

14.451 Problem Set 1

Fall 2009

Due in class on September 22.

1 Preliminaries

1. Banach spaces. Exercises from SLP: 3.2.c, 3.2.e, 3.3.d, 3.4.d, 3.5 and 3.6. (You should cover all the parts of 3.2, 3.3 and 3.4 but you don't need to hand them in).
2. Contraction mapping. Exercise 3.9 from SLP.
3. Theorem of Maximum. Exercise 3.16 from SLP.

2 Optimal saving in finite time

Consider the T -period optimal saving problem seen in class. Assume that the (gross) interest rate is equal to the inverse of the discount factor: $\beta(1+r) = 1$.

We want to prove that the value function takes the following form

$$V_t(a) = \frac{1 - \beta^{T-t+1}}{1 - \beta} u \left(y + \frac{1 - \beta}{1 - \beta^{T-t+1}} (1+r)a \right)$$

and the optimal policy is to set consumption and next period bond holdings following the rules

$$C_t(a) = y + \frac{1 - \beta}{1 - \beta^{T-t+1}} (1+r)a,$$

$$A_t(a) = (1+r)a + y - C_t(a).$$

1. Prove that the value function and policy above are correct for $t = T$ (it's trivial, sure, but we need to start somewhere).
2. Prove by induction that the value function and the policy are correct for all $t < T$. That is, assume they are correct for t and prove they are correct for $t - 1$.
3. What happens as $T \rightarrow \infty$?

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