

# 14.452 Economic Growth: Lecture 11, Human Capital, Technology Diffusion and Interdependencies

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# The Role of Human Capital

- We have already seen the importance of human capital in the context of the regression analyses and calibration exercises of the Solow growth model.
- Key questions: incentives to accumulate and invest in human capital.
  - See labor economics.
- Here: how does human capital affect growth?
  - Increasing efficiency units of labor.
  - Improving technology.

# Human Capital and Technology

- Perhaps major role of human capital is in improving technology.
- Why? Two possibilities
- Skilled workers are the “innovators”—we can model this once we understand endogenous technological progress.
- Skilled workers are better at adopting technologies or adapting to change, in particular, technological change.
  - Ted Schultz in the context on agricultural innovations.
  - Nelson and Phelps: a simple model.
    - Our first model of “endogenous technology”.

# Nelson-Phelps Model of Human Capital I

- Major role of human capital is not to increase productivity in existing tasks, but to enable workers to cope with change, disruptions and especially new technologies.
- Continuous time model.
- Output is given by

$$Y(t) = A(t)L, \quad (1)$$

- $L$  is the constant labor force, supplying its labor inelastically, and  $A(t)$  is the technology level of the economy.
- No capital and also no labor supply margin.
- The only variable that changes over time is technology  $A(t)$ .
- World technological frontier is given by  $A_F(t)$ .

## Nelson-Phelps Model of Human Capital II

- $A_F(t)$  evolves exogenously according to the differential equation

$$\frac{\dot{A}_F(t)}{A_F(t)} = g_F,$$

with initial condition  $A_F(0) > 0$ .

- Human capital of the workforce denoted by  $h$ .
- This human capital does not feature in the production function, (1).
- Evolution of the technology in use,  $A(t)$ , is governed by the differential equation

$$\dot{A}(t) = gA(t) + \phi(h)A_F(t),$$

with initial condition  $A(0) \in (0, A_F(0))$ .

- Parameter  $g$  is strictly less than  $g_F$  and measures the growth rate of technology  $A(t)$ .

## Nelson-Phelps Model of Human Capital III

- Assume that  $\phi(\cdot)$  is increasing, with

$$\phi(0) = 0 \text{ and } \phi(h) = g_F - g > 0 \text{ for all } h \geq \bar{h},$$

where  $\bar{h} > 0$ .

- Since  $A_F(t) = \exp(g_F t) A_F(0)$ , the differential equation for  $A(t)$  can be written as

$$\dot{A}(t) = gA(t) + \phi(h) A_F(0) \exp(g_F t).$$

- Solving this differential equation,

$$A(t) = \left[ \left( \frac{A(0)}{g} - \frac{\phi(h) A_F(0)}{g_F - g} \right) \exp(gt) + \frac{\phi(h) A_F(0)}{g_F - g} \exp(g_F t) \right],$$

- Thus growth rate of  $A(t)$  is faster when  $\phi(h)$  is higher.

## Nelson-Phelps Model of Human Capital IV

- Moreover, it can be verified that

$$A(t) \rightarrow \frac{\phi(h)}{g_F - g} A_F(t),$$

- Thus ratio of the technology in use to the frontier technology is also determined by human capital.
- This role of human capital is undoubtedly important in a number of situations:
  - educated farmers are more likely to adopt new technologies and seeds (e.g., Foster and Rosenzweig, 1995).
  - stronger correlation between economic growth and levels of human capital than between economic growth and changes in human capital.
- Human capital could be playing a more major role in economic growth and development than the discussion so far has suggested.

# Nelson-Phelps Model of Human Capital V

- But:
  - If taking place within the firm's boundaries, this will be reflected in the marginal product of more skilled workers and taken into account in estimations.
  - If at the level of the labor market, this would be a form of local human capital externalities and it should have shown up in the estimates on local external effects of human capital.
  - So unless is also external and these external effects work at a global level, should not be seriously underestimating the contribution of human capital.

