14.01 Principles of Microeconomics, Fall 2007
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Lecture 6

Optimization, Revealed Preference, and Deriving Individual Demand

Outline

1. Chap 3: *Corner Solution of Optimization*
2. Chap 3: *Revealed Preference*
3. Chap 4: *Deriving Individual Demand, Engle Curve*

1 Corner Solution of Optimization

When we have an interior solution,

$$\frac{P_x}{P_y} = \frac{U_x}{U_y}$$

must be satisfied. However, sometimes a consumer gets highest utility level when $x = 0$ or $y = 0$. If that's the case, we have corner solutions, and

$$\frac{P_x}{P_y} \neq \frac{U_x}{U_y},$$

as shown in Figure 1.

In Figure 1, because people cannot consume negative amounts of goods (bundle A), their best choice is to consume bundle B, so the quantity of y consumed is zero. Conditions for corner solutions:

- $$MRS = \frac{U_x}{U_y} > \frac{P_x}{P_y} \text{ when } y = 0.$$
- $$MRS = \frac{U_x}{U_y} < \frac{P_x}{P_y} \text{ when } x = 0.$$

Example (An example of consumer's problem). The parameters are

$$P_x = 1,$$

$$P_y = 1,$$

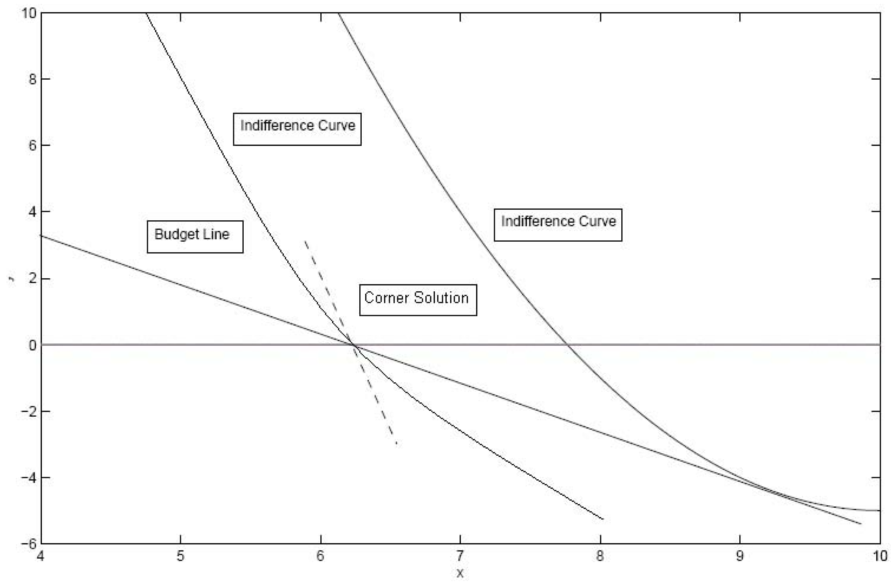

Figure 1: Corner Solution to Consumer's Problem.

$$I = 2.$$

The utility function is

$$U(x, y) = x + 2\sqrt{y}.$$

The budget constraint is

$$x + y = 2.$$

According to the condition for an interior solution:

$$\frac{P_x}{P_y} = \frac{U_x}{U_y}.$$

$\Longrightarrow$

$$\frac{1}{1} = \frac{1}{\frac{1}{\sqrt{y}}}.$$

$\Longrightarrow$

$$y = 1 \Longrightarrow x = 1.$$

If the price y changes to 1:

$$P_y = 1,$$

then the solution is

$$y = 4 \Longrightarrow x = -3 < 0,$$

which is impossible.
Then we have the corner solution:

$$x = 0, y = 2.$$

$x = 0$ since consumer wants to consume as little as possible.

