Ocean Basins

We now want to consider instability, transition to turbulence, and turbulence in an ocean basin. There are lots of systems to choose from. The Stommel model appears to be stable even at strong forcing.

$$\frac{\partial}{\partial t}q + J(\psi, y + \delta_i^2 q) = -\sin(\pi y) - \delta_f q$$

$$q = \nabla^2\psi$$

For these, $\delta_f = 0.032$. `Demos, Page 1: Stommel 1 gyre` *<linear model>* *<zeta>* *<Stommel di=0.023>* *<zeta>* *<di=0.071>* *<zeta>* *<di=0.23>* *<zeta>*

`Demos, Page 1: Two gyre` *<di=0.0093>* *<q>* *<di=0.030>* *<q>* *<di=0.093>* *<q>*

The Munk model

$$\frac{\partial}{\partial t}q + J(\psi, y + \delta_i^2 q) = -\sin(\pi y) + \delta_f^3\nabla^2 q$$

is livelier.

$$\delta_f = \left(\frac{\kappa}{\beta L^3}\right)^{1/3} \quad . \quad \delta_i = \left(\frac{fW/H}{\beta L^3}\right)^{13}$$

`Demos, Page 1: Munk` *<di=0.0037>* *<di=0.0053>* *<di=0.0064>* *<di=0.0074>* *<di=0.0083>* *<di=0.010>* *<di=0.012>* *<summary>*

`Demos, Page 1: Munk two gyre` *<di=0.0037>* *<di=0.0053>* *<di=0.0064>* *<di=0.0074>* *<di=0.0083>* *<di=0.010>* *<di=0.012>* *<summary>*

Stability and Bifurcations

Steady states

Cessi and Ierley (*JPO*, **25**, 1196-1205) found steady states by discretizing (using Chebyshev polynomials) and using Newton's method to find conditions where $\frac{\partial}{\partial t}q = 0$. Some solutions can be found by starting with the viscous solution and gradually shifting the parameters; ohters require initial guesses. Once found, these can then be continued.

Markings indicate number of steady states found;

(1) one antisymmetric

(3) one anti + 2 non

(5a) one anti + 4 non

(5b) three anti + 1 non

`Demos, Page 1: Regimes Cessi and Ierley` *<overall: xv -geometry -0+0 ~glenn/12.822t/graphics/ci1.jpg>* closeupci2.jpg *<df=4e-2,di=2e-2>* *<di=3e-2>* *<di=4e-2>* *<di=5e-2>* *<di=6e-2>* *<summary>* *<A1 and N1>* *<N2>* *<A2 and A3>*

The experiments shown have $\delta_f = 4 \times 10^{-2}$ and $\delta_i = 1 \ldots 6 \times 10^{-2}$ so that they cross the regimes shown.

Bifurcations and stability

The bifurcation sequence can be inferred by looking at where the roots coalesce; stability can be found by discretizing the linearize equations and looking for eigenvalues with positive real parts. Alternatively one can integrate the linearized equations starting with random perturbations and look for exponentially growing modes to appear.

`Demos, Page 2: bifurcations` *<df=0.075>* *<df=0.0478>* `Demos, Page 2: stability` *<regions>*

Baroclinic case

The two-layer problem allows for baroclinic instability as well, with significant changes in the dynamics. In principle the two-layer system allows an upper-layer only solution, like the Munk solution but with the addition of a deformation radius to the PV. If the diffusion acts only on the relative vorticity, the R_d term would drop out in steady state; in our model, the diffusivity acts on the PV, so it would still appear, But the spin-up to this state is complicated. We tend to think of each mode independently in the linear problem, so that the barotropic wave crosses the basin, followed much more slowly by the baroclinic wave which wipes out the deep signal, leaving the Sverdrup flow in the upper layer only. But, this picture does not account for the boundary conditions. Cane and Sarachik (1976, *JMR*, **34**, 629-665) showed that the spin-up involves Kelvin waves crossing to the east along the equator or souther boundary, moving up the eastern wall, and generating Rossby waves which then cross the basin to the west.

The QG model has a representation of this process in the baroclinic boundary condition. If we take the transformed geostrophic momentum, interface, and mass equations

$$\frac{\partial}{\partial t}\mathbf{u} + \hat{\mathbf{z}} \times \mathbf{u}\,q + f_0 \hat{\mathbf{z}} \times \mathbf{u}^\dagger = -\nabla p^\dagger + \mathcal{F} - \mu \mathbf{u}$$

$$\nabla \cdot \mathbf{u}^\dagger \mp \frac{w^\dagger}{H} = 0$$

$$\frac{\partial}{\partial t} h + w^\dagger = \mathcal{H} - \mu h$$

and note that the transformed variables still satisfy $\mathbf{u}^\dagger \cdot \hat{\mathbf{n}} = 0$ on the walls, we find

$$\int w^\dagger = 0 \quad \Rightarrow \quad \int h = 0$$

The mass of each layer must be conserved. In addition, this implies that the circulation around the basin is altered only by direct forcing and dissipation. From these statements, we can augment the $\psi_j = c_j(t)$ conditions with a forcula for $c_1 - c_2$ (the barotropic part does not matter). Alternatively, when we invert the PV, we add sufficient free solution

$$(\nabla^2 - F_1 - F_2)\phi = 0 \quad , \quad \phi = 1 \textit{ on bound.}$$

so that the integrals of

$$\psi_1^{(p)} + C\phi + D \quad \textit{and} \quad \psi_2^{(p)} - \delta C\phi + D$$

are zero.

`Demos, Page 2: bc spinup` *<correction field>* *<no corr>* *<corr>*

We can avoid this problem by using the double-gyre.

Berloff and Meacham (*JPO*, **28**, 361-388) have explored the two-layer, single-gyre version of the same problem.

`Demos, Page 3: Berloff and Meacham` *<bifurcation>* *<limit cycle>* *<more complex>* *<chaos>*

We can see the transitions in the two-layer double gyre model. By comparison, the case with infinitely deep lower layer has steady solutions

`Demos, Page 3: Transition` *<tau 0.003>* *<tau 0.004>* *<tau 0.005>* *<tau 0.004 del=0>* *<q>* *<q tot>*

`Demos, Page 3: Example` *<single layer>* *<two layer>* *<q1>* *<qt1>* *<qt2>* *<psi1>* *<psi2>* *<h>*