14.01 Principles of Microeconomics, Fall 2007
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Lecture 11

Production Functions

Outline

1. Chap 6: *Short Run Production Function*
2. Chap 6: *Long Run Production Function*
3. Chap 6: *Returns to Scale*

1 Short Run Production Function

In the short run, the capital input is fixed, so we only need to consider the change of labor. Therefore, the production function

$$q = F(K, L)$$

has only one variable L (see Figure 1).

Average Product of Labor.

$$AP_L = \frac{\text{Output}}{\text{Labor Input}} = \frac{q}{L}.$$

Slope from the origin to (L,q).

Marginal Product of Labor.

$$MP_L = \frac{\partial\text{Output}}{\partial\text{Labor Input}} = \frac{\partial q}{\partial L}.$$

Additional output produced by an additional unit of labor.

Some properties about AP and MP (see Figure 2).

- When
  $$MP = 0,$$
  Output is maximized.
- When
  $$MP > AP,$$
  AP is increasing.

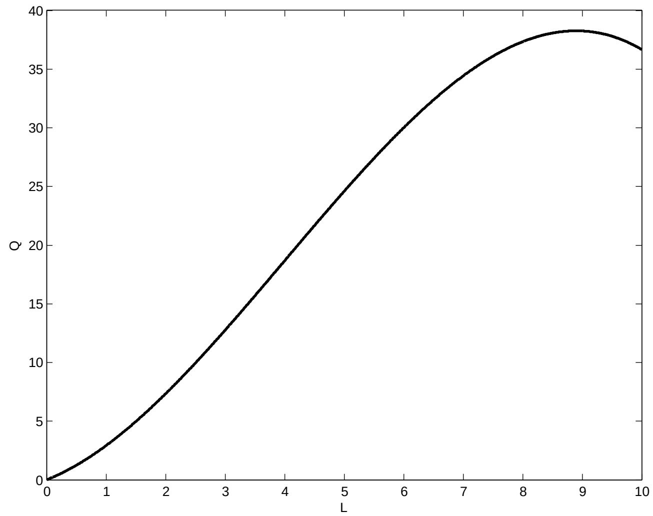

Figure 1: Short Run Production Function.

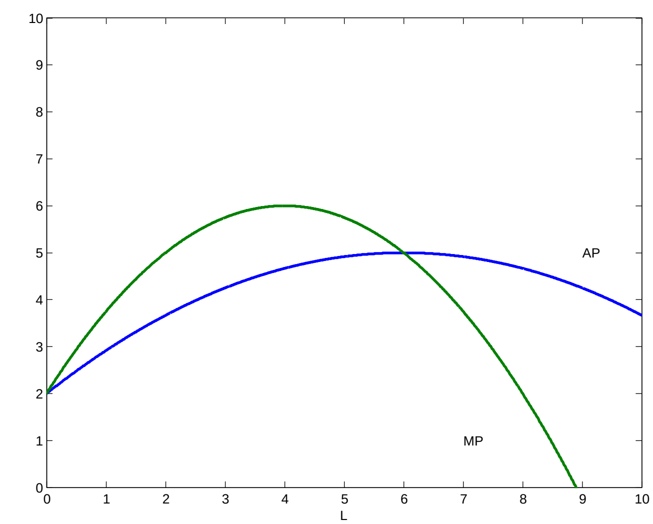

Figure 2: Average Product of Labor and Marginal Product of Labor.

- When
$$MP < AP,$$
AP is decreasing.
- When
$$MP = AP,$$
AP is maximized.

To prove this, maximize AP by first order condition:

$$\frac{\partial}{\partial L}\frac{q(L)}{L} = 0$$

$\Longrightarrow$

$$\frac{\partial q}{\partial L}\frac{1}{L} - \frac{q}{L^2} = 0$$

$\Longrightarrow$

$$\frac{\partial q}{\partial L} = \frac{q}{L}$$

$\Longrightarrow$

$$MP = AP.$$

Example (Chair Production.). Note that here AP_L and MP_L are not continuous, so the condition for maximizing AP_L we derived above does not apply.

