

14.122 Final Exam

**Answer all questions.** You have 3 hours in which to complete the exam. The exam is pretty long so I've put stars next to questions I guessed might be the hardest. You may want to leave these until the end if you are worried that you may end up being short on time.

1. (55 Minutes – 36 Points) Answer each of the following subquestions briefly. Please show your calculations and provide rough explanations where you can't give formal statements so I can give you partial credit.

(a) Give a formal definition as you can of what it means for a multistage game with observed actions to be continuous at infinity? Give an example of a game that is not continuous at infinity.

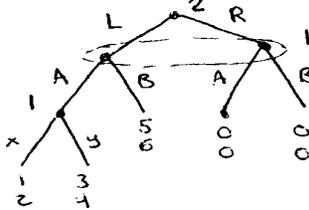
(b) Give as formal a definition as you can of what it means for a property to hold generically. What theorems that we saw in class were about properties that only hold generically.

(c) In class I defined payoff functions differently when defining finite extensive form games and infinite horizon multistage games with observed actions. How did the domains of the payoff functions differ and why did I make the change?

(d) Give a formal definition of a strategy in an extensive form game.

(e) Give an example of a game in which an outcome which does not seem reasonable is possible in a subgame perfect equilibrium.

(f) How many pure strategies does player 1 have in the extensive form game shown below. Write out the normal form payoff matrix. How many subgames does the game have?



(g) Find all of the Nash equilibria of the following game.

	A	B	C
U	0, 5	3, 1	3, 0
M	3, 1	1, 3	0, 1
D	2, 0	0, 0	0, 2

(h) Find the subgame perfect equilibrium of a two stage game where player 1 chooses

$a$  at cost  $a/16$  and players 1 and 2 then play the simultaneous move game shown below.

	$H$	$T$
$H$	$1+a, -1$	$-1, 1$
$-H$	$-1, 1$	$1, -1$

(i) Find the Nash equilibrium of the simultaneous move game where player 1 chooses  $a_1 \in \mathfrak{R}$ , player 2 chooses  $a_2 \in \mathfrak{R}$ , and the payoffs are

$$u_1(a_1, a_2) = (3 - a_2)a_1 - a_1^2$$

and

$$u_2(a_1, a_2) = (4 - a_1)a_2 - a_2^2.$$

2. (30 Minutes – 18 Points)

Consider the following game about trust that has been played in many experiments. The experimenter starts by giving player 1 ten dollars and player 2 zero dollars. The experimenter then asks player 1 how many dollars he is willing to give back to help player 2. If he chooses to give  $x$  dollars back to the experimenter, then the experimenter gives player 2  $3x$  dollars. Subsequently, player 2 has the opportunity to give any or all (or none) of the money he has received to player 1.

(a) Assuming that the two players are risk neutral and care only about their own payoff find the subgame perfect equilibrium of this game. (The subgame perfect equilibrium, by the way, is not what seems to happen in experiments. Usually player 1 gives some, but not all of the money back to the experimenter.)

(b) Does the game have a Nash equilibrium in which the players receive higher payoffs?

(c) Suppose we modified the game so that after the two stages above player 1 had the chance to punch player 2. Suppose that this would reduce player 1's utility by 1 dollar and reduce player 2's utility by 5 dollars. Would this change your answers to parts (a) and (b)? How about if we instead had the players play the game shown below after the second stage? Do these predictions seem reasonable to you.

	A	B
A	5,5	-5,-5
B	-5,5	5,5

(d\*) An alternate explanation for the experimental results is that the players may be altruistic. Show that the simplest representation of altruism — each player maximizing a weighted sum of his own dollar payoff and the other player's dollar payoff — can not account for the experimental regularity except for one very special (and not compelling) choice of the weights. Can you think of utility functions that might be used to account for the experimental result?

3. (60 Minutes – 28 Points)

Two drivers are side-by-side in adjacent lanes on the highway. The two lanes are continually narrowing and the two cars getting closer and closer together as they approach a construction site. If neither driver slows down and merges in behind the other they will crash at some point within the next minute.

(a) To model this situation as a simple normal form game, suppose each player simultaneously chooses the time  $t_i \in [0, 1]$  at which he or she will slow down if the other driver has not yet done so. Assume that each player  $i$  gets a payoff of -1 if the cars crash, 0 if they do not crash and player  $i$  ends up behind the other player, and 1 if they do not crash and player  $i$  ends up in front of the other player.

