Notes on Tax Implementation

Iván Werning

1 Certainty

- utility

$$U(x, \theta)$$

  where $x \in X$ is a vector and $\theta \in \Theta$ is worker types

- Example 1: Mirrlees (1971) has $x = (c, -y)$ where c is consumption y is effective labor; in this case we want to know study the non-linear income tax schedule.

- Example 2: Two-period model with labor in the first period and consumption in both periods: $U(c_0, c_1, y_0)$; in this example we'd like to study the nonlinear taxation of income and the taxation of savings [Atkinson-Stiglitz applies if U is separable]

- at this stage: no assumption on preferences (conavity, dimentionality of Θ, single crossing, etc.) needed

- define MRS

$$MRS_{ij}(x, \theta) = \frac{U_i(x, \theta)}{U_j(x, \theta)}$$

- when $\hat{x}(\theta)$ is an optimal allocation, it to look at the "wedges" or "implicit marginal taxes" defined by either

$$MRS_{ij}(\hat{x}(\theta), \theta) - \frac{p_j}{p_i} \qquad \text{or} \qquad \frac{MRS_{ij}(\hat{x}(\theta), \theta)}{p_j / p_i}$$

  we want to understand to what extent these measures are related to explicit taxes

1.1 The Problem of Implementation...

- incentive compatible allocation is a function $\hat{x} : \Theta \to X$ such that

$$U(\hat{x}(\theta), \theta) \geq U(\hat{x}(\theta'), \theta) \qquad \forall \theta, \theta' \in \Theta^2 \tag{1}$$

- implementability question: what budget sets B can we confront agent with and get $\hat{x}$ allocation?

$$\hat{x}(\theta) \in \arg\max_{x \in B} U(x, \theta) \tag{2}$$

- Note: B is independent of θ
...captures anonymous taxation

1.2 ...Its Solution...

- smallest set that works...

$$\underline{B} \equiv \{x \mid \exists \theta \in \Theta \quad \text{s.t.} \quad x = \hat{x}(\theta)\}$$

note that: incentive compatibility (1) implies (2) with $\underline{B}$

- this gives as much choice as the direct mechanism!...
... not a lot of choice if X has high dimension and Θ is low dimension

- largest set?

$$\bar{B} \equiv \{x \mid U(x, \theta) \leq U(\hat{x}(\theta), \theta) \; \forall \theta \in \Theta\}$$

equivalently

$$\bar{B} \equiv \{x \mid U(x, \theta) \leq \hat{v}(\theta) \; \forall \theta \in \Theta\}$$

where $\hat{v}(\theta) \equiv U(\hat{x}(\theta), \theta)$

- full characterization: any set B such that

$$\underline{B} \subseteq B \subseteq \bar{B}$$

also implements $\hat{x}$

1.3 ...In Terms of Taxes

- to think of taxation...

- benchmark budget without tax:

$$p \cdot x \leq 0$$

- $T(x)$ function such that

$$p \cdot x + T(x) \leq 0$$

is equivalent to $x \in B$ where B implements $\hat{x}$

- for lowest possible taxes use $B = \bar{B}$

- numeraire good: $x = (x_1, x_{-1})$ with $p_1 = 1$ then

$$x_1 + T(x_{-1}) + p_{-1} \cdot x_{-1} \leq 0$$

- retention function...

$$x_1 \leq R(x_{-1}) = -(T(x_{-1}) + p_{-1} \cdot x_{-1})$$

- to implement we need

$$\hat{x}_1(\theta) = R(\hat{x}_{-1}(\theta)) \qquad \theta \in \Theta$$

and

$$R(x_{-1}) \leq \hat{R}(x_{-1}) \equiv \max_{x_1} x_1 \quad \text{s.t.} \quad U(x_1, x_{-1}, \theta) \leq \hat{v}(\theta) \quad \forall \theta \in \Theta$$

- equivalently: need $R(x_{-1}) \leq \hat{R}(x_{-1})$ for all $x \in X$ and $R(x_{-1}) = \hat{R}(x_{-1})$ for $x \in \underline{B}$.

- invert...

$$U(x_1, x_{-1}, \theta) \leq \hat{v}(\theta)$$

to write...

$$x_1 \leq U^{-1}(\hat{v}(\theta), x_{-1}, \theta)$$

- then

$$\hat{R}(x_{-1}) \equiv \min_{\theta \in \Theta} U^{-1}(\hat{v}(\theta), x_{-1}, \theta)$$

1.4 Some Properties of the Solution

- idea: since $\hat{R}$ defined as optimization, we can apply Maximum and Envelope Theorems

