Development of anisotropic structure in the Earth's lower mantle by solid-state convection

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Is there evidence for the localization of dislocation creep in the lowermost mantle?

- As we discussed last class, LPO of (Mg,Fe)O may be a viable candidate mechanism for D'' anisotropy.
- But: LPO requires a "kick" from the diffusion creep regime (most of the lower mantle) to the dislocation creep regime.
- This paper: implement a numerical model for deformation due to a downgoing slab. Is there evidence for this transition from the model?

A quick review: diffusion creep vs. dislocation creep

- DIFFUSION CREEP
  - High T
  - Small grain size
  - Low stress
  - Low strain rate
  - Newtonian viscosity
  - DOES NOT result in LPO or (macroscopic) seismic anisotropy
  - DOES erase previous LPO.
- DISLOCATION CREEP
  - Low T
  - Large grain size
  - High stress
  - High strain rate
  - Non-Newtonian viscosity
  - DOES result in LPO and (macroscopic) seismic anisotropy (IF STRAINS HIGH ENOUGH, >100-200%)

Model Details (I)

(Details mostly from *McNamara et al.* [2001])

Utilizes a *composite* rheology incorporating both diffusion & dislocation creep, with:

$$\dot{\varepsilon} = \dot{\varepsilon}_{\text{diff}} + \dot{\varepsilon}_{\text{disl}} \tag{1}$$

$$\dot{\varepsilon}_{\text{diff}} = A'_{\text{diff}} \left(\frac{b}{d}\right)^m \exp\left(-\frac{g_{\text{diff}}\, T_{\text{m}}}{T_{\text{dim}}}\right) \frac{\sigma}{\mu} \tag{2}$$

$$\dot{\varepsilon}_{\text{disl}} = A'_{\text{disl}} \exp\left(-\frac{g_{\text{disl}}\, T_{\text{m}}}{T_{\text{dim}}}\right) \left(\frac{\sigma}{\mu}\right)^n \tag{3}$$

where A'_{diff} and A'_{disl} are prefactors, μ and b are reference values for the rigidity and Burgers vector, g_{diff} and g_{disl} are activation coefficients, T_{m} is the dimensional melting temperature, d is the grain size, σ is the stress, and m and n are constants. Because $\sigma = \eta\dot{\varepsilon}$, these may be rearranged

The transition stress, σ_{t}, is defined as the stress at which the material flows equally by diffusion and dislocation creep:

$$\sigma_{\text{t}} = \left[\left(\frac{A'_{\text{disl}}}{A'_{\text{diff}}}\right) \left(\frac{d}{b}\right)^m \mu^{(1-n)} \exp \left(\frac{T_{\text{m}}}{T}\,(g_{\text{diff}} - g_{\text{disl}})\right)\right]^{\frac{1}{1-n}} \tag{7}$$

Model Details (II)

The physics: conservation of **mass**, **momentum**, and **energy** in the extended Boussinesq approximation.

Solve w/ finite element code in 2D cylindrical geometry.

2.1. Model setup

The numerical calculations are performed by solving the non-dimensional conservation equations of mass, momentum, and energy in the extended Boussinesq approximation [27]. The equation for mass conservation in incompressible flow is:

$$\nabla \cdot \boldsymbol{u} = 0 \qquad (12)$$

where $\boldsymbol{u}$ is the velocity vector. The momentum equation is:

$$\nabla P + \nabla \cdot (\eta \underline{\dot{\varepsilon}}) = \alpha Ra T \hat{r} \qquad (13)$$

where $\hat{r}$ is the radial unit vector directed toward the center, P is the dynamic pressure, η is the effective viscosity, $\dot{\varepsilon}$ is the deviatoric strain rate tensor, α is the non-dimensional thermal expansivity, Ra is the Rayleigh number, and T is the temperature. The energy equation includes viscous dissipation and adiabatic (de)compression:

