

# Lecture Note: Market Signaling – Theory and Evidence

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## 1 INTRODUCTION

George Akerlof's 1970 paper on 'Lemons' was the first to formalize the adverse selection problem. Three key ingredients of this paper are:

1. Goods on the market are of heterogeneous quality
2. Sellers are better informed than buyers about the quality of their goods
3. Sellers have a reservation price for their goods

It is this reservation price that generates an adverse selection equilibrium. In particular, Akerlof's model shows that one can have a market equilibrium with no trade even when:

1. Every seller wants to sell at a positive price
2. There are buyers willing to pay the seller's reservation price for each good

The reason that no trade may occur despite these conditions is that sellers hold reservation prices above the price buyers are willing to pay for the goods available at that price. Because the owner of an inferior good will always be willing to sell for the price of a superior good, there is adverse selection of the goods for sale at a given price. In the extreme case, there is no equilibrium price where the value of goods on the market at given price is equal to that price.

### 1.1 REVIEW: THE AKERLOF MODEL

A simple example.

- There are 2 types of new cars available at dealerships: good cars and lemons, which break down often.
- The fraction of lemons at a dealership is  $\lambda$ .
- Dealers do not distinguish among good cars versus lemons – they sell what's on the lot at the sticker price.

- Buyers cannot tell good cars from lemons, but they know that some fraction  $\lambda$  will be lemons.
- After buyers have owned the car for any period of time, they can tell whether or not they have bought a lemon.
- Assume that good cars are worth \$2,000 and lemons are worth \$1,000.
- For simplicity (and without loss of generality), assume that cars do not deteriorate and that buyers are risk neutral.

What is the equilibrium price for new cars? Clearly this must be

$$P^N = (1 - \lambda) \cdot 2,000 + \lambda \cdot 1,000.$$

Since dealers sell all cars at the same price, buyers pay the expected value of a new car.

Now, consider the used car market. Assume that used car sellers are willing to part with their cars at 20% below their new value. So,

$$P_G^U = \$1,600 \text{ and } P_L^U = \$800.$$

- Since cars don't deteriorate, used car buyers will be willing to pay \$2,000 and \$1,000 respectively for used good cars and lemons. There is a \$400 or \$200 gain from trade from each sale.

What is the equilibrium price of used cars?

- The natural answer is

$$P^U = (1 - \lambda) \cdot 1,600 + \lambda \cdot 800,$$

but this is not necessarily correct.

- Recall that buyers cannot distinguish good cars from lemons whereas owners of used cars know which is which. Assuming sellers are profit maximizing, this means that at any  $P^U \geq 800$ , owners of lemons will gladly sell them. But at  $P^U < 1,600$ , owners of good cars will *keep* their cars.

- So, the equilibrium price of used cars will depend on  $\lambda$ .
- Take the case where  $\lambda = 0.4$ . In this case, the expected value (to a buyer) of a randomly chosen used car—assuming both good cars and lemons are sold—would be

$$P_{\lambda=0.4}^U = (1 - 0.4) \cdot 2000 + 0.4 \cdot 1000 = 1,600.$$

Here, used cars sell at exactly the average price at which potential sellers value them. Owners of good cars are indifferent and owners of lemons get a \$800 surplus.

- But now take the case where  $\lambda = 0.5$ . If both good cars and lemons were sold, would the expected value of a randomly chosen used car be

$$P_{\lambda=0.5}^U = (1 - .5) \cdot 2000 + .5 \cdot 1000 = 1,500?$$

- The answer is clearly no. Since owners demand \$1,600 for their good used cars, they will not sell at this price. The only used cars on the market at  $P_{\lambda=0.5}^U = 1,500$  are lemons.
- But then the equilibrium price of used cars cannot be \$1,500. It must be  $P_{\lambda=0.5}^U = \$1,000$ .
- In other words, if the share of lemons in the overall car population is high enough ( $\lambda > .4$  in this example), then bad products drive the good ones out of the market.
- The key point: market quality is endogenous; it depends on price. When sellers have private information about products' intrinsic worth, they will only bring good products to market when prices are high. Buyers understand this, and so must adjust the price they are willing to pay to reflect the quality of the goods they expect to buy at that price.
- In equilibrium, goods available at a given price must be worth that price.

## 1.2 HOW SPENCE'S MODEL DIFFERS

Spence's 1973 (QJE) Nobel prize-winning paper considered a labor market *without* adverse selection. At the time of its writing, it was believed that the Akerlof model produced a perverse equilibrium because of seller reservation prices. If sellers of used cars did not have reservation

prices that depended positively on their cars' quality, then all goods would enter the market simultaneously, and adverse selection would be eliminated. Problem solved?