# Introduction I

- In the models thus far each country is treated as an “island”; its technology is either exogenous or endogenously generated within its boundaries.
- A framework in which *frontier* technologies are produced in advanced economies and then copied or adopted by “follower” countries provides a better approximation.
- Thus, should not only focus on differential rates of endogenous technology generation but on *technology adoption* and *efficient technology use*.
- Exogenous growth models have this feature, but technology is exogenous. Decisions in these models only concern investment in physical capital. In reality, technological advances at the world level are not “mana from heaven”.

# Introduction II

- Technology adoption involves many challenging features:
  - Even within a single country, we observe considerable differences in the technologies used by different firms.
  - It is difficult to explain how in the globalized world some countries may fail to import and use technologies.

# Review: Productivity and Technology Differences within Narrow Sectors I

- Longitudinal micro-data studies (often for manufacturing): even within a narrow sector there are significant and persistent productivity differences across plants.
- Little consensus on the causes.
  - Correlation between plant productivity and plant or firm size, various measures of technology (in particular IT technology), capital intensity, the skill level of the workforce.
  - But these correlations cannot be taken to be causal.
- But technology differences appear to be an important factor.
- A key determinant seems to be the skill level of the workforce, though adoption of new technology does not typically lead to a significant change in employment structure.

## Review: Productivity and Technology Differences within Narrow Sectors II

- Productivity differences appear to be related to the entry of new and more productive plants and the exit of less productive plants (recall Schumpeterian models).
- But entry and exit account for only about 25% of average TFP growth, with the remaining accounted for by continuing plants.
- Thus models in which firms continually invest in technology and productivity are important for understanding differences across firms and plants and also across countries.

# Technology Diffusion I

- Despite technology and productivity differences among firms in similar circumstances, cross-sectional distributions of productivity and technology are not stationary.
- New and more productive technologies diffuse over time.
- Griliches's (1957) study of the adoption of hybrid corn in the US (findings confirmed by others):
  - Slow diffusion affected by local economic conditions.
  - Likelihood of adoption related to the contribution of the hybrid corn in a particular area, the market size and the skill level.
  - S-shape of diffusion.

# Technology Diffusion II

- Important lessons:
  - Differences are not only present across countries, but also within countries.
  - Even within countries better technologies do not immediately get adopted by all firms.
- But note causes of within-country and cross-country productivity and technology differences might be different:
  - e.g., within-countries might be due to differences in managerial ability or to the success of the match between the manager and the technology.

# Technology Diffusion: Exogenous World Growth Rate I

- Endogenous technological change model with expanding machine variety and lab equipment specification.
- Aggregate production function of economy  $j = 1, \dots, J$  at time  $t$ :

$$Y_j(t) = \frac{1}{1-\beta} \left[ \int_0^{N_j(t)} x_j(v, t)^{1-\beta} dv \right] L_j^\beta, \quad (2)$$

- $L_j$  is constant over time,  $x$ 's depreciate fully after use.
- Each variety in economy  $j$  is owned by a technology monopolist; sells machines embodying this technology at the profit maximizing (rental) price  $\chi_j(v, t)$ .
- Monopolist can produce each unit of the machine at a cost of  $\psi \equiv 1 - \beta$  units on the final good.

## Technology Diffusion: Exogenous World Growth Rate II

- No international trade, so firms in country  $j$  can only use technologies supplied by technology monopolists in their country.
- Each country admits a representative household with the same preferences as before except  $n_j = 0$  for all  $j$ .
- Resource constraint for each country:

$$C_j(t) + X_j(t) + \zeta_j Z_j(t) \leq Y_j(t), \quad (3)$$

- $\zeta_j$ : potential source of differences in the cost of technology adoption across countries (institutional barriers as in Parente and Prescott, subsidies to R&D and to technology, or other tax policies).