Assume that the probability that the cars crash is  $\min(t_1, t_2)$ , i.e. it is as if the time at which a crash occurs would be uniform on  $[0, 1]$  if neither car ever slows down. (Note for a crash to occur with probability one as it does in this specification the hazard rate at  $t$  (the likelihood of a crash occurring between  $t$  and  $t+dt$  conditional on there being no crash before  $t$ ) must go to infinity as  $t$  approaches 1. Here the hazard rate is  $1/(1-t)$  as long as both cars haven't yet slowed down and zero if one car has slowed down.)

Show that the game has an infinite number of asymmetric pure strategy Nash equilibria. If you can, show that there are no other pure strategy Nash equilibria.

(b\*) Find a symmetric mixed strategy Nash equilibrium in which the players mix on some interval  $[0, \bar{t}]$ .

(c) Consider an incomplete information version of this game. To simplify things, assume that there are only two times at which drivers can slow down. Each driver  $i$  must pick either  $t_i = 0$  or  $t_i = \frac{1}{2}$ . Assume that driver  $i$  gets a payoff of  $\theta_i$  if there is a crash, 0 if there is no crash and driver  $i$  slows down before *or at the same time as* the other driver, and 1 if there is no crash and driver  $i$  slows down second (which can only happen here if  $t_i = \frac{1}{2}$  and  $t_{-i} = 0$ .) Consistent with the previous formulation assume that the probability of a crash is 0 if at least one driver slows down at  $t = 0$  and  $\frac{1}{2}$  if both drivers slow down at  $t = \frac{1}{2}$ .

Suppose that  $\theta_i$  is known only to player  $i$ . Suppose that  $\theta_1$  and  $\theta_2$  are independent. Suppose that  $\theta_1$  is equal to -0.5 with probability  $\frac{1}{3}$  and equal to -2 with probability  $\frac{2}{3}$ . Suppose that  $\theta_2$  equals -3 with probability  $\frac{2}{5}$  and that with probability  $\frac{3}{5}$  player 2 actually likes crashing more than slowing down and has  $\theta_2 = 0.2$ .

Find the Bayesian Nash Equilibrium of this game.

4. (35 Minutes – 18 Points)

Consider the following game. There are two players. Player 1 is Mike Tyson. Player 2 is another heavyweight boxer, e.g. Buster Douglas. For the purposes of this question assume that Mike Tyson is good enough to defeat and perhaps seriously injure Buster Douglas if he trains hard and shows up for a fight in good shape. He is not good enough to beat Buster Douglas if he shows up fat and out of shape. Whether Mike Tyson trains hard is not a strategic choice that he makes. It depends entirely on his state of mind between the time the contract is signed and the time of the fight and this is beyond everyone's control.

Suppose (and here the question gets less realistic) that Mike Tyson is very aware of the current state of his psychological problems and the temptations he will face during training. Specifically, suppose that when the contract for the fight is about to be signed Mike Tyson already knows whether he will train hard. Buster Douglas does not. Douglas' prior is that the probability that Mike Tyson will train hard is  $\frac{2}{3}$ .

Suppose that just before the contract is signed, Lenox Lewis shows up at the press conference and offers Buster Douglas \$1 million dollars if he gives up his right to fight Mike Tyson (so that Tyson will surely be the champion when he later fights Lenox Lewis.) Mike Tyson then has the opportunity to make one of two speeches to Buster Douglas "you should take his money because I'm going to train hard and whip your ..." or "I will show up fat and out of shape so you'd be better off not taking the money, beating me in this fight, and getting \$10 million for a later rematch." Buster Douglas then has to decide whether to accept Lewis's offer and not fight Tyson or to decline the offer and fight Tyson.

Assume that if Douglas accepts, the payoffs are 0 utils for Tyson and 1 for Douglas. Assume that if Douglas declines Lewis's offer and fights Tyson the payoffs will be 2 utils for Tyson and -2 for Douglas if Tyson trains hard and -2 utils for Tyson and 10 utils for Douglas if Tyson doesn't train hard.

(a) Describe formally how you would represent this situation as a dynamic game of incomplete information between Mike Tyson and Buster Douglas. (Nature would be included also in the game tree, but don't include Lenox Lewis). Be clear what the type spaces are and draw a game tree.

(b) Show that there is no separating equilibrium where Mike Tyson truthfully makes the first speech if and only if he will train hard.

(c) Is there a pooling equilibrium where Mike Tyson claims that he will train hard regardless of whether this is true?

(d\*) Try to describe completely the set of semiseparating equilibria of the model. Are the semiseparating equilibria here like either of the semiseparating equilibria I discussed in class?