This observation is partly correct – the no trade result depends on these reservation prices. But Spence noticed that a perverse outcome is still possible even absent reservation prices. If owners of superior goods (let's say better workers) can take observable actions that less-able workers cannot, they will often choose to do it, and wages will be paid differentially for this observed characteristic. Spence's insight was that these actions can occur in equilibrium, even if they are costly and unproductive. Trade may occur with gross inefficiencies because of unproductive signaling.

The Spence paper is quite simple, and the model would not pass muster in an advanced undergraduate game theory class nowadays. But that doesn't detract from the insight. For edification, let's do a modern, continuous version of the Spence model.

### 1.3 A SIGNALING MODEL – SEPARATING EQUILIBRIUM WITH MANY ABILITY TYPES

Let's consider a labor market with a continuum of types  $\theta \in [\underline{\theta}, \bar{\theta}]$ . The productivity of each type  $\theta$  is  $\theta$ . Each type  $\theta$  must choose an education level  $e$ . The endogenous variables (actually functions) in these model will be  $w(e)$  and  $e^*(\theta)$ . These are the wage for a given education level and the optimal choice of education for each value of  $\theta$ .

- The Worker's Problem: Each  $\theta$  will maximize over  $e$ .

$$\begin{aligned} \max_e w(e) - c(e, \theta) &\Rightarrow \\ \frac{\partial w(e^*)}{\partial e} - c_e(e^*, \theta) &= 0, \forall \theta. \end{aligned} \tag{1}$$

- The Firm's Problem: Assuming that the first condition results in a distinct  $e$  for each  $\theta$ , it needs to be the case that wages  $w(e)$  equal productivity at every  $e$ .

$$w(e^*(\theta)) = \theta, \forall \theta, \tag{2}$$

where  $e^*(\theta)$  is the solution to (1). Notice that because employers cannot observe  $\theta$ , the wage schedule must be entirely a function of  $e$ , which will itself depend on  $\theta$  and on  $w(e)$ .

- So, we have two unknown functions:  $e^*(\theta)$ ,  $w(e^*(\theta))$ .
- Let's solve a concrete example. Assume

$$c(e, \theta) = k \frac{e}{\theta}, \quad (3)$$

$k > 0$ , and  $\underline{\theta} = 1$ . Assume also that the lowest type gets no education:  $e(\underline{\theta}) = e(1) = 0$ . (Probably a costless normalization.)

- The crucial feature of this cost function (3) is that its degree of convexity is declining in  $\theta$ . The higher is  $\theta$ , the lower is the cost of education *and* the less rapidly its cost rises:

$$c_\theta = -k \frac{e}{\theta^2} < 0, \quad c_e = \frac{k}{\theta} > 0, \quad c_{e\theta} = -\frac{k}{\theta^2} < 0.$$

- Applying the cost function to the worker's problem, equation (1), we get

$$\frac{\partial w}{\partial e} = \frac{k}{\theta}.$$

- Applying the cost function to the firm's problem, (2), we get

$$\frac{\partial w}{\partial e} \cdot \frac{\partial e^*}{\partial \theta} = 1.$$

In words, worker ability must increase 1-for-1 with wages – otherwise wages are too high or too low.

- Putting put these conditions together gives,

$$\frac{\partial e^*}{\partial \theta} = \frac{\theta}{k}.$$

- We now have the derivative but not the function. To find  $e^*(\theta)$ , we need to integrate the derivative from  $\underline{\theta}$  to  $\theta$ :

$$e^*(\theta) - e^*(\underline{\theta}) = \int_{\underline{\theta}}^{\theta} \frac{\theta}{k} d\theta = \frac{\theta^2}{2k} - \frac{\underline{\theta}^2}{2k}.$$

- Recalling that the lowest type gets no education ( $e^*(\underline{\theta}) = 0$ ) and that  $\underline{\theta} = 1$ , we obtain an expression for  $e^*(\theta)$ :

$$e^*(\theta) = \frac{\theta^2}{2k} - \frac{1}{2k}.$$

- Finally, we can obtain the function  $w(e)$  by inverting  $e^*(\theta)$  to get the expectation of  $\theta$  as a function of observable expectation and substitute this back into the equilibrium expressions.

$$e = e^*(\theta) = \frac{\theta^2}{2k} - \frac{1}{2k},$$

$$w(e^*(\theta)) = E(\theta|e) = \sqrt{e^*(\theta)2k + 1}.$$

- Note that this solution depended on the existence of a complete separating equilibrium: each ability level  $\theta$  gets a different  $e^*$ . This equilibrium need not exist for all problems.
- **[Algebra check.** Take  $k = \frac{1}{2} \Rightarrow c(e) = \frac{1}{2}\frac{e}{\theta}$ ,  $w(e) = \sqrt{e + 1}$ .

