Lecture Notes on Corrective Taxation

- simplest departure from welfare theorems
- very relevant: environmental policy, congestion, rent seeking
- today: avoid redistribution issues

1 Pigouvian Taxation with a Single Agent

- First assume
  - single agent
  - lump sum taxation
  - but externalities
- Utility
$$u(x, \bar{x})$$
concave in both x and $\bar{x}$
- technology is constant returns, resource constraint:
$$F(x+g) = 0$$
- by Euler's theorem this is equivalent to
$$F_x(x+g)\cdot(x+g) = 0$$
letting
$$p_j = F_{x_j}$$
this is simply
$$p\cdot(x+g) = 0$$
- Social optimum
$$\max_x u(x,x) \qquad F(x+g) = 0$$

$$\Rightarrow u_x(x^*,x^*) + u_{\bar{x}}(x^*,x^*) = \gamma F_x(x^*+g)$$
$$\Rightarrow \frac{p_i}{p_j} = \frac{F_{x_i}}{F_{x_j}} = \frac{u_{x_i}(x^*,x^*)+u_{\bar{x}_i}(x^*,x^*)}{u_{x_j}(x^*,x^*)+u_{\bar{x}_j}(x^*,x^*)}$$

- in equilibrium
  - government budget constraint

$$(q-p)\cdot x + T = p \cdot g$$

    implies consumer budget constraint

$$q \cdot x + T = 0$$

  - consistency

$$\bar{x} = x$$

    but do not impose this on agent optimization!
  - agent solves, takes $\bar{x}$ and T as given:

$$\max_x u(x, \bar{x}) \qquad q \cdot x + T = 0$$

$$\Rightarrow u_x(x^e, x^e) = \lambda q$$

$$\Rightarrow \frac{q_i}{q_j} = \frac{u_{x_i}(x^e, x^e)}{u_{x_j}(x^e, x^e)}$$

- To make

$$x^e = x^*$$

  a necessary condition is that both conditions hold, implying

$$\frac{p_i/q_i}{p_j/q_j} = \frac{1 + \frac{u_{\bar{x}_i}(x^*,x^*)}{u_{x_i}(x^*,x^*)}}{1 + \frac{u_{\bar{x}_j}(x^*,x^*)}{u_{x_j}(x^*,x^*)}}$$

- Theorem: if p and q to satisfy this equation, then there exists an income I (i.e. lump sum tax/transfer) so that the agent chooses $x = x^*$.
  Proof: Compute p from optimum, compute q from conditions above, then just compute T from budget constraint. Apply Lagrangian sufficiency theorem.

- Remarks:
  - no explicit role for elasticity of demand
  - correct damage per unit consumed, not target reduction

- reinterpretation: “internality” or Paternalism

- suppose no externality, but agent maximizes wrong utility

$$\hat{u}(x)$$

paternistic planner cares about

$$u^*(x)$$

then we can think of above model as one where agent has utility

$$u(x) + u^*(\bar{x}) - u(\bar{x})$$

- behavioral biases can sometimes lead us to something similar
- internalities and naivety
  * you ignore (or downplay) some effects from your consumption, such as health from smoking
  * you are optimistic about certain probabilities that affect your saving decisions

2 Pigouvian Taxation with Many Agents

- Now:
  - suppose many agent types $i \in I$ with weights π^i
  - agent specific lump sum taxes T^i
  - without externalities: first best
  - with externalties: need Pigouvian taxes

- Utility

$$u^i(x^i, \bar{x})$$

concave in both x^i and

$$\bar{x} = \sum x^i \pi^i$$

assume agent type i takes x^i as given (atomostic within each group)

- implicit assumption: everyone contributes the same to externalities; relax later.