- economic questions...
  - how much more choice?
  - marginal taxes exist?
  - do they equal wedges?
- Maximum Theorem: Assume $U : X \times \Theta \to \mathbb{R}$ is continuous, then $\hat{v}(\theta)$ and $\hat{R}(x_{-1})$ are continuous functions; the set

$$M(x_{-1}) \equiv \arg\min_{\theta \in \Theta} U^{-1}(\hat{v}(\theta), x_{-1}, \theta)$$

  is upper hemi continuous correspondence (note that $\theta \in M(\hat{x}_{-1}(\theta))$)
- this means we never impose sharp penalties in the sense of discontinuous taxes;
- In contrast, the direct mechanism implicitly imposes infinite taxes for any allocation outside $\underline{B}$! In this sense, Taxes are very discontinous.
- Envelope Theorem: Suppose U is differentiable w.r.t. x and $M(x_{-1})$ is single valued, then

$$\frac{\partial}{\partial x_{-1}} \hat{R}(x_{-1}) \equiv \frac{\partial}{\partial x_{-1}} U^{-1}(\hat{v}(M(x_{-1})), x_{-1}, M(x_{-1})) = -\frac{\frac{\partial}{\partial x_{-1}} U(\hat{R}(x_{-1}), x_{-1}, M(x_{-1}))}{\frac{\partial}{\partial x_1} U(\hat{R}(x_{-1}), x_{-1}, M(x_{-1}))}$$

  That is,

$$\frac{\partial}{\partial x_{-1}} \hat{R}(x_{-1}) = MRS_{x_1, x_{-1}}(\hat{R}(x_{-1}), x_{-1}, M(x_{-1}))$$

- $M(x_{-1})$ is single valued means that only one type θ is indifferent to $(\hat{R}(x_{-1}), x_{-1})$.
- This provides a condition for the marginal tax to exist and equal the tax wedge along the equilibrium set $\underline{B}$.
- If $M(x_{-1})$ is not single valued then we candidate MRSs...
  ...this actually implies kinks in $\hat{R}$
  ...we can still compute left and right derivatives
- for example: static Mirrlees (1971) when bunching occurs we get a convex kink in income tax schedule

1.5 Linear Taxes?

- can we choose a subset of goods to be taxed linearly? (not taxed is particular case, e.g. Atkinson-Stiglitz)
- suppose we can divide goods $x = (x^a, x^b)$ so that

$$\hat{x}^b(\theta) = \hat{x}^b(\theta') \implies \hat{x}^a(\theta) = \hat{x}^a(\theta') \qquad \forall \theta, \theta' \in \Theta^2$$

  i.e. x^b identifies x^a, write

$$x^a = \hat{\alpha}(x^b)$$

- typically

$$\dim \Theta = \dim X^b \leq \dim X$$

  so this can be done

- define support of x^b

$$B^b \equiv \{x^b \mid \exists \theta \in \Theta \quad x^b = \hat{x}^b(\theta)\}$$

- now, for given x^b consider the set

$$B(x^b) \equiv \{x^a \mid U(x^a, x^b, \theta) \leq \hat{v}(\theta) \quad \forall \theta \in \Theta\} = \{x^a \mid (x^a, x^b) \in \bar{B}\}$$

- given $x^b \in \bar{B}^b$ define a linear set

$$B^L(x^b) = \{x^b \mid q(x^b) \cdot (x^a - \hat{\alpha}(x^b)) \leq 0\}$$

  for some consumer prices q which may depend on x^b

- note: $\hat{\alpha}(x^b) \in B^L(x^b)$
- "mixed taxation"...

$$B = \{x \mid x^a \in B^L(x^b) \quad \text{and} \quad x^b \in B^b\}$$

- Question: can this implement $\hat{x}$?
- Yes, if and only if

$$B^L(x^b) \subseteq B(x^b)$$

- sufficient condition: holds if $[B(x^b)]^c$ is convex
- in terms of taxes:

$$p \cdot x + T(x^a, x^b) \leq 0$$

given x^b can we make $T(\cdot, x^b)$ linear? i.e.

$$T(x^a, x^b) = t(x^b) + \tau(x^b) \cdot x^a$$

- sufficient condition: if $\bar{T}(\cdot, x^b)$ is convex then we use linear tangent
- Example: two-period consumption, linear tax on savings that depends on income
- with finite types and binding IC constraints:

  1. kinks! linear tax not possible
  2. but as types are closer: kinks get smaller
  3. near optimal allocation do not require kinks: linear tax possible

- with continuum of types: possible

1.6 Interdependence of Taxation

- note the tradeoff: linear tax but dependent on x^b
- sometimes possible to separate taxes...

$$T(x^a, x^b) = t^b(x^b) + t^a(x^a)$$

- Example: consumption two periods, nonlinear tax on income and savings (Estate Taxation paper Farhi-Werning)

2 Uncertainty

- opens many possibilities...
  general implementation: a dynamic choice problem
- Today: less general
- only uncertainty is θ_1 at $t = 1$

