14.01SC Principles of Microeconomics, Fall 2011
Transcript – Problem 2-4 Solution Video

PROFESSOR: Today, we're going to be working on fall 2010, problem set number 2, and we're going to be doing problem number 4. I'm going to start off by reading the problem. There's a lot of information here, so I've written up some of the key points on the board for you guys already. It is exactly 24 hours before Lauren's physics final. She has an economics final directly after her physics final, and has no time to study in between. Lauren wants to be a physicist, so she places more weight on her physics test score. Her utility function is given right here. Where p is the score of her physics final and e is the score of her economics final.

Although, she cares more about physics, she is better at economics. For each hour spent studying economics, she will increase her score by three points. But her physics score will only increase by two points for every hour spent studying physics. Studying zero hours results in a score of zero on both subjects. Although natural log of zero is not defined, assume her utility for a score of zero is negative infinity. Now, we're going to go ahead and we're going to work through parts A, B, and C, A, B, C, and D and then we'll do part E, which is a new scenario afterwards.

Part A, we're going to find the constraints that Lauren faces in her test score maximization problem. And part B, we're going to find how many hours does Lauren optimally spend studying physics, how many hours does she spent studying economics. And hours are divisible, so we don't need whole number solutions for the hours spent studying physics and the hours spent studying economics.

So for part A, all we're looking for is we're looking for the constraints. And it sounds like the constraint that Lauren is really facing in this scenario is the amount of time that she has. It seems like Lauren's pretty intense about studying, so in the 24 hours before the test, she's not getting any sleep. She's going to spend all of this time studying. So we're going to have two hours variables to represent the hours she spends studying physics and the hours she spends studying economics.

And we know that the hours, when we add them together, they're going to have to be less than or equal to 24. Now, if this is our constraint, we really know that if she's trying to maximize her scores and if that's all she really cares about, she's not going to spend less than 24 hours studying. So we can actually just say that the sum of those two hours-- her hours spent studying physics and her hours spent studying economics are going to be equal to 24. So that's the first constraint that Lauren is going to face.

The second constraint that she's going to face, and it's not necessarily an intuitive one-- or it is actually really intuitive, but it's also a trivial one. It's just that she can't spend less than zero hours studying physics or studying economics. So we're going to add those constraints in as well. And the final constraints are actually production constraints. If the hours spent studying-- these aren't actually what she's interested in. What she's interested in are the p and the e. That's how she's going to get her utilities-- through the test scores.

So what we need is we need to find how she produces her physics score. If she gets two points for every hour she spent studying, her physics score is going to be p equals 2 times Hp-- that's the production of her physics score. The production of her economics score is going to equal 3 times He, since we know that she's a little bit better at economics than physics. So these are the constraints that we're going to face in the problem.

Let's go ahead and move on to part B. And for part B, what we're going to try to do is we're going to try to take these constraints and we're trying to maximize the amount of utility she's going to get from studying. So what we're doing for part B is we're going to maximize utility, and we're going to plug in some of these constraints. Before we can do this, we really need to get down to first-order conditions. We need to be able to take the derivative with regards to only one variable. So we have to get-- instead of p and e here, we're going to try to get only one variable. And the variable we're going to get in there is going to be the hours spent study economics. So we're going to get it all in terms of He.

So we need to find a way to replace both e and p with He. In this case, replacing e with He is really easy. We can just say that e is going to equal 3He. Replacing p with the He or the hours spent studying economics is a little bit more difficult. We're going to go to this equation. And we can say that Hp is going to be equal to 24 minus He. And then for this Hp, we're going to plug this into our production function for the physics score. So we get that p is equal to 2 times 24 minus He. So we're going to take these two equations that we have in and we're going to substitute for e and p in our utility function.

And so we're going to maximize, and the way we're going to maximize is by just taking the first-order conditions or the FOC. And the FOC in this case is just going to be the derivative with respect to He. When you take the derivative with respect to He, what you're going to get is 0.6 times negative 2 all over (48 minus 2 He) plus 0.4 times 3 all over 3He. And the reason this first-order condition is going to help us is because we know to maximize. We just set the first-order condition or the derivative with respect to He equal to zero.

And from here, we can solve out for He. And when we solve for He, we find that the hour she spends studying economics is going to equal to 9.6. And we know that any time she spends not studying the economics, she's studying physics, since all she cares about is her test scores. She's going to spend the remaining 14.4 hours studying physics. This is our answer for part B.