- technology

$$F(x+g) = 0$$

- Social optimum

$$\max_x \sum \lambda^h u^h(x^h, \bar{x})\pi^h \qquad F(\bar{x}+g) = 0 \qquad \bar{x} = \sum x^h \pi^h$$

first order conditions are

$$\lambda^h u^h_{x_j}(x^h, \bar{x}) = \eta_j$$

$$\sum_h \lambda^h u^h_{\bar{x}_j}(x^h, \bar{x})\pi^h + \eta_j = \gamma F_j(\bar{x}+g)$$

- substituting gives

$$\sum_h \frac{\lambda^h u_{\bar{x}_j}(x^h,\bar{x})\pi^h}{\lambda^h u_{x_j}(x^h,\bar{x})}\eta_j + \eta_j = \gamma F_j(\bar{x}+g)$$

  canceling

$$\eta_j \sum_h \frac{u^h_{\bar{x}_j}(x^h,\bar{x})}{u^h_{x_j}(x^h,\bar{x})}\pi^h + \eta_j = \gamma F_j(\bar{x}+g)$$

- dividing

$$\Rightarrow \frac{p_i}{p_j} = \frac{F_{x_i}}{F_{x_j}} = \frac{u^h_{x_i}(x^h,\bar{x})}{u^h_{x_j}(x^h,\bar{x})} \frac{1+\sum_h \pi^h \frac{u^h_{\bar{x}_i}(x^h,\bar{x})}{u^h_{x_i}(x^h,\bar{x})}}{1+\sum_h \pi^h \frac{u^h_{\bar{x}_j}(x^h,\bar{x})}{u^h_{x_j}(x^h,\bar{x})}} = \frac{\eta_i}{\eta_j} \frac{1+\frac{\gamma F_j - \eta_j}{\eta_j}}{1+\frac{\gamma F_i - \eta_i}{\eta_i}}$$

- in equilibrium
  - government budget constraint

$$(q-p)\cdot x + \sum_i T^i \pi^i = p \cdot g$$

  - implied by consumer budget constraints

$$q \cdot x^i + T^i = 0$$

  - consistency

$$\bar{x} = x$$

    but do not impose this on agent optimization!
  - agent solves, takes $\bar{x}$ and T^i as given

$$\max_x u^i(x^i,\bar{x}) \qquad q\cdot x^i + T^i = 0$$

$$\Rightarrow u^i_x(x^i,\bar{x}) = \lambda q$$

$$\Rightarrow \frac{q_i}{q_j} = \frac{u^i_{x_i}(x^i,\bar{x})}{u^i_{x_j}(x^i,\bar{x})}$$

- To make

$$x^j = x^{j*}$$

  a necessary condition is that both conditions hold, implying

$$\frac{q_i}{q_j} = \frac{\eta_i}{\eta_j}$$

$$\frac{p_i/q_i}{p_j/q_j} = \frac{1 + \sum_h \pi^h \frac{u^h_{\bar{x}_i}(x^h,\bar{x})}{u^h_{x_i}(x^h,\bar{x})}}{1 + \sum_h \pi^h \frac{u^h_{\bar{x}_j}(x^h,\bar{x})}{u^h_{x_j}(x^h,\bar{x})}} = \frac{1 + \frac{\gamma F_j - \eta_j}{\eta_j}}{1 + \frac{\gamma F_i - \eta_i}{\eta_i}}$$

- Theorem: set p and q to satisfy this equation, then there exists an income I (i.e. lump sum tax/transfer) so that the agent chooses $x = x^*$.
  Proof: Same as before.

- Remarks

  - role of λ^i is very implicit, not in formula; lump sum taxes help here;
  - in general $T^i \neq T^j$ but this has nothing to do with externalities;
  - there are optima with $T^i = T^j = T$ for the right λ^i; (e.g. solve equilibrium fixed point with $T^i = T$ and setting p/q as in above formula then compute λ^i from social planner optimum);

- Conclusion: corrective tax on goods restores efficiency

3 More General

- Utility

$$u^i(x^i, \bar{x})$$
$$\bar{x} = \sum x^i \pi^i$$

- Technology

$$F(x + g, \bar{x} + g) = 0$$

  assume firms maximize profits using first argument

- Define

$$\bar{F}(\bar{x}) = F(\bar{x}, \bar{x})$$

- Social optimum

$$\max_x \sum \lambda^h u^h(x^h, \bar{x})\pi^h \qquad \bar{F}(\bar{x} + g) = 0 \qquad \bar{x} = \sum x^h \pi^h$$

  first order conditions are

$$\lambda^h u^h_{x_j}(x^h, \bar{x}) = \eta_j$$
$$\sum_h \lambda^h u^h_{\bar{x}_j}(x^h, \bar{x})\pi^h + \eta_j = \gamma \bar{F}_j(\bar{x} + g)$$