# Technology Diffusion: Exogenous World Growth Rate III

- *Innovation possibilities frontier:*

$$\dot{N}_j(t) = \eta_j \left( \frac{N(t)}{N_j(t)} \right)^\phi Z_j(t), \quad (4)$$

where  $\eta_j > 0$  for all  $j$ , and  $\phi > 0$  and is common to all economies.

- World technology frontier of varieties expands at an exogenous rate  $g > 0$ , i.e.,

$$\dot{N}(t) = gN(t). \quad (5)$$

- Flow profits of a technology monopolist at time  $t$  in economy  $j$ :

$$\pi_j(t) = \beta L_j.$$

# Steady State Equilibrium I

- Suppose a steady-state (balanced growth path) equilibrium exists in which  $r_j(t)$  is constant at  $r_j^* > 0$ . Then the net present discounted value of a new machine is:

$$V_j^* = \frac{\beta L_j}{r_j^*}.$$

- If the steady state involves the same rate of growth in each country, then  $N_j(t)$  will also grow at the rate  $g$ , so that  $N_j(t) / N(t)$  will remain constant, say at  $v_j^*$ .

## Steady State Equilibrium II

- In that case, an additional unit of technology spending will create benefits equal to  $\eta_j (v_j^*)^{-\phi} V_j^*$  counterbalanced against the cost of  $\zeta_j$ . Free-entry (with positive activity) then requires

$$v_j^* = \left( \frac{\eta_j \beta L_j}{\zeta_j r^*} \right)^{1/\phi}, \quad (6)$$

where given the preferences, equal growth rate across countries implies that  $r_j^*$  will be the same in all countries ( $r^* = \rho + \theta g$ ).

## Steady State Equilibrium III

- Higher  $v_j$  implies that country  $j$  is technologically more advanced and thus richer
- Thus (6) shows that countries with higher  $\eta_j$  and lower  $\zeta_j$ , will be more advanced and richer.
- A country with a greater labor force will also be richer (scale effect): more demand for machines, making R&D more profitable.

## Summary of Equilibrium

**Proposition** Consider the model with endogenous technology adoption described in this section. Suppose that  $\rho > (1 - \theta)g$ . Then there exists a unique steady-state world equilibrium in which relative technology levels are given by (6) and all countries grow at the same rate  $g > 0$ .

Moreover, this steady-state equilibrium is globally saddle-path stable, in the sense that starting with any strictly positive vector of initial conditions  $N(0)$  and  $(N_1(0), \dots, N_J(0))$ , the equilibrium path of  $(N_1(t), \dots, N_J(t))$  converges to  $(v_1^* N(t), \dots, v_J^* N(t))$ .

# Technology Diffusion: Endogenous World Growth Rate I

- More satisfactory to derive the world growth rate from the technology adoption and R&D activities of each country.
- Modeling difficulties:
  - Degree of interaction among countries is now greater.
  - More care needed so that the world economy grows at a constant endogenous rate, while there are still forces that ensure relatively similar growth rates across countries. Modeling choice:
    - Countries grow at permanently different long run rates, e.g. to approximate long-run growth differences of the past 200 or 500 years
    - Countries grow at similar rates, e.g. like the past 60 years or so.
- Since long-run differences emerge straightforwardly in many models, focus here on forces that will keep countries growing at similar rates.

## Technology Diffusion: Endogenous World Growth Rate II

- Replace the world growth equation (5) with:

$$N(t) = \frac{1}{J} \sum_{j=1}^J N_j(t). \quad (7)$$

- $N(t)$  is no longer the “world technology frontier”: it represents average technology in the world, so  $N_j(t) > N(t)$  for at least some  $j$ .
- Disadvantage of the formulation: contribution of each country to the world technology is the same. But qualitative results here do not depend on this.
- Main result: pattern of cross-country growth will be similar to that in the previous model, but the growth rate of the world economy,  $g$ , will be endogenous, resulting from the investments in technologies made by firms in each country.

