14.01 Principles of Microeconomics, Fall 2007
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Lecture 13

Cost Functions

Outline

1. Chap 7: *Short-Run Cost Function*
2. Chap 7: *Long-Run Cost Function*

Cost Function

Let w be the cost per unit of labor and r be the cost per unit of capital. With the input Labor (L) and Capital (K), the production cost is

$$w \times L + r \times K.$$

A cost function $C(q)$ is a function of q, which tells us what the minimum cost is for producing q units of output. We can also split total cost into fixed cost and variable cost as follows:

$$C(q) = FC + VC(q).$$

Fixed cost is independent of quantity, while variable cost is dependent on quantity.

1 Short-Run Cost Function

In the short-run, firms cannot change capital, that is to say,

$$r \times K = \text{const.}$$

Recall the production function given fixed capital level K in the short run (refer to Lecture 11) (see Figure 1). Suppose $w = 1$, the variable cost curve can be derived from Figure 1. Adding $r \times K$ to the variable cost, we obtain the total cost curve (see Figure 2). Average total cost is

$$ATC = \frac{TC}{q} = \frac{FC + VC}{q} = \frac{rK}{q} + \frac{wL(q;K)}{q}.$$

With the definition of the average product of labor:

$$AP_L = \frac{q}{L},$$

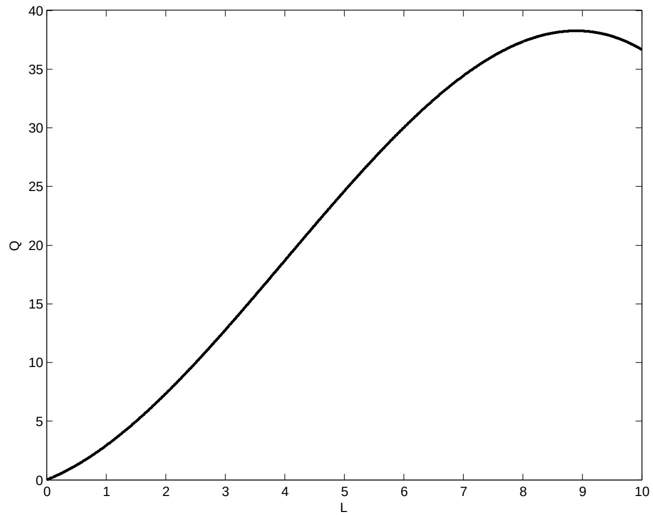

Figure 1: Short Run Production Function.

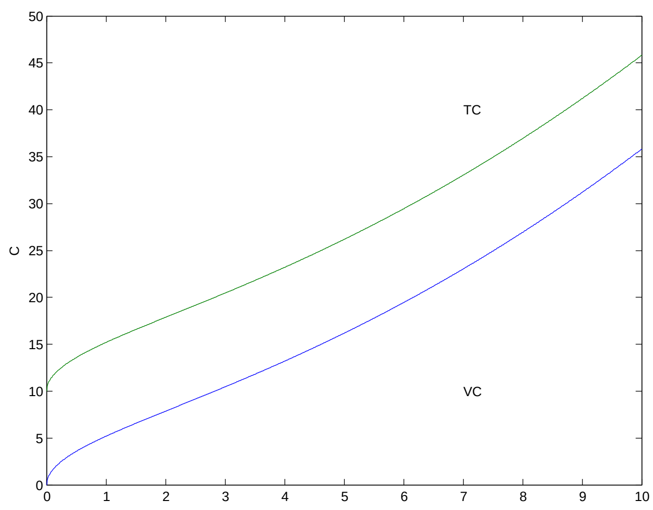

Figure 2: Short Run Cost Function.

we can rewrite ATC as

$$ATC = \frac{rK}{q} + \frac{w}{AP_L},$$

in which the average variable cost is

$$\frac{VC}{q} = \frac{wL(q;K)}{q} = \frac{w}{AP_L}.$$

Likewise, we rewrite the marginal cost:

$$MC = \frac{dTC}{dq} = \frac{dVC}{dq} = w\frac{dL(q)}{dq} = \frac{w}{\frac{\partial q}{\partial L}} = \frac{w}{MP_L}.$$

In Lecture 11, we discussed the relation between average product of labor and marginal product of labor (see Figure 3). We draw the curves for AVC and

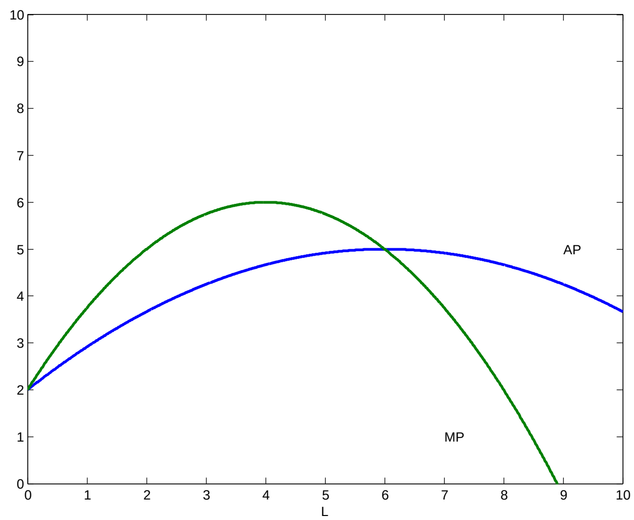

Figure 3: Average Product of Labor and Marginal Product of Labor.

