Formula Sheet Exam 2

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$$\text{Var}(X) = \sigma^2 = \; E[(X-\mu)^2] \qquad \mu = E(X). \tag{1}$$

$$\text{Cov}(X,Y) = E[(X-\mu_X)(Y-\mu_Y)] \tag{2}$$

$$\rho(X,Y) = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y} \tag{3}$$

$$\text{Var}(X) = E[\text{Var}(X|Y)] + \text{Var}[E(X|Y)]. \tag{4}$$

$$P(X \geq t) \leq \frac{E(X)}{t} \; . \tag{5}$$

$$P(|X - E(X)| \geq t) \leq \frac{Var(X)}{t^2} \; . \tag{6}$$

$$f_{\mathbf{Y}}(y_1, y_2, ..., y_m) = \sum_{\substack{(x_1,...,x_n) \;:\; r_i(x_1,...,x_n)=y_i \\ \forall i=1..m}} f_{\mathbf{X}}(x_1, ..., x_n) \tag{7}$$

$$f_{\mathbf{Y}}(y_1, y_2, ..., y_n) = \begin{cases} f_{\mathbf{X}}\big(s_1(), s_2(), ..., s_n()\big) \; |J| \;, & \text{for } (y_1, y_2, ..., y_n) \in \mathcal{Y} \subseteq R^n; \\ 0 \;, & \text{otherwise.} \end{cases} \tag{8}$$

where,

$$\begin{array}{rcl} Y_1 = r_1(X_1, ...X_n) & & X_1 = s_1(Y_1, ...Yn) \\ Y_2 = r_2(X_1, ...X_n) & \textbf{unique} & X_2 = s_2(Y_1, ...Yn) \\ \vdots & \textbf{transformation} & \vdots \\ Y_n = r_n(X_1, ...X_n) & \longrightarrow & X_n = s_n(Y_1, ...Y_n); \end{array} \tag{9}$$

and

$$J = det \begin{bmatrix} \frac{\partial s_1}{\partial y_1} & \cdots & \frac{\partial s_1}{\partial y_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial s_n}{\partial y_1} & \cdots & \frac{\partial s_n}{\partial y_n} \end{bmatrix} \qquad (Jacobian); \tag{10}$$

and

$\mathcal{X}$ is the support of $X_1, ...X_n : \mathcal{X} = \{\mathbf{x} : f_{\mathbf{X}}(\mathbf{x}) > 0\}$.

$\mathcal{Y}$ is the induced support of $Y_1, ...Y_n : \mathcal{Y} = \{\mathbf{y} : \mathbf{y} = r(\mathbf{x}) \text{ for all } \mathbf{x} \in \mathcal{X}\}$.

$$(x_1, ...x_n) \in \mathcal{X} \Longleftrightarrow (y_1, ...y_n) \in \mathcal{Y}. \tag{11}$$

If $X_i \sim N(\mu_i, \sigma_i^2)$ and all n X_i are mutually independent, then

$$H = \sum_{i=0}^{n} a_i X_i + b_i \sim N(\sum_{i=0}^{n} a_i \mu_i + b_i \ , \ \sum_{i=0}^{n} a_i^2 \sigma_i^2). \tag{12}$$

If $Y \sim N(0, 1)$, then

$$Z = Y^2 \sim \chi^2_{(1)} \tag{13}$$

If $X_1 \sim \chi^2_{(p)}$ and $X_2 \sim \chi^2_{(q)}$ are independent, then

$$H = X_1 + X_2 \sim \chi^2_{(p+q)} \tag{14}$$