2 Revealed Preference

In the former chapters, we discussed how to decide optimal consumption from utility function and budget constraint:

$$\begin{matrix} \text{Utility Function} & \\ & \Longrightarrow \text{Optimal Consumption} \\ \text{Budget Constraint} & \end{matrix}$$

And now we discuss how to know consumer's preference from budget constraint and consumption:

$$\begin{matrix} \text{Budget Constraint} & \\ & \Longrightarrow \text{Preference} \\ \text{Consumption} & \end{matrix}$$

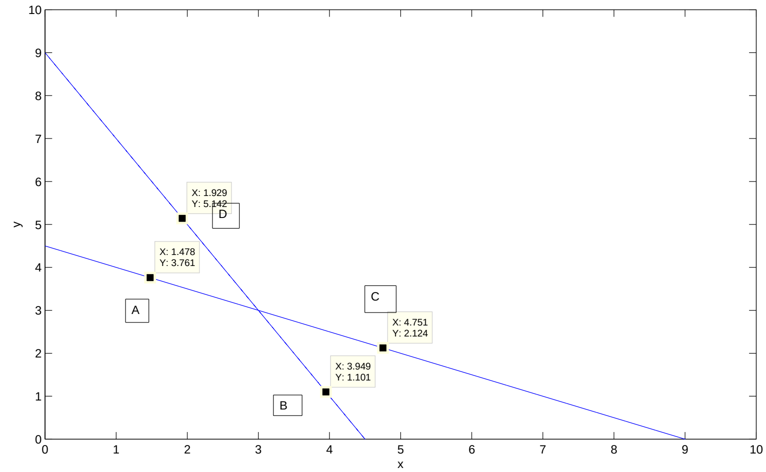

Figure 2: A Contradiction of Preference. A and B are the Choices.

Example (Revealed preference). In Figure 2, two budget constraint lines intersect. Assume one person's choices are A and B respectively. Then we have

$$A \succcurlyeq C,$$

$$B \succcurlyeq D.$$

And Figure 2 obviously shows that

$$C \succ B,$$

$$D \succ A.$$

Thus,

$$A \succcurlyeq C \succ B \succcurlyeq D \succ A,$$

which is a contradiction, which means utility does not optimized and the choice is not rational.

3 Deriving Individual Demand, Engle Curve

Use the following utility function again:

$$U(x, y) = x + 2\sqrt{y},$$

with a budget constraint:

$$P_x x + P_y y = I.$$

When

$$I \geqslant \frac{P_x^2}{P_y},$$

we have an interior solution. $MRS = P_x/P_y$. Thus,

$$x = \frac{I}{P_x} - \frac{P_x}{P_y},$$

$$y = \left(\frac{P_x}{P_y}\right)^2.$$

When

$$I \leqslant \frac{P_x^2}{P_y},$$

we have a corner solution.

$$x = 0,$$

$$y = \frac{I}{P_y}.$$

- Figure 3 shows a demand function of y and P_y as an example. (Assume that I, x and P_x are held constant.)
- Engle Curve describes the relation between quantity and income. Figure 4 shows the relation between x and income, and Figure 5 shows that between y and income.

  Normal good. Quantity demanded of good increases with income.

  Inferior good. Quantity demanded of good decreases with income.

  Substitutes. Increase in price of one leads to an increase in quantity demanded of the other.

  Complements. Increase in price of one leads to an decrease in quantity demanded of the other.

For this problem,

- if $I < \frac{P_x^2}{P_y}$, x and y are neither substitutes nor complements, and x is a normal good.
- if $I \geqslant \frac{P_x^2}{P_y}$, x and y are substitutes, and y is a normal good.