- substituting gives

$$\eta_j \sum_h \frac{u^h_{\bar{x}_j}(x^h, \bar{x})}{u^h_{x_j}(x^h, \bar{x})} \pi^h + \eta_j = \gamma \bar{F}_j(\bar{x} + g)$$

$$\Rightarrow \frac{\bar{F}_{x_i}}{\bar{F}_{x_j}} = \frac{u^h_{x_i}(x^h, \bar{x})}{u^h_{x_j}(x^h, \bar{x})} \frac{1 + \sum_h \pi^h \frac{u^h_{\bar{x}_i}(x^h,\bar{x})}{u^h_{x_i}(x^h,\bar{x})}}{1 + \sum_h \pi^h \frac{u^h_{\bar{x}_j}(x^h,\bar{x})}{u^h_{x_j}(x^h,\bar{x})}}$$

  and

$$\frac{\bar{F}_{x_i}}{\bar{F}_{x_j}} = \frac{F_{x_i}}{F_{x_j}} \frac{\frac{\bar{F}_{x_i}}{F_{x_i}}}{\frac{\bar{F}_{x_j}}{F_{x_j}}} = \frac{p_i}{p_j} \frac{\frac{\bar{F}_{x_i}}{F_{x_i}}}{\frac{\bar{F}_{x_j}}{F_{x_j}}}$$

- agent solves, takes $\bar{x}$ and T^i as given

$$\max_x u^i(x^i, \bar{x}) \qquad q \cdot x^i + T^i = 0$$

$$\Rightarrow u^i_x(x^i, \bar{x}) = \lambda q$$

$$\Rightarrow \frac{q_i}{q_j} = \frac{u^i_{x_i}(x^i, \bar{x})}{u^i_{x_j}(x^i, \bar{x})}$$

- To make

$$x^j = x^{j*}$$

  a necessary condition is that both conditions hold, implying

$$\frac{q_i}{q_j} = \frac{\eta_i}{\eta_j}$$

$$\frac{p_i/q_i}{p_j/q_j} = \frac{\frac{\bar{F}_{x_j}}{F_{x_j}}}{\frac{\bar{F}_{x_i}}{F_{x_i}}} \frac{1 + \sum_h \pi^h \frac{u^h_{\bar{x}_i}(x^h,\bar{x})}{u^h_{x_i}(x^h,\bar{x})}}{1 + \sum_h \pi^h \frac{u^h_{\bar{x}_j}(x^h,\bar{x})}{u^h_{x_j}(x^h,\bar{x})}}$$

4 Optimal Public Goods

- Samuelson (1969)
- we omitted g in utility function; let us add it

$$u^i(x^i, g)$$

  we ignore externalities now

- optimality condition
$$\sum_i \lambda^i u_g^i(x^i, g)\pi^h = \gamma F_g(x+g)$$

- rearranging
$$\sum_i \frac{u_g^h(x^h, g)}{u_{x_i}^h(x^h, g)}\pi^h = \frac{F_g(x+g)}{F_{x_i}(x+g)} = \frac{p_g}{p_{x_j}}$$

  avoids λ^i weights

5 Non Pigouvian Correction

- Diamond (1973) Bell journal paper
- relax assumption that externality is just aggregate consumption
  - congestion: some slow and some fast drivers
  - environment: clean and dirty cars
- simplifying assumptions:
  - ignore redistribution issues with quasilinear utility
  - focus on single good
  - constant producer price p
- Utility for agent h
$$U^h(\alpha^1, \alpha^2, \cdots, \alpha^H) + \mu^h$$
- note: initially not assumed atomostic (can approach that by making H very large)

5.1 Additively Separable

- suppose externality is additive
$$U^h = u^h(\alpha^h) + v^h(\alpha^1, \alpha^2, \cdots, \alpha^{h-1}, \alpha^{h+1}, \cdots, \alpha^n) + \mu^h$$
- agent has some demand
$$\max_{\alpha^h} \left(u^h(\alpha^h) + v^h(\alpha^1, \alpha^2, \cdots, \alpha^{h-1}, \alpha^{h+1}, \cdots, \alpha^n) + \mu^h\right)$$
$$(p+t)\alpha^h + \mu^h = m^h$$
$$\Rightarrow u^{h\prime}(\alpha^h) = p + t$$
$$\Rightarrow \alpha^{h*} = \alpha^h(p+t)$$