$$e^*(1) = 0, e^*(2) = 3,$$

$$w^*(0) = 1, w^*(3) = 2.$$

Now check the self-selection ('separating') constraints: Does any  $\theta$  wish to change her education level? Recall that cost of education is  $k\frac{e}{\theta}$ .

$$w(3) - c(3,1) = 2 - 3/2 = 1/2 < w(0) - c(0,1) = 1,$$

$$w(3) - c(3,2) = 2 - 3/4 = 5/4 > w(0) - c(0,2) = 1.$$

**The equilibrium functions work.]**

- Hence, an equilibrium must have:
  - Rational expectations: Along the equilibrium wage schedule, firms receive workers whose productivity is equal to the wage they are paid.
  - Self-selection: Workers have no incentive to take actions that would cause productivity to deviate from the wage schedule.

## 2 EMPIRICAL TESTS OF MARKET SIGNALING

- Does the signaling model share any implications with the Human Capital model?
  1. People who attend additional years of schooling are more productive. YES.

2. People who attend additional years of schooling receive higher wages. YES.
  3. The rate of return to schooling should be roughly equal to the rate of interest. NO PREDICTION.
  4. People will attend school while they are young, i.e., before they enter the workforce. YES.
- How do you empirically distinguish the human capital and signaling models?
    1. Measure whether more educated people are more productive? (Would be true for either model.)
    2. Measure people's productivity before and after they receive education – see if it improves. (Conceptually okay, very difficult to do.)
    3. Test whether higher ability people go to school? (Could be true in either case—certainly true in the signaling case.)
    4. Find people of identical ability and randomly assign some of them to go to college. Check if the college educated ones earn more? (Both models say they would.)
    5. Find people of identical ability and randomly assign them a diploma. See if the ones with the diploma earn more. (A pure test of signaling.)
  - Because their empirical implications appear so similar, many economists had begun to conclude that these models could not be empirically distinguished. The empirical papers on the syllabus by Lang and Kropp (1986) and Bedard (2001) offer some evidence. Weiss (1995) provides a survey.
  - Both of the empirical papers implement closely related indirect tests of the signaling model along the following lines. If we are initially at a separating equilibrium, how does an exogenous decline in the cost of schooling for one group affect the education choices of other worker groups who are not directly affected by the price change? Lang and Kropp look at college going as a function of school leaving laws (which primarily directly affect secondary school attendance). Bedard looks at high school dropout rates as a function of

the cost of attending college. Both purport to find that the ‘indirectly affected groups’ behave as the signaling model would predict: students attend college to ‘separate’ from the high school grads (Lang and Kropp); more students drop out of high school because the quality of the high school graduate pool is diluted by greater college attendance – and so its not worth the cost of education to pool with them (Bedard).

- These tests are pretty weak – the empirical equivalent of aiming for the capillaries. They indirectly test second order predictions of the signalling model.
- A first order prediction of the model is this: A ‘signal’ (such as education) commands a positive price in equilibrium, even when acquiring that signal has no impact on human capital production. To test this prediction, one would have to randomly assign signals to individuals of otherwise identical expected productivity. Tyler, Murnane and Willett identify one such quasi-experiment.

### 3 THE TYLER, MURNANE AND WILLETT STUDY

- TMW are interested in knowing whether the General Educational Development certificate (GED) raises the subsequent earnings of recipients.
- This question is important for educational policy:
  - By 1996, 9.8% of those ages 18 – 24 had completed High School via the the GED versus 76.5% via a HS diploma.
  - See Table I. Notice that between 1990 and 1996, HS Diploma rates actually fell dramatically for Black, Non-Hispanics. The rise in the GED just offset this. Hence, we ought to hope that these GED holders are doing at least somewhat better than HS dropouts.
- In 1996, 759,000 HS Dropouts attempted the GED and some 500,000 passed.
- The monetary cost of taking the GED is \$50 and the exam lasts a full day.

- The average person spends 20 hours studying for the GED (though some spend much more and some spend zero).
- See Table II. GED holders earn substantially less than HS graduates, but somewhat more than HS Dropouts.
- Self-selection (endogenous choice):
  - GED holders probably would have earned less than HS Diploma holders regardless. These are not typically the cream of your HS class.
  - GED holders probably would have earned more than other HS dropouts regardless. Relative to other dropouts, GED holders have:
    - \* More years of schooling prior to dropout.
    - \* Higher measured levels of cognitive skills.
    - \* Their parents have more education.
- So, simple comparisons of earnings among dropouts/ GED holders/ HS diploma holders tell us nothing about the causal effect of a GED for a person who obtains it.