# Steady State Equilibrium I

- Suppose there exists a steady-state world equilibrium in which each country grows at the rate  $g$ .
- Then, (7) implies  $N(t)$  will also grow at  $g$ .
- The net present discounted value of a new machine in country  $j$  is

$$\frac{\beta L_j}{r^*},$$

- No-arbitrage condition in R&D investments: for given  $g$ , each country  $j$ 's relative technology,  $v_j^*$ , should satisfy (6).

## Steady State Equilibrium II

- Dividing both sides of (7) by  $N(t)$  implies that in the steady-state world equilibrium:

$$\frac{1}{J} \sum_{j=1}^J v_j^* = 1$$
$$\frac{1}{J} \sum_{j=1}^J \left( \frac{\eta_j \beta L_j}{\zeta_j (\rho + \theta g)} \right)^{1/\phi} = 1, \quad (8)$$

which uses  $v_j^*$  from (6) and substitutes for  $r^*$  as a function of the world growth rate.

- The only unknown in (8) is  $g$ .
- Moreover, the left-hand side is clearly strictly decreasing in  $g$ , so it can be satisfied for at most one value of  $g$ , say  $g^*$ .

## Steady State Equilibrium III

- A well-behaved world equilibrium would require the growth rates to be positive and not so high as to violate the transversality condition. The following condition is necessary and sufficient for the world growth rate to be positive:

$$\frac{1}{J} \sum_{j=1}^J \left( \frac{\eta_j \beta L_j}{\zeta_j \rho} \right)^{1/\phi} > 1. \quad (9)$$

- By usual arguments, when this condition is satisfied, there will exist a unique  $g^* > 0$  that will satisfy (8) (if this condition were violated, (8) would not hold, and we would have  $g = 0$  as the world growth rate).

# Summary of Steady State Equilibrium

**Proposition** Suppose that (9) holds and that the solution  $g^*$  to (8) satisfies  $\rho > (1 - \theta)g^*$ . Then there exists a unique steady-state world equilibrium in which growth at the world level is given by  $g^*$  and all countries grow at this common rate. This growth rate is endogenous and is determined by the technologies and policies of each country. In particular, a higher  $\eta_j$  or  $L_j$  or a lower  $\zeta_j$  for any country  $j = 1, \dots, J$  increases the world growth rate.

## Remarks

- Taking the world growth rate given, the structure of the equilibrium is very similar to that before.
- The same model now gives us an “endogenous” growth rate for the world economy. Growth for each country appears “exogenous”, but the growth rate of the world economy is endogenous.
- Technological progress and economic growth are the outcome of investments by all countries in the world, but there are sufficiently powerful forces in the world economy through technological spillovers that pull relatively backward countries towards the world average, ensuring equal long-run growth rates for all countries in the long run.
- Equal growth rates are still consistent with *large level differences* across countries.
- Several simplifying assumptions: same discount rates and focus on steady-state equilibria (transitional dynamics are now more complicated, since the “block recursiveness” of the dynamical system is lost).

# Trade, Specialization and the World Income Distribution

- Similar interdependences because of trade.
- Model based on Acemoglu and Ventura (2001). Ricardian features: each country will specialize in subset of available goods and affect their prices.
- Hence each country's terms of trade will be endogenous and depend on the rate at which it accumulates capital.
- Model can allow for differences in discount (and saving) rates and has richer comparative static results.
- Also now exhibit endogenous growth, determined by the investment decisions of all countries.
- International trade (without any technological spillovers) will create sufficient interactions to ensure a common long-run growth rate.

# Basics I

- $J$  of “small” countries,  $j = 1, \dots, J$ .
- Continuum of intermediate products  $\nu \in [0, N]$ .
- Two final products used for consumption and investment.
- Free trade in intermediate goods and no trade in final products or assets (rule out international borrowing and lending).
- Each country has constant population normalized to 1.
- Country  $j$  will be defined by  $(\mu_j, \rho_j, \zeta_j)$ , vary across countries but constant over time:
  - $\mu$ : indicator of how advanced the technology of the country is,
  - $\rho$ : rate of time preference, and
  - $\zeta$ : measure the effect of policies and institutions on the incentives to invest.