- Welfare criterion

$$W = \sum_h U^h(\alpha) + \sum \mu^h$$

subject to

$$p \sum \alpha^h + \sum \mu^h = \sum m^h$$

- indirect welfare

$$W(t) = \sum_h U^h(\alpha^1(p+t), \alpha^2(p+t), \ldots, \alpha^H(p+t)) - p \sum \alpha^h(p+t) + \sum_h m^h$$

$$\begin{aligned} W'(t) &= \sum_h \sum_i \frac{\partial U^h}{\partial \alpha_i} \alpha^{i\prime}(p+t) - p \sum \alpha^{h\prime}(p+t) \\ &= \sum_h \sum_{i \neq h} \frac{\partial U^h}{\partial \alpha_i} \alpha^{i\prime} - t \sum \alpha^{h\prime} \end{aligned}$$

setting $W'(t) = 0$ gives

$$t = \frac{-\sum_h \sum_{i \neq h} \frac{\partial U^h}{\partial \alpha_i} \alpha^{i\prime}}{\sum_h \alpha^{h\prime}}$$

weighted average of externalities $\sum_{i \neq h} \frac{\partial U^h}{\partial \alpha_i} \alpha^{i\prime}$, weighted by consumer demand derivatives

5.2 Demand Interactions

- without separability

$$\frac{\partial U^h}{\partial \alpha^h} = p + t$$

$$\Rightarrow \alpha^{h*} = \alpha^h(p+t, \alpha^1, \ldots, \alpha^{h-1}, \alpha^{h+1}, \ldots, \alpha^H)$$

demands affect each other:

  - fewer drivers on highway increase individual demand for driving

- for given price we need to think of the fixed point

$$\alpha^{h*} = \alpha^h(p+t, \alpha^{1*}, \ldots, \alpha^{h-1*}, \alpha^{h+1*}, \ldots, \alpha^{H*}) \qquad \forall h$$

a system of H equations in H unknowns

- same first order condition then applies for this fixed point relation
  - beware! not pure price elasiticies
  - some could be positive, so not weighted sum

- example

$$U^1(\alpha_1, \alpha_2) + \mu_1 = \alpha_1^{\frac{1}{2}} - \tfrac{1}{3}\alpha_2 + \mu_1$$
$$U^2(\alpha_2, \alpha_2) + \mu_2 = 0.3 \log (\alpha_2 + 0.9\alpha_2) - \alpha_1 + \mu_2.$$
$$\alpha_1 = 0.25(p + t)^{-2}$$
$$\alpha_2 = \text{Max}\{0.3(p + t)^{-1} - 0.9\alpha_1, 0\}.$$
$$W(t) = m_1 + m_2 + 0.3 \log 0.3 - 0.3 \log (p + t) + 0.1(p + t)^{-1} - 0.2(p + t)^{-2}.$$

  - with $p = 1$ maximum is at $t = 0$!
  - changing parameters, could be negative t...
  - Not weighted sum of externalities

5.3 An Aggregator

- imagine externality affects everyone equally

$$\gamma = \Gamma(\alpha^1, \ldots, \alpha^H)$$

  and utility

$$U^h(\alpha^h, \gamma) + \mu^h$$

- demand is then

$$\alpha^h(p + t, \gamma)$$

- fixed point again

$$\gamma = \Gamma(\alpha^1(p + t, \gamma), \ldots, \alpha^H(p + t, \gamma))$$

  one equation in one unknown

- implicit function theorem

$$\frac{d\gamma}{dt} = \frac{\sum \Gamma_h \frac{\partial \alpha^h}{\partial t}}{1 - \sum \Gamma_h \frac{\partial \alpha^h}{\partial \gamma}}$$

  denominator is feedback effect:

  - tax increases cost of driving...
  - ... but less drivers makes driving more attractive

- if agents take into account their own effect on γ then this is just a special case of what we did before