Now, part C just asks us, "What economics and physics test scores will she achieve?" so we're looking for the e star and the p star. We can just take the hour she spent studying physics and the hours she spends studying economics and we can plug these back into her production functions for her physics score and an economics score. So, for part C, her economics score is going to be equal to 3 times 9.6. She's going to get a 28.8 on her economics test.

And for her physics test, she's also going to get a 28.8. Probably the not the best test scores in the world. Let's go ahead and look at part D. Part D asks us, "What utility level will she achieve with these test scores that she has?" Now, to find the utility levels, all we have to do is we have to go back to our utility function that we wrote down in part A, and we're just going to plug in the two test scores that she received, and we can solve through for overall utility.

And so when you grab a calculator, and you solve through for this equation, what you're going to find is you're going to find her overall utility level is going to be equal to 3.36. And when we're measuring utility, we don't have any units on this. The units, we usually think of as utils, but we can just keep it like this. Just know that in relativistic terms, her utility is that 3.36.

And this is going to be useful for part D. Because in part E, we have the option of sending Lauren off to an economics tutor. And we're going to be comparing her utility after going to the economics tutor to this utility of 3.36. And by comparing her new utility with the old utility, we can see if her utility's higher after going the economics tutor. We can see if that's a good decision for her.

So now that we finished calculating the utility for Lauren in the initial case where she had 24 hours and no help, now we're going to move on to part E. Part E says, "Suppose Lauren can get an economics tutor. If she goes to the tutor, she will increase her economics test score by five points for every hour spent studying, instead of three points. But will lose four hours of study time by going to the tutor. She cannot study while at the tutor, and going to the tutor does not directly improve her test score. Should Lauren go to the tutor?"

Now, for this problem, we can go back to our initial scenario. Where we had our constraint for the amount of time Lauren can spend. If Lauren goes to the tutor, it sounds like she's going to spend about four hours in the car driving to the tutor's house and back. So instead of having 24 total hours of study time, her new constraint is that her total study time Hp plus He is equal to 20.

That's the downside. The upshot of it is that her production function for economics test scores is no longer three points for every hour she spent studying, now she gets five point for every hour she spends studying. So we're going to go through the exact same process for A, B, C, and D that we did before, only now we're going to use our new constraints to solve for the maximization problem.

Before we start off for our maximization problem and solving for the first-order conditions, what we have to do is we have to get it all in terms of one variable. And the variable we're going to get it in terms of is the hours of economic studying or He. So from the utility function, to solve or to plug in for e, the He, we have that e is going to equal five times He.

For p, we know that Hp is equal to 20 minus He. That just says that any time she's not studying economics, she's going to be studying physics. We can plug this into our p equals 2 times Hp. So we know that p is going to equal 2 times 20 minus He. Let's go ahead and plug this and this into our utility maximization problem and solve for the first-order conditions. So we're maximizing utilities-- so we're going to take the derivative with respect to He again. When we do that, we get our FOC.

And again, for the maximization problem, we can just set that equal to 0. Solving out for He, we find that she's going to spend eight hours studying economics. And we also find that any time she's not spending studying economics, she's going to be studying physics. So she's going to spend 12 hours studying physics.

Now, we're not done with part E because really all we have to do for this problem is we have to solve for her new utility using these hours that she spends studying. And we have to compare her new utility to her old utility of 3.36 to see if she's better off or worse with the economics tutor.

In her production function for economics test score, we know that for each of those eight hours you spend studying, her test score's going to improve eight points, so her new economic score is 40. And we know that for each of those 12 hours she spends studying physics, her score's going to improve two points. So her physics score has dropped a little bit to 24. When we plug in for e and p into our utility function, we can find her utility level with the economics tutor.

When we solve through for these natural logs, we're going to find that her new utility is equal to 3.38. And since the problem asks us to actually make a qualitative judgment-- is she better off or worse off, we need to compare her utility before of 3.36 to her new utility of 3.38. This is the comparison that we're interested in.

And when we make this comparison, we can see that in relative terms, she's a little bit better off going to be economics tutor because her utility has risen two hundredths of a point. So in this case, for part E, we found that her utility increases even though she's lost time because her production function for her economics to score has gone up, and that she's better off going to the economics tutor because of that increase in utility.

So the point of this problem was to look at the different constraints that consumers face when deciding where to spend their time or where to spend their money, and to see how those different constraints affect their utility levels.