1. Answer two questions out of the three.

14.382 Econometrics I
Final Examination
Spring, May 2004
(Professor Jerry Hausman)

INSTRUCTIONS: (2 hour final exam)

- 2. Let  $y = \beta_1 x_1 + \epsilon$  where  $x_1 = x_1^* + v$  where  $Ev = 0, E(x_{1i}v_i) = 0, E(E_iv_i) = 0, E(\epsilon_i x_{1i}^*) = 0$ .
  - (i) Suppose you do least squares. Derive the plim of  $\hat{\beta}$  and demonstrate "attenuation bias." ("iron law" of econometrics)
- (ii) Suppose you have an instrument z. What properties must z have to be a valid instrument? Give a proof that the IV estimation is consistent.
- (iii) Suppose the specification is  $y_1 = \beta_1 x_1 + \beta_2 x_2 + \epsilon$ , where  $Cov(x_1, x_2) \neq 0$  and  $E(x_{2i}v_i) = 0$ ,  $E(x_{2i}\epsilon_i) = 0$ . Determine the large sample bias in  $\hat{\beta}_1$  and  $\hat{\beta}_2$ . (Hint: partial out  $x_2$ ).
- (iv) Does the "iron law" of econometrics hold for  $\hat{\beta}_1$  (downward bias in magnitude). Does the presence of  $x_2$  lead to less or more large sample bias in  $\hat{\beta}_1$ ?
- 3. You have a panel data model:

$$y_{it} = X_{it}\beta + Z_i\gamma_i + \alpha_i + \eta_{it}$$
$$i = 1, ..., N; t = 1, ..., T$$

Where N is large and T is small.

- (i) How should you test Ho:  $E(\alpha_i | X_{it}, Z_i) = 0$ ?
- (ii) You run fixed effects estimation and do an F test that Ho:

$$\alpha_1 = \alpha_2 = \ldots = \alpha_N = 0$$

Specify the test. What should you conclude about your estimates of  $\beta$  and  $\gamma$  if you reject Ho?

- (iii) Suppose you think you may have errors in variables (EIV) in one of the  $X_{it}$ 's:  $X_{1it} = X_{1it}^* + v_{it}$ , where  $Ev_{it} = 0$ ,  $Ev_{it}v_{i\tau} = 0$  for  $t \neq \tau$  and  $E(X_{1it}^*v_{it}) = 0$ . What effect could EIV have on your fixed effects estimates and your test of  $E(\alpha_i|X_{it},Z_i) = 0$ ?
- (iv) How could you test if you do have an EIV problem? Can you give a consistent estimator if you do have an EIV problem?
- 4. You have a Tobit Model:

$$y_i^* = X_i \beta + \epsilon_i, \quad \epsilon_i \sim N(0, \sigma^2), \quad i = 1, ..., N.$$
 and

$$y_i = y_i^*$$
  $if y_i^* < S_i$   
 $y_i = S_i$   $if y_i^* \ge S_i$ 

- (i) Write down the likelihood function (LF) where  $S_i = S_j$  for all i,j. Then generalize the LF where  $S_i \neq S_j$ .
- (ii) Demonstrate "Fisher Consistency" for the situations where  $S_i \neq S_j$ .
- (iii) Suppose you observe the  $S_i$ 's with error:  $S_i = S_i^* + v_i$ , where the  $S_i^*$  are not observed and  $E(S_i^*v_i) = 0$ ,  $E(\epsilon v_i) = 0$ , and  $E(v_i) = 0$ . What is the effect on the ML estimates?
- (iv) Suppose you decide to test the model specification. You do a probit model for  $y_i < S_i$  or  $y_i = S_i$ . You compare these results to the ML Tobit Model estimate. Give a test and

determine its properties.