## Basics II

- All countries admit a representative household with utility function:

$$\int_0^{\infty} \exp(-\rho_j t) \ln C_j(t) dt, \quad (10)$$

- Country  $j$  starts with a capital stock of  $K_j(0) > 0$  at time  $t = 0$ .
- Budget constraint of representative household in country  $j$  at time  $t$ :

$$\begin{aligned} p_j^I(t) \dot{K}_j(t) + p_j^C C_j(t) &= Y_j(t) \\ &= r_j(t) K_j(t) + w_j(t), \end{aligned} \quad (11)$$

- Because consumption and investment goods are not traded, their prices might differ across countries.
- Notice equation (11) imposes no depreciation.
- Consumption and investment goods have different production technologies and thus their prices will differ.

## Basics III

- *Armington* preferences or technology:  $N$  intermediates partitioned such that each intermediate can only be produced by one country.
- While each country is small in import markets, it will affect its own terms of trades by the amount of the goods it exports.
- Denoting the measure of goods produced by country  $j$  by  $\mu_j$ :

$$\sum_{j=1}^J \mu_j = N. \quad (12)$$

- A higher level of  $\mu_j$  implies country  $j$  has the technology to produce a larger variety of intermediates.

## Basics IV

- Intermediates produced competitively.
- In each country one unit of capital produces one unit of any of the intermediates that the country is capable of producing.
- Free entry to the production of intermediates.
- Hence prices of all intermediates

$$p_j(t) = r_j(t), \quad (13)$$

# The AK Model I

- Simplified version where capital is the only factor of production.
- In (11) we have  $w_j(t) = 0$ :

$$Y_j(t) = r_j(t) K_j(t).$$

- Consumption and investment goods produced using domestic capital and a bundle of all the intermediate goods in the world.
- Production function for consumption goods:

$$C_j(t) = \chi K_j^C(t)^{1-\tau} \left( \int_0^N x_j^C(t, \nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\frac{\tau\varepsilon}{\varepsilon-1}}. \quad (14)$$

# The AK Model II

- Note:

- $K_j^C$  = “non-traded” component; if a country has low  $K_j^C$ , relative price of capital will be high and less of it will be used
- Term in parentheses represents bundle of intermediates purchased from the world economy.
- Throughout assume

$$\varepsilon > 1,$$

which avoids the counterfactual and counterintuitive pattern of “immiserizing growth”.

- Exponent  $\tau$  ensures constant returns to scale.  $\tau$  is also the share of trade in GDP for all countries.
  - $\chi$  is introduced for normalization.
- Production function for investment goods:

$$I_j(t) = \zeta_j^{-1} \chi K_j^I(t)^{1-\tau} \left( \int_0^N x_j^I(t, \nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\frac{\tau\varepsilon}{\varepsilon-1}}, \quad (15)$$

## The AK Model III

- Term  $\zeta_j$  allows differential levels of productivity in production of investment goods across countries:
  - Consistent with results on relative prices of investment goods
  - May think of greater distortions as higher  $\zeta_j$  (higher  $\zeta_j$  reduce output and increase relative price of investment goods).
- Market clearing for capital:

$$K_j^C(t) + K_j^I(t) + K_j^H(t) \leq K_j(t), \quad (16)$$

where  $K_j^H(t)$  capital used in the production of intermediates and  $K_j(t)$  is total capital stock of country  $j$  at time  $t$ .

- *AK* version: production goods uses capital and intermediates that are produced from capital. Doubling capital stock will double the output of intermediates and of consumption and investment goods.