MC in the same way (see Figure 4). The relation between MC and AVC is:

- If
$$MC < AVC,$$
AVC decreases;

- if
$$MC > AVC,$$
AVC increases;

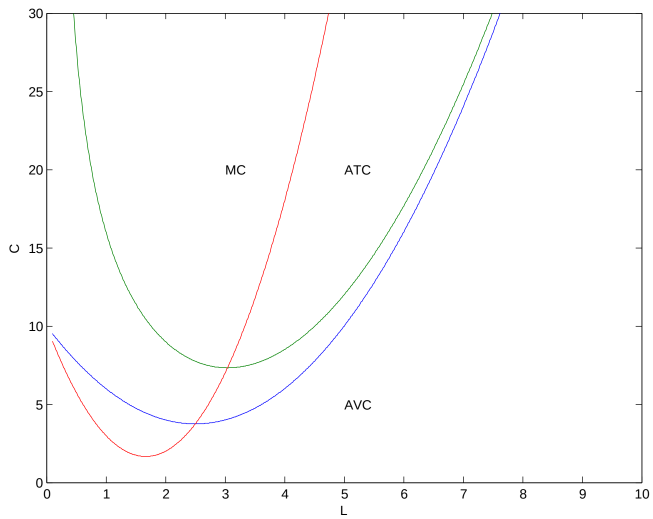

Figure 4: Average Cost, Average Variable Cost, and Marginal Cost.

- if
$$MC = AVC,$$
AVC is minimized.

Now consider the total cost. Note that the difference between ATC and AVC decreases with q as the average fixed cost term dies out (see Figure 4). The relation between MC and ATC is:

- If
$$MC < ATC,$$
ATC decreases;
- if
$$MC > ATC,$$
ATC increases;
- if
$$MC = ATC,$$
ATC is minimized.

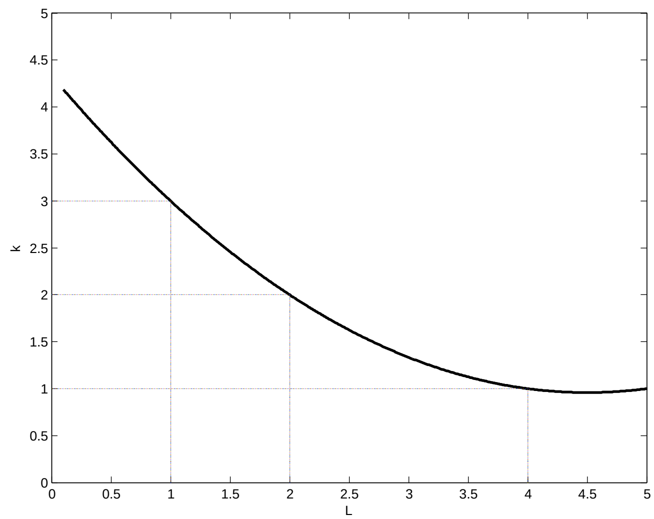

Figure 5: Isoquant Curve.

2 Long-Run Cost Function

In the long-run, both K and L are variable. The isoquant curve describes the same output level with different combination of K and L (see Figure 5). The slope of an isoquant curve is

$$-MRTS = -\frac{MP_L}{MP_K}.$$

Similarly, the isocost curve is constructed by different (K, L) with the same cost (see Figure 6). The isocost curve equation is:

$$rK + wL = \text{const},$$

therefore, it is linear, with a slope $-\frac{w}{r}$.
Now we want to minimize the cost $rK+wL$ subject to an output level $Q(K, L) = q$. This minimum cost can be obtained when the isocost curve is tangent to the isoquant curve (see Figure 7). Thus the slopes of these two curves are equal:

$$MRTS = \frac{MP_L}{MP_K} = \frac{w}{r}.$$

Now consider an increase in wage (w). The slope of the isocost curve increases (see Figure 8), and the firm use more capital and less labor. The firm's choice of input moves from A to B in the figure.

The expansion path shows the minimum cost combinations of labor and capital at each level of output (see Figure 9).

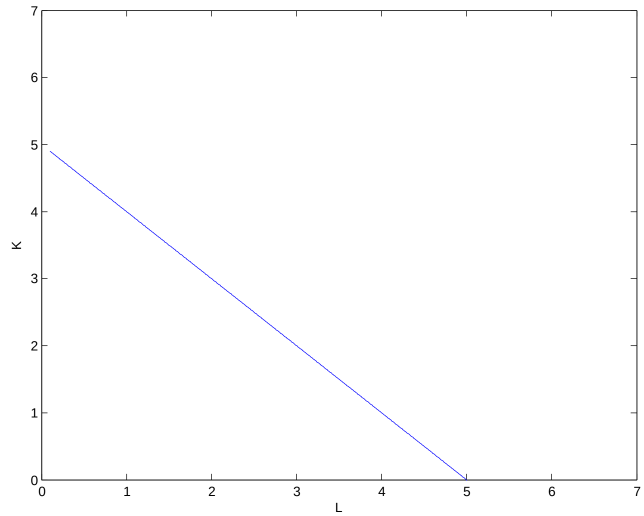

Figure 6: Isocost Curve.