### 3.1 STRATEGY

- GED passing standards differ by U.S. state. Some test takers who would receive a GED in Texas with a passing score of 40 – 44 would not receive a GED in New York, Florida, Oregon or Connecticut with the identical scores.
- But if GED score is a good measure of a person’s ability/productivity, then people with same ‘ability’ (40 – 44) are assigned a GED in Texas but not in New York.
- This quasi-experiment effectively randomly assigns the GED ‘signal’ to people with the same GED scores across different U.S. states.
- If we could determine who these marginal people are, we could identify the pure signaling effect of the GED, holding ability constant.

- What does the signaling model predict in this case?
- Since some dropouts obtain the GED and some do not, it's plausible that the market is in some type of 'separating' equilibrium (i.e., not everyone gets the signal).
- For the GED to perform as a signal, it needs to be the case that the cost of obtaining it is lower for more productive workers (otherwise everyone or no one would get it). This seems plausible; you cannot pass the GED without some education and study.
- In equilibrium, the following must be true for individuals:

$$w_{GED} - w_{NO-GED} \geq C_{GED} \Rightarrow \text{obtain,}$$

$$w_{GED} - w_{NO-GED} < C_{GED} \Rightarrow \text{don't obtain,}$$

where  $C_{GED}$  is the direct and indirect costs of obtaining the GED.

- And the following must be true for employers:

$$w_{GED} = E(\text{Productivity} | C_{GED} \leq w_{GED} - w_{NO-GED}),$$

$$w_{NO-GED} = E(\text{Productivity} | C_{GED} > w_{GED} - w_{NO-GED}).$$

- If these conditions are satisfied, firms will be willing to pay the wages  $w_{GED}, w_{NO-GED}$  to GED and non-GED holders respectively, and workers will self-select to obtain the GED accordingly.
- We are assuming that firms cannot observe worker ability independent of the GED.
- So, given the quasi-experimental setup, the signaling model predicts that workers with GED scores of 40 – 44 will earn more if they receive the GED certificate than if they do not.
- By contrast, the Human Capital model implies that since ability is comparable among these groups, wages will also be comparable.

### 3.2 DIFFERENCE-IN-DIFFERENCE

- The econometric strategy for this paper is similar to the Card-Krueger minimum wage study: differences-in-differences.

- TMW choose a control group of GED test takers with scores just about the cutoff for both groups of states. Hence, the GED treatment works as follows:

	Low Passing Standard	High Passing Standard
Low Score (treatment group)	GED	NO GED
High Score (control group)	GED	GED

- The outcome variable will be earnings for each of these four groups:

	Low Passing Standard	High Passing Standard
Low Score (treatment group)	$\$L, L$	$\$L, H$
High Score (control group)	$\$L, H$	$\$H, H$

- Notice the following contrasts

$$(1) \ \$L, L - \$L, H \quad = \quad \text{earnings diff of GED/non-GED holders with same scores.}$$

$$(2) \ \$L, H - \$H, H \quad = \quad \text{earnings diff of GED holders with same scores across these states.}$$

$$(1) - (2) \quad = \quad \text{difference-in-difference estimate (netting out state diffs)}$$

- See Table V.
- See Figures I-III.

### 3.3 TMW: CONCLUSIONS

- Large signaling effects for whites, estimated at 20% earnings gain after 5 years.
- Does this prove that GED holders are *not* more productive than non-GED holders?
  - No. Just the opposite.
  - For there to be a signaling equilibrium, it must be the case that GED holders *are* on average more productive than otherwise similar HS dropouts who do not hold a GED.
- Do these results prove that the GED is productive?

- No. They are not inconsistent with that fact, but they offer no evidence one way or another.
- Do these results prove that education is unproductive?
  - No, they also have nothing to say on this because education/skill is effectively held constant by this quasi-experiment.
- What the study shows unambiguously is that the GED is taken as a positive signal by employers. And this can only be true if:
  1. GED holders are on average more productive than non-GED holders.
  2. The GED is in some sense more expensive for less productive than more productive workers to obtain. This probably has to do with maturity, intellect, etc.
  3. Employers are unable to perfectly distinguish productivity directly and hence use GED status as one signal of expected productivity.
- This is a superb paper.

#### 4 CONCLUSIONS

To the disappointment of some, signaling models have not dramatically changed the way most economists view education. In fact, economic interest in the efficiency of educational production has burgeoned in the last decade – which would not be true if economists generally believed that schooling is about testing how long students can tolerate sitting in a chair.

Nevertheless, it's reasonably clear that signals do carry value in some markets, and that this could possibly give rise to distortions like those envisioned by Spence. It's unclear whether those distortions are economically significant.

Image removed due to copyright considerations.

Please see Tables 1, 2, 5 and Figures I-III in Tyler, John, Richard J. Murnane, and John Willett. "Estimating the Labor Market Signaling Value of the GED." *Quarterly Journal of Economics* 115, no. 2 (2002).