- pre-committed goods z (scalar; to simplify)
- ex-post goods $x(\theta)$ (vector)

- Resource constraint:

$$z + \frac{1}{R} p_x \cdot \int x(\theta) dF(\theta) \leq e$$

with first element being numeraire: $p_{x,1} = 1$

- Utility

$$\mathbb{E}[U(z, x, \theta)] = \int U(z, x(\theta), \theta) dF(\theta)$$

- Example: two period Inverse euler example $z = c_0$ and $x = (c_1, y_1)$

$$U(c_0, (c_1, y_1), \theta) = u(c_0) + \beta u(c_1) - h(y_1; \theta)$$

- we take as given allocation $\hat{z}$ and $\hat{x}(\theta)$ and try to implement it

- intertemporal wedge

$$(1 + \tau)\mathbb{E}[U_z(\hat{z}, \hat{x}(\theta), \theta)] = R\mathbb{E}[U_{x_1}(\hat{z}, \hat{x}(\theta), \theta)]$$

- Incentive compatibility...

$$U(\hat{z}, \hat{x}(\theta), \theta) \geq U(\hat{z}, \hat{x}(\theta'), \theta) \qquad \theta', \theta \in \Theta^2$$

- Budget constraint

$$z + s + T^s(s) \leq \hat{z}$$

$$p_x \cdot x + T^x(x_{-1}) \leq Rs$$

- note that T^s does not depend on θ

- we want to implement $s = 0$ (by "Ricardian equivalence" we could also do things with for any $s \neq 0$)

- Define $T^x(x_{-1}) \equiv -R_1^x(x_{-1}) - p_{-1} \cdot x_{-1}$

$$R^x(x_{-1}) \equiv \min_{\theta} U^{-1}(x_{-1}, \hat{z}, \theta)$$

- utility given this is...

$$v(z,s,\theta) \equiv \max_x U(z,x,\theta)$$
$$\text{s.t.} \quad p_x \cdot x + T^x(x_{-1}) \leq Rs$$

- This function $v(z,s,\theta)$ is continuos and differentiable in regions where the maximum is unique

- the Envelope condition at the proposed solution...

$$v_z(\hat{z},0,\theta) = U_z(\hat{z},\hat{x}(\theta),\theta)$$

$$v_s(z,0,\theta) = U_{x_1}(\hat{z},\hat{x}(\theta),\theta)$$

- expected utility is

$$V(z,s) \equiv \int v(z,x,\theta)dF(\theta)$$

- this function shares properties with v; it may be smoother even due to the averaging across θ...

- ...if θ is continouosly distributed, $\frac{\partial}{\partial s}V(z,s)$ and $\frac{\partial}{\partial z}V(z,s)$ exist and

$$\frac{\partial}{\partial s}V(z,s) = \int v_s(z,s,\theta)dF(\theta)$$

$$\frac{\partial}{\partial z}V(z,s) = \int v_z(z,s,\theta)dF(\theta)$$

  since the countable kinks in v do not matter when we average

- Now at $t=0$ we want $(z,s) = (\hat{z},0)$ so that

$$\bar{B}^z(z,s) = \{(z,s) \mid V(z,s) \leq V(\hat{z},0)\}$$

  defines the largest set of pairs (z,s) that can be offered. Then

$$V(z,s) = V(\hat{z},0)$$

  defines the frontier of this set. In terms of taxes

$$V(\hat{z} - s - T^s(s), s) = V(\hat{z},0)$$

- Differentiating the definition of T at equilibrium then gives

$$\left(1 + \frac{\partial}{\partial s} T^s(s)\right) \mathbb{E}[U_{z_1}(\hat{z}, \hat{x}(\theta), \theta)] = \mathbb{E}[U_{x_1}(\hat{z}, \hat{x}(\theta), \theta)]$$

- if $F(\theta)$ is *not* continuos then we may have kinks in T^s

2.1 Alternative: State Contingent Linear Taxes

- separable utility case
- Kocherlakota proposes state dependent taxes
- define state dependent wedges:

$$U_{z_1}(\hat{z}, \hat{x}(\theta'), \theta) = (1 - \tau(\theta')) R U_{x_1}(\hat{z}, \hat{x}(\theta'), \theta)$$

with separability only depends on θ'

- Budget constraint then

$$z + s \leq \hat{z}$$

$$x_1(\theta') = (1 - \tau(\theta')) Rs + \hat{x}_1(\theta')$$

$$x_{-1}(\theta') = \hat{x}_{-1}(\theta')$$

- note: we can turn this into

$$z + s \leq \hat{z}$$

$$x_1 + p \cdot x_{-1} + T(x_{-1}) = (1 - \tau(x_{-1})) Rs$$

14.471 Public Economics I
Fall 2012