- assume agents ignore their effect on γ (justification: atomistic within agent h types as before)

$$\frac{\partial U^h}{\partial \alpha^h} = p + t$$

- welfare

$$W(t) = \sum_h U^h(\alpha^h(p+t), \gamma) - p\sum_h \alpha^h(p+t,\gamma) + \sum_h m^h$$

$$W'(t) = \sum_h \left(\frac{\partial}{\partial \alpha^h}U^h - p\right)\left(\frac{\partial}{\partial t}\alpha^h + \frac{d\gamma}{dt}\frac{\partial}{\partial \gamma}\alpha^h\right) + \frac{d\gamma}{dt}\sum_h \frac{\partial}{\partial \gamma}U^h$$

using first order condition

$$W'(t) = t\left(\sum_h \frac{\partial}{\partial t}\alpha^h + \frac{d\gamma}{dt}\sum_h \frac{\partial}{\partial \gamma}\alpha^h\right) + \frac{d\gamma}{dt}\sum_h \frac{\partial}{\partial \gamma}U^h$$

- hence at $t = 0$ we have $W'(0) > 0$ if $\frac{d\gamma}{dt}\sum_h \frac{\partial}{\partial \gamma}U^h > 0$
- solving gives

$$t^* = -\left(\sum \frac{\partial U^h}{\partial \gamma}\right)\left(\frac{\sum \Gamma_h \frac{\partial \alpha_h}{\partial t}}{\sum \frac{\partial \alpha_h}{\partial t}}\right)\frac{\sum \frac{\partial \alpha_h}{\partial t}}{\sum \frac{\partial \alpha_h}{\partial t} - \Delta}$$

and

$$\begin{aligned}\Delta &\equiv \sum \Gamma_h \frac{\partial \alpha_h}{\partial t}\sum \frac{\partial \alpha_h}{\partial \gamma} - \sum \Gamma_h \frac{\partial \alpha_h}{\partial \gamma}\sum \frac{\partial \alpha_h}{\partial t} \\ &= \left(\sum \frac{\partial \alpha_h}{\partial \gamma}\right)\left(\sum \frac{\partial \alpha_h}{\partial t}\right)\left(\sum \Gamma_h \frac{\frac{\partial \alpha_h}{\partial t}}{\sum \frac{\partial \alpha_h}{\partial t}} - \sum \Gamma_h \frac{\frac{\partial \alpha_h}{\partial \gamma}}{\sum \frac{\partial \alpha_h}{\partial \gamma}}\right)\end{aligned}$$

so Δ is the difference of two covariances of Γ_h with either $\frac{\frac{\partial \alpha_h}{\partial t}}{\sum \frac{\partial \alpha_h}{\partial t}}$ or $\frac{\frac{\partial \alpha_h}{\partial \gamma}}{\sum \frac{\partial \alpha_h}{\partial \gamma}}$.

- since Δ is typically not zero then tax is not weighted sum of Pigouvian tax

6 Rent Seeking

- we modeled externalities as affecting utility directly
- sometimes can think of affecting income
- example: rent seeking
  - output in some sector is $\mu(E)$ where E is total labor effort
  - payment (wage) for unit of effort is

$$\frac{\mu(E)}{E}$$

- actual marginal benefit is

$$\mu'(E) \neq \frac{\mu(E)}{E}$$

unless $\mu(E) = \bar{\mu}E$

- application: fishing in a particular bay, overfishing without tax
- here, formally, the distortion is that without taxes prices agents face are not equal to F_{x_j}
  - but similar conclusions to utility case
  - sometimes can substiute income into utility

7 Imperfect Instruments

- In Diamond we have **imperfect instruments** in that taxes are not agent specific
  - some agents may not pollute as much
  - but we charge everone the same tax
  - this leads to a 2nd best problem, unlike before where we attained the first best
- Another source of imperfect instruments: taxes are coarse
  - can only tax gasoline, or miles driven, not use of a particular road
  - can only tax total fish caught, not fishing in particular locations
- Rotschild-Scheuer (2011): application to rent seeking
- important: take into account "general equilibrium" effects from fixed point
- example: it may be that $d\gamma/dt = 0$
- Suppose constant output in rent-seeking sector

$$\mu(E) = \bar{\mu} = 1$$

- Suppose two types of agents
  - producers: $\theta = 1$ and $\varphi = 1$ (they can do both things)
  - rent-seekers: $\theta = 0$ and $\varphi = \varphi_R \in (0, 1$ (they can only rent seek)
- total rent-seeking effort is

$$E = \varphi_R e_R + \lambda_P e_P$$

where λ_P is the fraction of producers that rent-seek and e_P is their effort (rent seekers only work in rent-seeking sector)