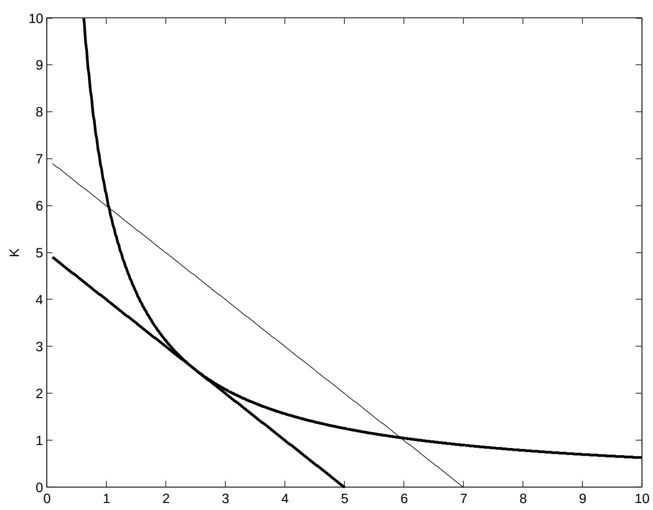

Figure 7: Minimize the Cost Subject to a Output Level.

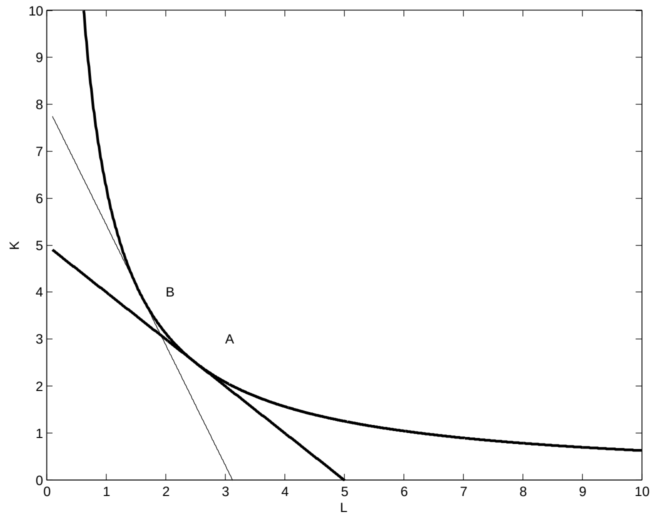

Figure 8: The Change of Cost Minimized Situation.

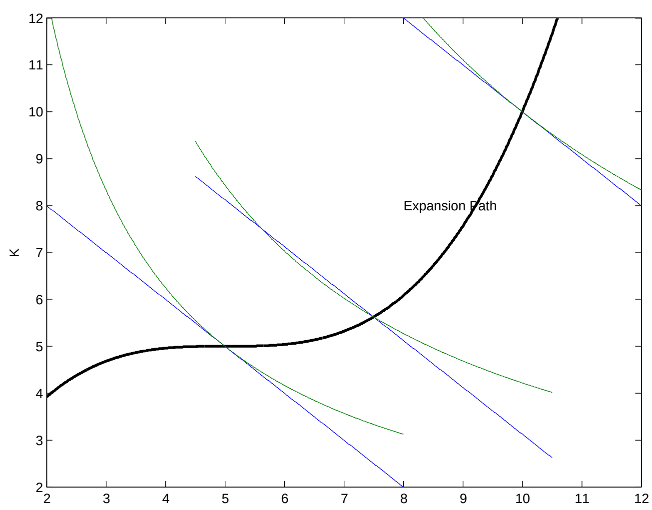

Figure 9: Expansion Path.

Example (Calculating the Cost.). Given the production function

$$q = L^{\frac{2}{3}} K^{\frac{2}{3}}.$$

In the short run,

$$C_{SR}(q; K) = rK + w\frac{q^{\frac{3}{2}}}{K},$$

where K is fixed.
In the long run, according to the equation

$$\frac{MP_L}{MP_K} = \frac{w}{r},$$

we have

$$\frac{K}{L} = \frac{w}{r}.$$

Then the expansion path is

$$K = \frac{w}{r}L.$$

Substituting this result into the production function, we obtain

$$L = q^{\frac{3}{4}}(\frac{r}{w})^{\frac{1}{2}},$$

$$K = q^{\frac{3}{4}}(\frac{w}{r})^{\frac{1}{2}}.$$

Hence, the long-run cost function is:

$$C_{LR}(q) = wL + rK = 2q^{\frac{3}{4}}\sqrt{wr}.$$