## The AK Model IV

- *Unit cost functions*: cost of producing one unit of consumption and investment goods in terms of the numeraire.
- Production functions (14) and (15) are equivalent to unit cost functions for consumption and production:

$$B_j^C \left( r_j(t), [p(t, \nu)]_{\nu \in [0, N]} \right) = r_j(t)^{1-\tau} \left[ \left( \int_0^N p(t, \nu)^{1-\varepsilon} d\nu \right)^{\frac{\tau}{1-\varepsilon}} \right], \quad (17)$$

$$B_j^I \left( r_j(t), [p(t, \nu)]_{\nu \in [0, N]} \right) = \zeta_j r_j(t)^{1-\tau} \left[ \left( \int_0^N p(t, \nu)^{1-\varepsilon} d\nu \right)^{\frac{\tau}{1-\varepsilon}} \right], \quad (18)$$

where  $p(t, \nu)$  is the price of the intermediate  $\nu$  at time  $t$  and the constant  $\chi$  in (14) and (15) is chosen appropriately.

- These prices not indexed by  $j$ , since there is free trade in intermediates.

# The AK Model V

- World equilibrium: sequence of prices, capital stock levels and consumption levels for each country, such that all markets clear and the representative household in each country maximizes his utility given the price sequences,

$$\left[ \left\{ p_j^C(t), p_j^I(t), r_j(t), K_j(t), C_j(t) \right\}_{j=1}^J, [p(t, \nu)]_{\nu \in [0, N]} \right]_{t \geq 0}.$$

- Steady-state world equilibrium defined as usual, in particular, requiring that all prices are constant.
- Maximization of the representative household, i.e. of (10) subject to (11) for each  $j$  yields Euler equation:

$$\frac{r_j(t) + \dot{p}_j^I(t)}{p_j^I(t)} - \frac{\dot{p}_j^C(t)}{p_j^C(t)} = \rho_j + \frac{\dot{C}_j(t)}{C_j(t)} \quad (19)$$

## The AK Model VI

- Euler requires (net) rate of return to capital to be equal to rate of time preference plus slope of the consumption path.
- Difference from standard Euler stems: potentially different technologies for producing consumption and investment, thus change in their relative price—term  $\dot{p}_j^I(t) / p_j^I(t) - \dot{p}_j^C(t) / p_j^C(t)$ .
- Transversality condition:

$$\lim_{t \rightarrow \infty} \exp(-\rho_j t) \frac{p_j^I(t) K_j(t)}{p_j^C(t) C_j(t)} = 0, \quad (20)$$

for each  $j$ .

- Integrating budget constraint and using the Euler and transversality conditions, consumption function:

$$p_j^C(t) C_j(t) = \rho_j p_j^I(t) K_j(t), \quad (21)$$

## The AK Model VII

- Individuals spend a fraction  $\rho_j$  of their wealth on consumption at every instant.
- Define the numeraire for this world economy as the ideal price index for the basket of all the (traded) intermediates:

$$\begin{aligned}
 1 &= \left[ \int_0^N p(t, \nu)^{1-\varepsilon} d\nu \right]^{\frac{1}{1-\varepsilon}} \\
 &= \sum_{j=1}^J \mu_j p_j(t)^{1-\varepsilon}.
 \end{aligned} \tag{22}$$

- Since each country is small it exports practically all of its production of intermediates and imports the ideal basket of intermediates.
- Thus  $p_j(t) = r_j(t)$  is not only the price of intermediates produced by  $j$ , but also its *terms of trade*.