Number of Workers	Number of Chairs Produced	AP_L	MP_L
0	0	N/A	N/A
1	2	2	2
2	8	4	6
3	9	3	1

Table 1: Relation between Chair Production and Labor.

2 Long Run Production Function

Two variable inputs in long run (see Figure 3).

Isoquants. Curves showing all possible combinations of inputs that yield the same output (see Figure 4).

Marginal Rate of Technical Substitution ($MRTS$). Slope of Isoquants.

$$MRTS = -\frac{dK}{dL}$$

How many units of K can be reduced to keep Q constant when we increase L by one unit. Like MRS, we also have

$$MRTS = \frac{MP_L}{MP_k}.$$

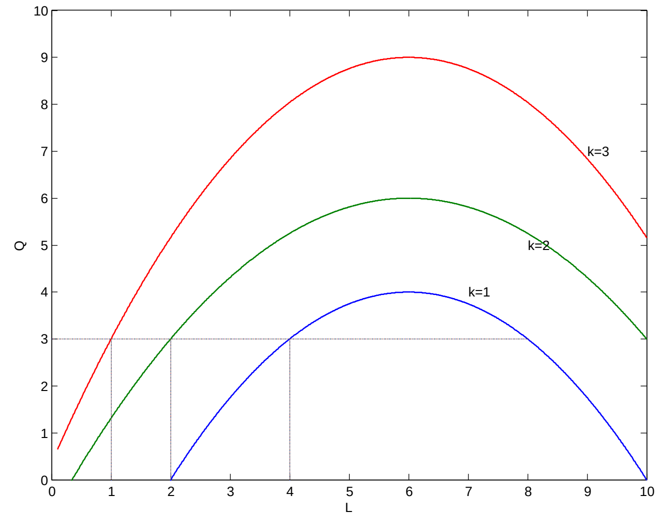

Figure 3: Long Run Production Function.

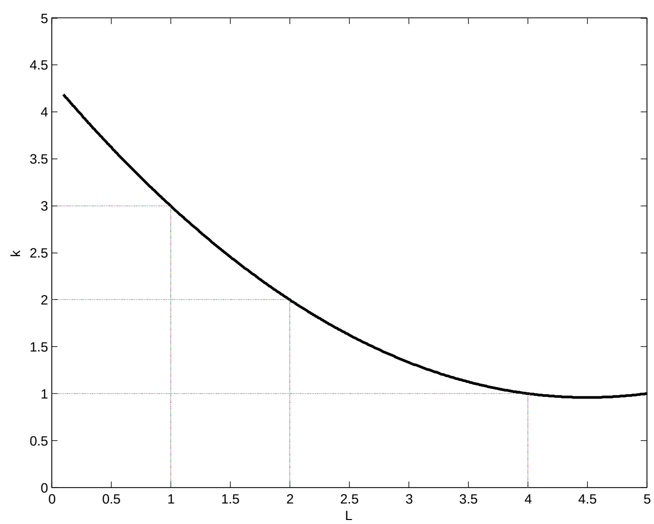

Figure 4: K vs L, Isoquant Curve.

Proof. Since K is a function of L on the isoquant curve,

$$q(K(L), L) = 0$$

$\Longrightarrow$

$$\frac{\partial q}{\partial L}\frac{dK}{dL} + \frac{\partial q}{\partial L} = 0$$

$\Longrightarrow$

$$-\frac{dK}{dL} = \frac{MP_L}{MP_K}.$$

□

Perfect Substitutes (Inputs). (see Figure 5)

