

Exam 1

14.30 Fall 2003

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Instructions: This exam is closed-book and closed-notes. You may use a simple calculator. Please read through the exam in order to ask clarifying questions and to allocate your time appropriately. You **must show** all your calculations. You have approximately 90 minutes to complete the exam (90 points). Good Luck!

1. (21 points, 3 each) True, False, or Uncertain. Please explain briefly (but, no explanation no points).

a. Sets must be either disjoint or exhaustive, but not both.

b. For every 3 non-zero probability events A , B , and C ,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cup B) - P(A \cup C) - P(C \cup B).$$

c. For two non-zero probability events to be independent they must be based in two different experiments. If not, they are dependent. For example, consider tossing two coins. The outcome of the first flip is independent of the outcome of the second flip, because they are based on two different and independent flips (experiments).

d. For every 2 non-zero probability events A and B , $P(A|B) + P(A^c|B) = 1$.

e. For every 2 non-zero probability events A and B , the probability that exactly one of the events occur is $P(A) + P(B) - P(A \cup B)$.

f. The pmf/pdf, $f(x)$, is the probability that outcome x occurs.

g. If the joint pdf of X, Y is $f(x, y) = ke^{xy}$ for $0 < x < 1$ and $0 < y < 1$, then X and Y are not independent.

2. (14 points) Suppose there are 30 MIT students (20 of them women), 40 Harvard students (20 of them women), and 60 BU students (10 of them woman).

a.(5 points) In how many ways can you form a committee of:

i) 6 students?

ii) 6 students, if 2 of them have to be from MIT, 2 from Harvard, and 2 from BU?

b.(4 points) Assume the students are chosen randomly. What is the probability of choosing a committee of 6 students where 2 are women?

c.(5 points) In how many ways can you split the students in three groups such that each group contains 10 MIT, 10 or 20 Harvard, and 20 BU students.

3. (12 points) Suppose you have 10 boxes. Each box contains the same 15 balls, all of different color. Assume you pick 10 balls (one from each box). In how many ways can you pick the balls without taking into consideration from which box were they taken? (For example, Red Blue Red is the same as Blue Red Red.)

4. (8 points) Gabriel and Daniela just start using a random device to decide who washes the dishes. They toss a fair coin 4 times. Gabriel washes with 3 heads or more, and Daniela washes with 3 tails or more; otherwise they leave it until tomorrow. The next day they repeat the procedure, but if no one is assigned in two days in a row with this procedure, Gabriel gives up and wash them in that day. Find the probability that:

a.(3 points) Gabriel will wash the second day given that nobody washed the first day.

b.(5 points) No one washed the first day given that Gabriel washed the second day.

(Do not forget Questions 5 and 6.)

5. (13 points)

a. (8 points) Write down the formula for the cdf drawn in Graph 5.1. Then, draw and write down the formula for the pdf of the random variable with the cdf drawn in Graph 5.1.

a. (5 points) Find A, B, C and D in Graph 5.2. Then write down the cdf of the random variable with the pdf drawn in Graph 5.2.

6. (22 points) Suppose that the joint pdf of two random variables X and Y is given by:

$$f(x, y) = \begin{cases} e^{-x-y} & \text{if } 0 < x < \infty \text{ and } 0 < y < \infty \\ 0 & \text{otherwise.} \end{cases}$$

a. (3 points) Check that the function $f(x, y)$ is a well defined pdf.

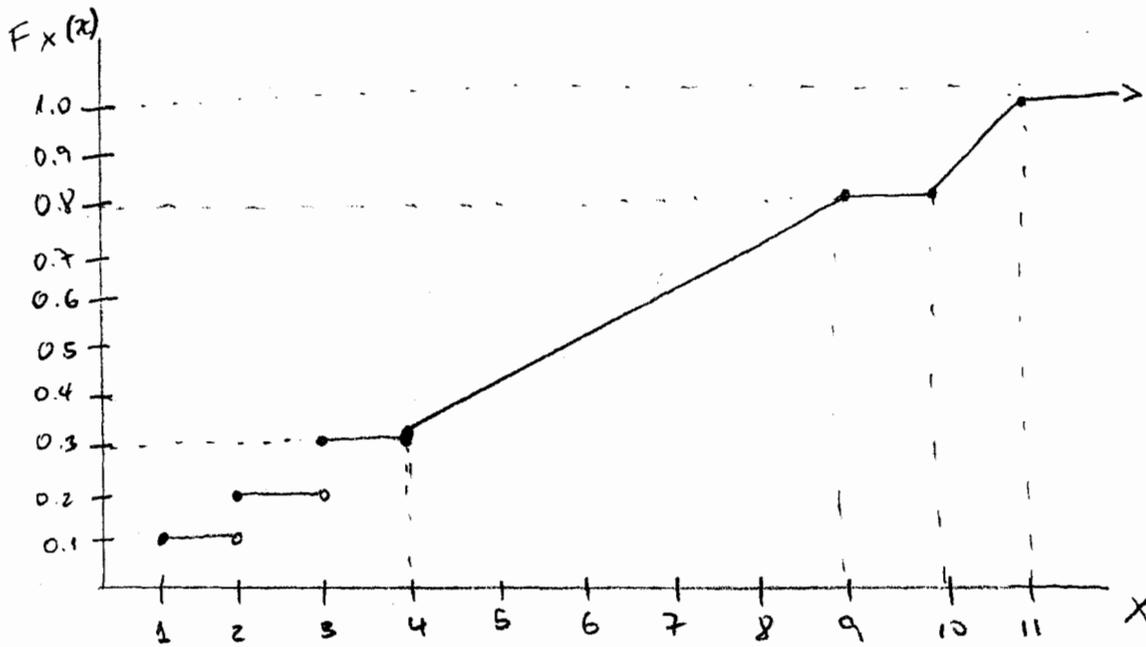
b. (4 points) Find the joint cdf of X and Y , $F(x, y)$. Compute $P(X < 1 \text{ and } Y < 1)$.

c. (4 points) Find the marginal distribution of X , $f(x)$, and the conditional distribution of X given $Y = y$, $f(x|y)$. Check that both are well defined pdfs.

d. (6 points) Find $P(X - Y > 1)$ and $P(X + Y > 1 \text{ and } X > Y)$. You **do not** need to solve the integrals, just define them correctly (**do not** forget the limits of integration).

e. (5 points) Find the cdf of the random variable $\frac{X}{Y}$.

graph 5.1



graph 5.2