## The AK Model VIII

- Using the price normalization in (22), (17) and (18) imply:

$$p_j^C(t) = r_j(t)^{1-\tau} \quad \text{and} \quad p_j^I(t) = \zeta_j r_j(t)^{1-\tau}. \quad (23)$$

- To compute the rate of return to capital, need to impose market clearing for capital in each country and have a trade balance equation for each country.
- By Walras' law enough to use the trade balance equation:

$$Y_j(t) = \mu_j r_j(t)^{1-\varepsilon} Y(t), \quad (24)$$

where  $Y(t) \equiv \sum_{j=1}^J Y_j(t)$  is total world income at time  $t$ . Here,

- Each country spends  $\tau$  of its income on intermediates, and, since it is small, on imports.
- The rest of the world spends a fraction  $\tau \mu_j r_j(t)^{1-\varepsilon}$  of its income on intermediates produced by country  $j$  (follows from CES and that  $p_j(t) = r_j(t)$  is the relative price of each country  $j$  intermediate and there are  $\mu_j$  of them).

# The AK Model IX

- (13), (21), (23) and (24) together with the resource constraint, (11), characterize the world equilibrium fully.
- Distribution of capital stocks across the  $J$  economies, combining (11), (21) and (23) on the one hand, and (11) and (24) on the other:

$$\frac{\dot{K}_j(t)}{K_j(t)} = \frac{r_j(t)^\tau}{\zeta_j} - \rho_j, \quad (25)$$

$$r_j(t) K_j(t) = \mu_j r_j(t)^{1-\varepsilon} \sum_{i=1}^J r_i(t) K_i(t). \quad (26)$$

## Proposition: Steady State Equilibrium

There exists a unique steady-state world equilibrium where

$$\frac{\dot{K}_j(t)}{K_j(t)} = \frac{\dot{Y}_j(t)}{Y_j(t)} = g^* \quad (27)$$

for  $j = 1, \dots, J$ , and the world steady-state growth rate  $g^*$  is the unique solution to

$$\sum_{j=1}^J \mu_j \left[ \zeta_j \left( \rho_j + g^* \right) \right]^{(1-\varepsilon)/\tau} = 1. \quad (28)$$

The steady-state rental rate of capital and the terms of trade in country  $j$  are given by

$$r_j^* = p_j^* = \left[ \zeta_j \left( \rho_j + g^* \right) \right]^{1/\tau}. \quad (29)$$

This unique steady-state equilibrium is globally saddle-path stable.

# Proof of Proposition (Sketch): Steady State Equilibrium I

- A steady-state equilibrium must have constant prices, thus constant  $r_j^*$ .
- This implies that in any state state, for each  $j = 1, \dots, J$ ,  $\dot{K}_j(t) / K_j(t)$  must grow at some constant rate  $g_j$ .
- Suppose these rates are not equal for two countries  $j$  and  $j'$ .
  - Taking the ratio of equation (26) for these two countries yields a contradiction, establishing that  $\dot{K}_j(t) / K_j(t)$  is constant for all countries.
- Equation (24) then implies that all countries also grow at this common rate, say  $g^*$ . Given this common growth rate, (25) immediately implies (29). Substituting this back into (26) gives (28).

## Proof of Proposition (Sketch): Steady State Equilibrium II

- Since these equations are all uniquely determined and (28) is strictly decreasing in  $g^*$ , thus has a unique solution, the steady-state world equilibrium is unique.
- To establish global stability, it suffices to note that (26) implies that  $r_j(t)$  is decreasing in  $K_j(t)$ .
- Thus whenever a country has a high capital stock relative to the world, it has a lower rate of return on capital, which from (25) slows down the process of capital accumulation in that country.

## Discussion of Proposition: Steady State Equilibrium I

- Despite the high degree of interaction among the various economies, there exists a unique globally stable steady-state world equilibrium.
- Equilibrium takes a relatively simple form.
- All countries grow at the same rate  $g^*$ .
  - Surprising, since each economy has  $AK$  technology, and without any international trade, each country would grow at a different rate (e.g., those with lower  $\zeta_j$ 's or  $\rho_j$ 's would have higher growth rates).
  - International trade keeps countries together, and leads to a stable world income distribution.
- Intuition of third result: terms of trade effects encapsulated in equation (26).

