

14.01 Principles of Microeconomics, Fall 2007

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Lecture 3

## Elasticities of Demand

**Elasticity.** Elasticity measures how one variable responds to a change in another variable, namely the percentage change in one variable resulting a one percentage change in another variable. (The percentage change is independent of units.)

### Outline

1. Chap 2: *Price Elasticity of Demand*
2. Chap 2: *Income Elasticity of Demand*
3. Chap 2: *Cross Price Elasticity of Demand*
4. Chap 2: *Comparison of Elasticity Over Short Run and Long Run*

## 1 Price Elasticity of Demand

**Price elasticity of demand.** Price elasticity of demand measures the percentage change in quantity demanded resulting from one percentage change in price.

$$E_E^D = \frac{\% \Delta Q_P}{\% \Delta P} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}}.$$

### Example Calculation

Figure 1 shows a demand curve:

$$Q(P) = 8 - 2P.$$

When the price changes from 2 to 1, the price elasticity of demand is:

$$E_P^D|_{p=2 \rightarrow 1} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{\frac{2}{4}}{\frac{-1}{2}} = -1.$$

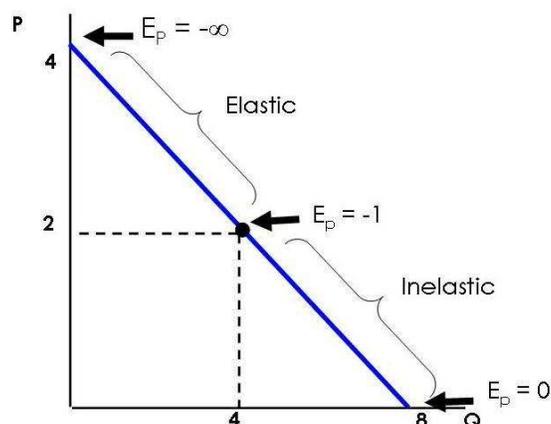


Figure 1: Price Elasticity of Demand.

If the direction of change is opposite, from 1 to 2, then the price elasticity of demand is:

$$E_P^D|_{P=1 \rightarrow 2} = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{\frac{-2}{6}}{\frac{1}{1}} = -\frac{1}{3}.$$

The two quantities are different. To solve this conflict, consider small changes in  $P$  and  $Q$ , and define:

$$E_P^D = \frac{\frac{dQ}{Q}}{\frac{dP}{P}} = \frac{P}{Q} \frac{dQ}{dP}.$$

Thus, at the point  $P = 2$ , the price elasticity of demand is:

$$E_P^D|_{P=2} = \frac{P}{Q} \frac{dQ}{dP} = \frac{2}{4} \times (-2) = -1.$$

## Properties of Price Elasticity of Demand

1. Price elasticity of demand is usually a negative number.
2.  $|E_P| > 1$  indicates that the good is price elastic, perhaps because the good has many substitutes;  $|E_P| < 1$  indicates that the good is price inelastic, perhaps because the good has few substitutes.
3. Given a linear demand curve,  $E_P$  is not a constant along the curve. For example, for curve in Figure 1,  $E_P = -\infty$  at top portion, but zero at bottom portion.
4. Discuss two extreme situations:  $|E_P| = 0$ , quantity independent of price Figure 2 and  $|E_P| = \infty$ , quantity very sensitive to price. See Figure 3.

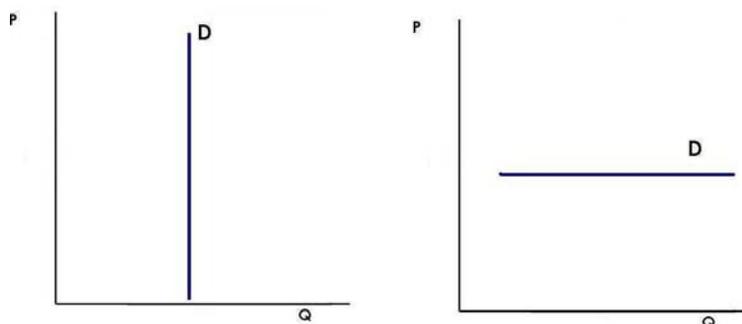


Figure 2: Extreme demand elasticity.  $|E_P| = 0$ , quantity independent of price.  
 Figure 3: Extreme demand elasticity.  $|E_P| = -\infty$ , quantity very sensitive to price.

5. The constant elasticity demand function is

$$Q = aP^b,$$

since

$$E_P = \frac{dQ}{dP} \frac{P}{Q} = abP^{b-1} \frac{P}{Q} = b \frac{aP^b}{Q} = b.$$

Refer to Figure 4.

6. How do total consumer expenditure change when the price of a good changes?

$$\frac{dExp}{dP} = \frac{d(PQ_D(P))}{dP} = Q + P \frac{dQ}{dP} = Q(1 + E_P) = Q(1 - |E_P|).$$

- If  $|E_P| > 1$ , total expenditure decreases when price increases;
- If  $|E_P| < 1$ , total expenditure increases when price increases.

*Example* (Cell phone). People need to do business in the morning, so  $E_P$  is low, so cell phone companies increase the rate while customers will expend more; but  $E_P$  is high in the evening since people do not have to talk, so cell phone companies lower the rate to encourage customer expenditure.

## 2 Income Elasticity of Demand

**Income elasticity of demand.** Income elasticity of demand measures the percentage change in quantity demanded resulting from one percentage change in income. Similarly,

$$E_I = \frac{\frac{dQ}{Q}}{\frac{dI}{I}} = \frac{I}{Q} \frac{dQ}{dI}.$$

The income elasticity of demand is usually positive.

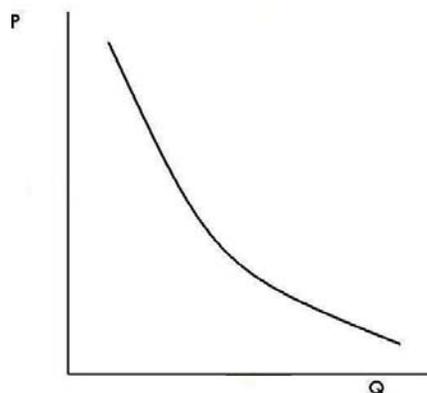


Figure 4: Constant Demand Elasticity.

### 3 Cross Price Elasticity of Demand

**Cross price elasticity of demand.** Cross price elasticity of demand measures the percentage change in quantity demanded of a good ( $x$ ) resulting from one percentage change in price of another good ( $y$ ).

$$E_{Q_x P_y} = \frac{\frac{dQ_x}{Q_x}}{\frac{dP_y}{P_y}} = \frac{P_y}{Q_x} \frac{dQ_x}{dP_y}.$$

- If  $y$  is a substitute of  $x$ , the cross price elasticity of demand is positive.
- If  $y$  is a complement of  $x$ , the cross price elasticity of demand is negative.

### 4 Comparison Between Elasticity Over Short Run and Long Run

Is demand more elastic in the long run or short run?

**Consumption goods.** For consumption goods, the demand is more elastic in the long run. Because people need goods for daily life and buy them constantly, the short run demand is inelastic. Faced with high prices in the long run, they may change habits or find more substitutes.

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**Durable goods.** For durable goods, the demand is more elastic in the short run. Consider cars. If price of cars increase, in the short run people might use their current cars longer. In the long run, though, people have to replace their cars.