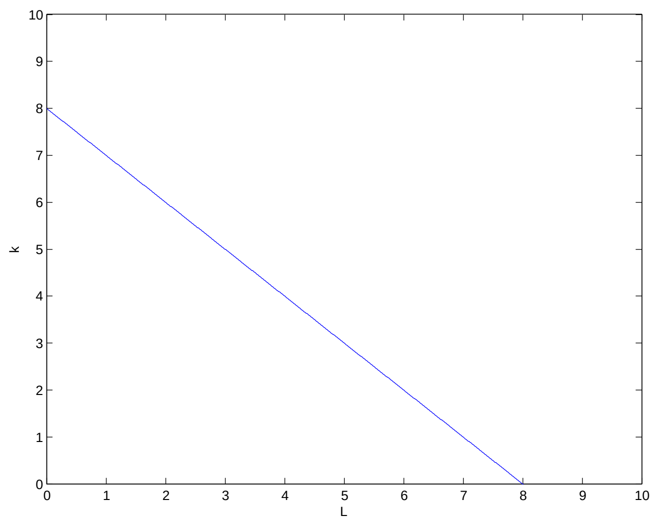

Figure 5: Isoquant Curve, Perfect Substitutes.

Perfect Complements (Inputs). (see Figure 6)

3 Returns to Scale

Marginal Product of Capital.

$$MP_K = \frac{\partial q(K, L)}{\partial K}$$

Marginal Product of Labor

$$K\ constant\ ,\ L \uparrow \rightarrow q?$$

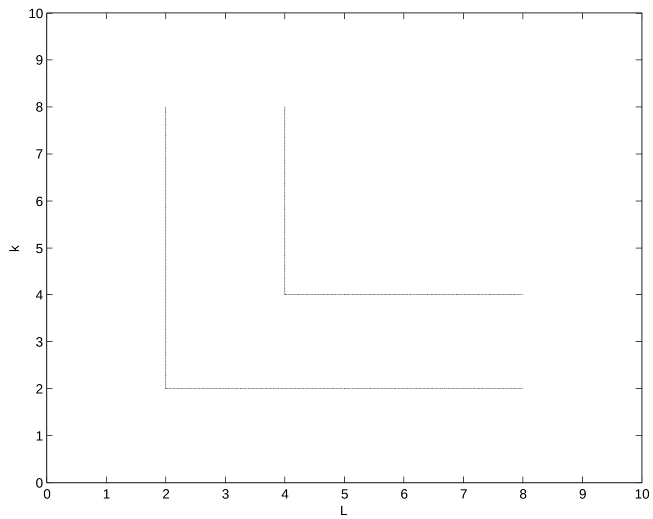

Figure 6: Isoquant Curve, Perfect Complements.

Marginal Product of Capital

$$L\ constant\ ,\ K \uparrow \rightarrow q?$$

What happens to q when both inputs are increased?

$$K \uparrow\ ,\ L \uparrow \rightarrow q?$$

Increasing Returns to Scale.

- A production function is said to have increasing returns to scale if

$$Q(2K, 2L) > 2Q(K, L),$$

or

$$Q(aK, aL) = 2Q(K, L),\ a < 2.$$

- One big firm is more efficient than many small firms.
- Isoquants get closer as we move away from the origin (see Figure 7).

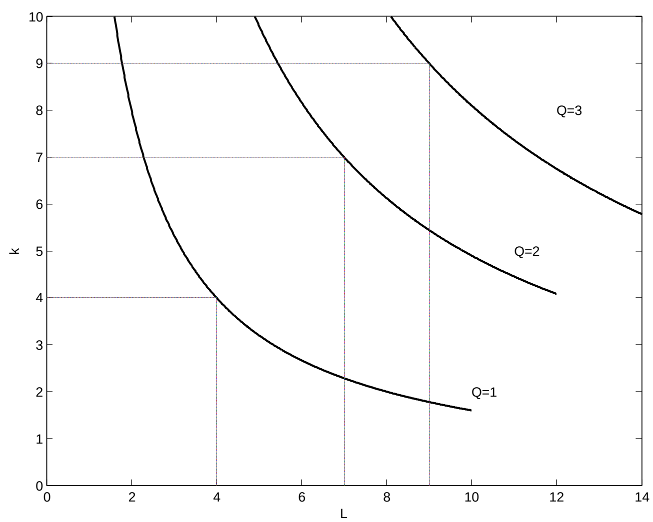

Figure 7: Isoquant Curves, Increasing Returns to Scale.