1 A Two Period Moral Hazard Model

- Rogerson (1985)
- moral hazard model
- two periods t = 0, 1
 - effort in first period e_0
 - consumption in both periods c_0 , c_1
 - stochastic output in second period $y_1 = \theta_1$ with denisty $f(\theta_1|e_0)$
- separable utility

$$U(c_0) - h(e) + \beta \int U(c_1(\theta_1)) f(\theta_1|e_0)$$

• incentive compatible $\{c_0, e_0, c_1(\theta_1)\}$ requires:

$$U(c_0) - h(e) + \beta \int U(c_1(\theta_1)) f(\theta_1|e_0) \ge U(c_0) - h(e') + \beta \int U(c_1(\theta_1)) f(\theta_1|e')$$

• rewrite in terms of utility assignements: $u_t = U(c_t)...$

$$u_0 - h(e) + \beta \int u_1(\theta_1) f(\theta_1|e) \ge u_0 - h(e') + \beta \int u_1(\theta_1) f(\theta_1|e')$$

• planning problem

$$\min \left\{ C(u_0) + q \int [C(u_1(\theta_1)) - y_1(\theta_1)] f(\theta_1|e_0) \right\}$$

$$u_0 - h(e) + \beta \int u_1(\theta_1) f(\theta_1|e) = v_0$$

$$u_0 - h(e) + \beta \int u_1(\theta_1) f(\theta_1|e) \ge u_0 - h(e') + \beta \int u_1(\theta_1) f(\theta_1|e')$$

• here $q = R^{-1}$

1.1 Savings Distortions

• if agent could save at safe rate of return $R = q^{-1}$ then we would have

$$U'(c_0) = \beta R \int U'(c_1(\theta_1)) f(\theta_1|e_0)$$

standard Euler equation.

- we will show this does not hold: there is a distortion in savings.
- fix e_0 and consider variations in consumption/utility:

$$\hat{u}_0 = u_0 - \beta \Delta$$

$$\hat{u}_1(\theta_1) = u_1(\theta_1) + \Delta$$

• no effect on utility or incentive constraint since:

$$u_0 - h(e') + \beta \int u_1(\theta_1) f(\theta_1|e') = \hat{u}_0 - h(e') + \beta \int \hat{u}_1(\theta_1) f(\theta_1|e')$$

for all e'

• we need to solve

$$\min_{\hat{u}_0, \hat{u}_1(\cdot), \Delta} \left\{ C(\hat{u}_0) + q \int C(\hat{u}_1(\theta_1)) f(\theta_1|e_0) \right\}$$

$$\hat{u}_0 = u_0 - \beta \Delta$$

$$\hat{u}_1(\theta_1) = u_1(\theta_1) + \Delta$$

• substituting

$$\min_{\Delta} \left\{ C(u_0 - \beta \Delta) + q \int C(u_1(\theta_1) + \Delta) f(\theta_1 | e_0) \right\}$$

- note: similarity with savings problem (Δ looks like assets; -C(-x) looks like the utility function)
- FOC is

$$C'(u_0 - \beta \Delta)\beta = q \int C'(u_1(\theta_1) + \Delta)f(\theta_1|e_0)$$

this condition is necessary and sufficient for an interior: we can use this to solve for $\boldsymbol{\Delta}$

• if original allocation was optimal then $\Delta=0$ and using that $C=U^{-1}$ we obtain

$$\frac{1}{U'(c_0)} = \frac{1}{\beta R} \int \frac{1}{U'(c_1(\theta_1))} f(\theta_1 | e_0)$$

Inverse Euler equation

- since $\frac{1}{r}$ is convex we can apply Jensen's inequality
- if $Var[c_1(\theta_1)] > 0$ then

$$U'(c_0) < \beta R \int U'(c_1(\theta_1)) f(\theta_1|e_0)$$

agents are "savings constrained"

wedge

$$U'(c_0) = \beta(1-\tau)R \int U'(c_1(\theta_1))f(\theta_1|e_0)$$

is positive

$$\tau \ge 0$$

1.2 Mirrlees Model

- Mirrlees model [Golosov et al]
- work time at t = 1

$$u(c_0) + \beta \int [u(c_1(\theta_1)) - h(y_1, \theta_1)] f(\theta_1)$$

- same perturbations
- same optimality conditions: Inverse Euler equation
- ullet true also for a mixed model of moral hazard and adverse selection where effort affects distribution of ullet

2 General Horizon and Welfare

• Utility

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}[U(c_{t})-h(y_{t},\theta_{t})]$$

- $\{\theta_t\}$ general stochastic process and private information
- again: rewrite in terms of u_t
- incentive constraint

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}[u(\theta^{t})-h(y(\theta^{t}),\theta_{t})] \geq \mathbb{E}\sum_{t=0}^{\infty}\beta^{t}[u(\sigma^{t}(\theta^{t}))-h(y(\sigma^{t}(\theta^{t})),\theta_{t})]$$

• Planner's net cost:

$$\mathbb{E}\sum_{t=0}^{\infty}q^{t}[C(u(\theta^{t}))-y(\theta^{t})]$$

• variations as before: [Farhi-Werning]

$$\hat{u}(\theta^t) = u(\theta^t) + \Delta(\theta^{t-1}) - \beta\Delta(\theta^t)$$

and a "No Ponzi" condition

$$\lim \beta^t \mathbb{E} \Delta(\sigma^t(\theta^t)) = 0$$

• preserve utility and incentive compatibility since

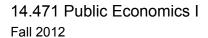
$$\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}u(\sigma^{t}(\theta^{t})) = \mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\hat{u}(\sigma^{t}(\theta^{t}))$$

• hence, must minimize cost

$$\mathbb{E}\sum_{t=0}^{\infty}q^{t}[C(u(\theta^{t})+\Delta(\theta^{t-1})-\beta\Delta(\theta^{t}))$$

- looks like savings problem: Farhi-Werning exploit this to solve this partial reform
- implementation: see slides

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