- productive agents may be...
  - indifferent, if $E = 1$, since then $\mu(E)/E = 1/E = 1$
  - not working in rent seeking sector if $E > 1$
  - working only in rent seeking sector if $E < 1$
- I'll focus on the interior equilibrium (one can show that this is where the optimum lies)

$$E = \varphi_R e_R + \mu_P e_P = 1$$
$$\Rightarrow \lambda_P(e_R, e_P) = \frac{1 - \varphi_R e_R}{e_P}$$

- output is the sum of rent seeking output production

$$\bar{\mu} + (1 - \lambda_P(e_R, e_P))e_P$$

- a Utilitarian will maximize output net of effort

$$W = \bar{\mu} + (1 - \lambda_P(e_R, e_P))e_P - \frac{1}{\gamma}e_P^\gamma - \frac{1}{\gamma}e_R^\gamma$$
$$= e_p + \varphi_R e_R - \frac{1}{\gamma}e_P^\gamma - \frac{1}{\gamma}e_R^\gamma$$

- Imagine it can control efforts, one motivation for this is that it controls a linear tax on each agent type (see below). Note, it cannot control the occupational choice.
- The first order conditions are then

$$e_p^{\gamma-1} = 1$$
$$e_R^{\gamma-1} = \varphi_R$$

  (this gives $e_p = 1$ and $e_R = \varphi_R^{\frac{1}{\gamma-1}} < 1$ so the equilibrium is indeed interior with $\lambda_P \in (0, 1)$)
- Since the wage is $w = 1$ for producers and $w = \varphi_R$ for rent seekers, these first order conditions coincide with that of the agent facing no distortionary taxes

$$\tau_p = 0$$
$$\tau_R = 0$$

8 A Decomposition

- Kopczuk (2003) provides a two step "as if" result:
  - first, correct the Pigouvian tax
  - then, solve the remaining tax problem
- A useful way to think or decompose things, even though it doesn't necessarily lead to any clear results without more structure
- Considers the following problem

$$\max_{t,T,\bar{x}} v(t, T; P, p; \bar{x})$$

$$X(t, T; P, p) = \bar{x}$$

$$R = I(t, T; P, p) + t\bar{x}$$

$$G(t, T) = 0$$

- The last constraint captures some constraints on taxes. For example, this could incorporate that taxes on some other goods have to equal t, so that the tax on X is not perfectly targeted as in Diamond (1973, RAND).
- For any baseline tax t^p we can change variables $t = t^p + s$ and write

$$\max_{s,T,\bar{x}} v(t^p + s, T; P, p; \bar{x})$$

$$X(t^p + s, T; P, p) = \bar{x}$$

$$R - t^p\bar{x} = I(t^p + s, T; P, p) + sX(t^p + s, T; P, p)$$

$$G(t^p + s, T) = 0$$

- Of course only $t^* = t^p + s^*$ is determined i.e. s^* varies one for one with t^p
- The first order condition for $\bar{x}$ is

$$v_x(t^*, T^*; P, p; \bar{x}) + \lambda^* + \mu^* t^p = 0$$

- Now <u>*define*</u> the tax t^p to ensure that $\lambda^* = 0$, so that

$$t^p = -\frac{v_x(t^*, T^*; P, p; \bar{x})}{\mu^*}$$

- With this value of t^p we get the same first order conditions if we look at the system

$$\max_{s,T,\bar{x}} v(t^p + s, T; P, p; \bar{x})$$

$$R - t^p\bar{x} = I(t^p + s, T; P, p) + sX(t^p + s, T; P, p)$$

$$G(t^p + s, T) = 0$$

- We can drop $\bar{x}$ from the maximization and we can rewrite this as

$$\max_{s,T} v(s, T; P, p + t^p; \bar{x})$$

$$R - t^p \bar{x} = I(t^p, T; P, p + t^p) + sX(s, T; P, p + t^p)$$
$$G(t^p + s, T) = 0$$

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