## Discussion of Proposition: Steady State Equilibrium II

- Consider special case  $\mu_j = \mu$  for all  $j$  and  $j$  has lower  $\zeta_j$  and  $\rho_j$  than the rest of the world.
  - Then (25) implies  $j$  will tend to accumulate more capital than others.
  - But (26) implies this cannot go on forever and  $j$ , being richer than the world average, will have a lower rate of return on capital.
  - This will compensate the greater incentive to accumulate and accumulation in  $j$  converges back to the rate of the world.
- Each country is “small” relative to the world, but has market power in the goods that it supplies.
- Hence when a country accumulates faster it will face *worsening terms of trades*.
  - This will reduce the income of the country that is accumulating faster.
  - Dynamic effects: (13) shows it also experiences a decline in the rate of return the capital and in the interest rate, that slows down its rate of capital accumulation.

## Discussion of Proposition: Steady State Equilibrium III

- Let  $y_j^* \equiv Y_j(t) / Y(t)$  the the relative income of country  $j$  in steady state. Then equations (24) and (29) immediately imply that

$$y_j^* = \mu_j \left[ \zeta_j \left( \rho_j + g^* \right) \right]^{(1-\varepsilon)/\tau} . \quad (30)$$

- Growth at a common rate does not imply same level of income:
  - Countries with better technology (high  $\mu_j$ ), lower distortions (low  $\zeta_j$ ) and lower discount rates (low  $\rho_j$ ) will be relatively richer.
  - Elasticity of income with respect to  $\zeta_j$  and  $\rho_j$  depends on elasticity of substitution between the intermediates,  $\varepsilon$ , and degree of openness,  $\tau$ .
  - When  $\varepsilon$  is high and  $\tau$  is relatively low, small differences in  $\zeta_j$ 's and  $\rho_j$ 's can lead to very large differences in income across countries.
- Recall that in a world with a Cobb-Douglas aggregate production function and no human capital differences, the Solow model implies:

$$y_j^* = A_j \left( \frac{s_j}{g^*} \right)^{\alpha/(1-\alpha)} , \quad (31)$$

## Discussion of Proposition: Steady State Equilibrium IV

- Equation (30) shows similar implications, except that:
  - the role of the labor-augmenting technologies is played by the technological capabilities of the country, which determine the range of goods in which it has a comparative advantage;
  - the role of the saving rate is played by the discount rate  $\rho_j$  and the policy parameter affecting the distortions on the production of investment goods,  $\zeta_j$ ;
  - instead of the share of capital in national income, the elasticity of substitution between intermediates and the degree of trade openness affects how spread out the world income distribution is.

# Main Lessons I

- We can make considerable progress in understanding technology and productivity differences across nations by positing a slow process of technology transfer across countries.
- It seems reasonable to assume that technologically backward economies will only slowly catch up to those at the frontier.
- An important element of models of technology diffusion is that they create a built-in advantage for countries (or firms) that are relatively behind
- This catch-up advantage for backward economies ensures that models of slow technology diffusion will lead to differences in income levels, not necessarily in growth rates.

## Main Lessons II

- Thus a study of technology diffusion enables us to develop a model of world income distribution, whereby the position of each country in the world income distribution is determined by their ability to absorb new technologies from the world frontier.
- This machinery is also useful in enabling us to build a framework in which, while each country may act as a neoclassical exogenous growth economy, importing its technology from the world frontier, the entire world behaves as an endogenous growth economy, with its growth rate determined by the investment in R&D decisions of all the firms in the world.
- Technological interdependences across countries implies that we should often consider the world equilibrium, not simply the equilibrium of each country on its own.

## Main Lessons III

- Similar issues, arise because of trade interactions. Ricardian trade leads to similar dynamics to those obtained from ecological interdependencies.
- More realistic and richer trade models lead to a more complex dynamics.
- Furthermore, trade and technological spillovers likely interact.

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## 14.452 Economic Growth

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