \\
&= (1+\xi)\left[c^{1,h} + \sum_{g=2}^{I^h} \frac{\mu^{g,h}}{\mu^{1,h}}c^{1,h}\right] \\
&= \frac{1+\xi}{\mu^{1,h}}c^{1,h}\sum_{i=1}^{I^h} \mu^{i,h},
\end{aligned}$$

which implies that

$$c^{i,h} = \frac{1}{1+\xi}\frac{\mu^{i,h}}{\sum_{g=1}^{I^h}\mu^{g,h}}\left(\sum_{g=1}^{I^h} w^{g,h}T^{g,h}+X^h\right) \tag{9}$$

$$\begin{aligned} l^{i,h} &= \frac{1}{w^{i,h}}\frac{\xi}{1+\xi}\frac{\mu^{i,h}}{\sum_{g=1}^{I^h}\mu^{g,h}}\left(\sum_{g=1}^{I^h} w^{g,h}T^{g,h}+X^h\right) \\ &= \frac{\xi}{1+\xi}\frac{\mu^{i,h}}{\sum_{g=1}^{I^h}\mu^{g,h}}\left(T^{g,h}+X^h+\sum_{g\neq i}\frac{w^{g,h}}{w^{i,h}}T^{g,h}+X^h\right). \end{aligned} \tag{10}$$

Let the policy increase the wages of individual i. From (9) and (10) it is seen that this wage increase

1. increases consumption of all individuals in the village.
2. keeps relative consumption levels across individuals constant as (9) implies that

$$\frac{c^{i,h}}{c^{m,h}} = \frac{\mu^{i,h}}{\mu^{m,h}}.$$

3. reduces leisure of individual i as (10) implies that leisure $l^{i,h}$ is decreasing in $w^{i,h}$
4. increases leisure of all other individuals as (10) implies that leisure $l^{-i,h}$ is increasing in $w^{i,h}$ (due to the income effect)

Most of the results are likely to be robust to different specifications. That leisure of individual i decreases is intuitive - only if there was a very strong wealth effect in leisure would we expect something else. That all consumption levels increase is also intuitive. As there will be more income, at least some consumption level has to increase. That all consumption levels increase can be expected from the risk-sharing intuition. That relative consumption of individuals is unaffected is of course special and depends crucially on the separability of preferences, which make relative cosumption levels only a function of the pareto weights μ. That other agents increase their consumption of leisure is also intuitive. With consumption being more abundant, the marginal utility of income decreases so that working becomes less attractive. At given wages, labor supply should go down because of wealth effects on the leisure choice.