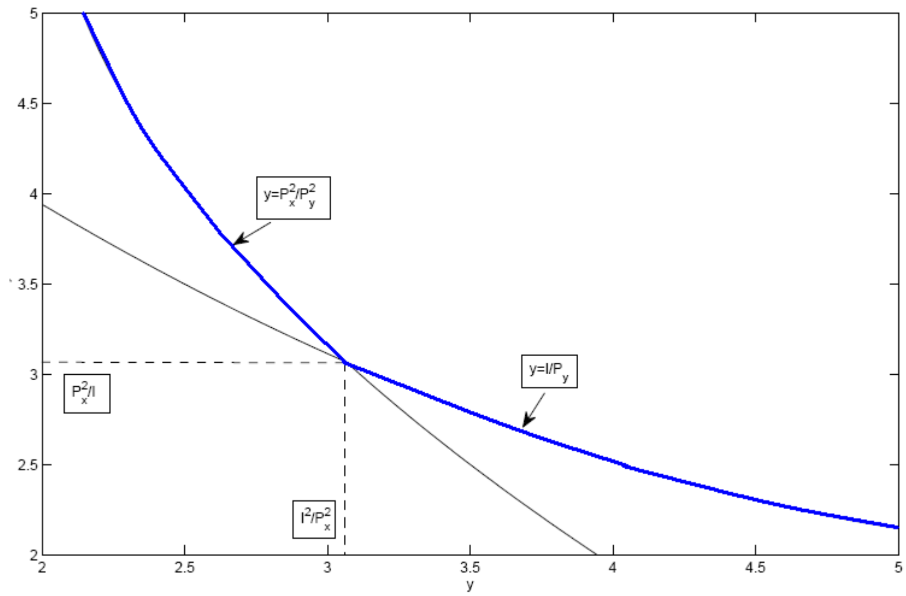

Figure 3: Demand Function for Goods 'y'.

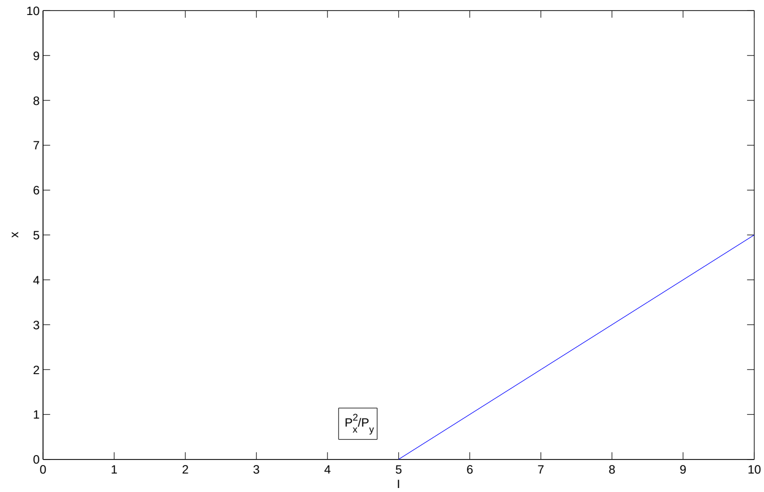

Figure 4: The Relation between Income and Quantity Demanded of 'x'. Engle curve of x.

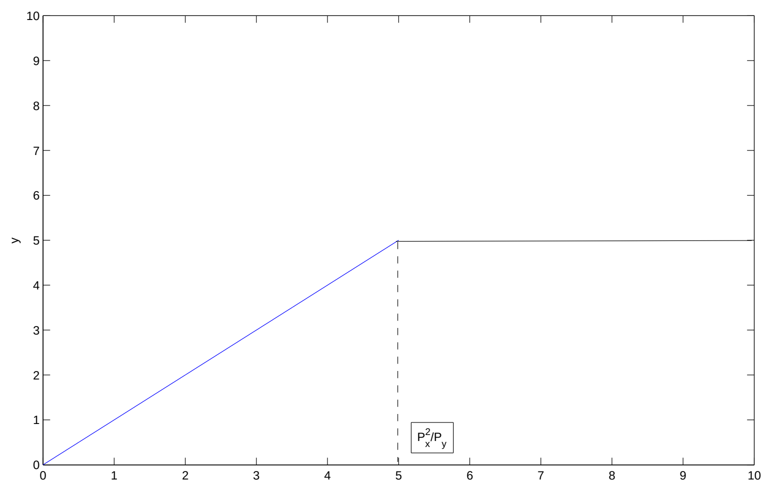

Figure 5: The Relation between Income and Quantity Demanded of 'y'. Engle curve of y.