$$\frac{\partial T}{\partial t} + (\boldsymbol{u} \cdot \nabla) T + \alpha \frac{T_{\mathrm{dim}}}{\Delta T} Di\, w = \nabla (k \cdot \nabla T) + \frac{Di}{Ra} \sigma_{ij} \frac{\partial u_i}{\partial x_j} \qquad (14)$$

where t is time, T_{dim} is the dimensional temperature, k is the non-dimensional thermal conductivity, ΔT is the temperature contrast across the model, Di is the dissipation number, w is the radial component of velocity, σ_{ij} are components of the stress tensor, and u_i and x_i indicate the ith component of the velocity and location vectors, respectively. The strain rate components are:

$$\dot{\varepsilon}_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \qquad (15)$$

The second invariant of the strain rate tensor is the effective strain rate and is represented as:

$$\dot{\varepsilon} = \left(\tfrac{1}{2} \Sigma \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij}\right)^{\frac{1}{2}} \qquad (16)$$

The Rayleigh number is given as:

$$Ra = \frac{\rho_0 g \alpha_0 \Delta T h^3}{\kappa_0 \eta_0} \qquad (17)$$

where ρ_0, α_0, κ_0, and η_0 are reference values of density, thermal expansivity, thermal diffusivity, and viscosity. h is the reference length scale corresponding to the depth of the mantle.

The non-dimensional viscosity is determined by dividing by the reference viscosity, η_0, which is defined as the diffusion creep viscosity of the olivine layer at $T_{\mathrm{dim}} = 1500$ K and $z = 140$ km. The dissipation number is given as:

Model Details (III)

Fixed parameters:

Table 1
Fixed parameters

Parameter	Description	Value	Units
ΔT	Temperature drop across mantle	3000	K
α_0	reference thermal expansivity	3×10^{-5}	K^{-1}
ρ_0	reference density	4500	kg m^{-3}
C_p	specific heat	1250	J kg^{-1} K^{-1}
h	mantle thickness	2.8×10^{6}	m
k_0	reference thermal conductivity	5.6	W m^{-1} K^{-1}
g	gravitational constant	9.8	m s^{-2}
κ_0	reference thermal diffusivity	$k_0\ \rho_0^{-1}\ C_p^{-1}$	
Di	dissipation number	0.5	
d_{um}	grain size	2.0	mm
d_{lm}	grain size	1.0	mm
m_{um}	grain size index	2.5	
m_{lm}	grain size index	2.5	
n_{um}	power-law index	3.0	
n_{lm}	power-law index	3.0	
$A'_{diff-um}$	olivine diffusion prefactor[a]	2.25×10^{14}	s^{-1}
$A'_{disl-um}$	olivine dislocation prefactor[a]	1.28×10^{22}	s^{-1}
$A'_{diff-lm}$	lower-mantle diffusion prefactor[a]	1.06×10^{14}	s^{-1}
μ	reference rigidity	300	GPa
b	Burgers vector	5.0×10^{-7}	mm
$g_{diff-um}$	olivine diffusion activation coefficient	17	
$g_{disl-um}$	olivine dislocation activation coefficient	31	
$g_{diff-lm}$	lower-mantle diffusion activation coefficient	10	
σ_d	ductile yield stress	400	MPa
σ'_b	brittle yield stress gradient	5.33	MPa km^{-1}
$R_{surface}$	non-dimensional surface radius	1.67813	
R_{bottom}	non-dimensional bottom radius	0.67813	
$R_{interface}$	non-dimensional upper–lower-mantle interface radius	1.42	
η_{max}	non-dimensional viscosity maximum	1.0	
η_{min}	non-dimensional viscosity minimum	10^{-4}	

Upper- and lower-mantle values are denoted by um and lm, respectively. The above radii are non-dimensionalized by dividing by the length scale, h.

[a]For the higher Rayleigh number case, $A'_{diff-um}=9.01\times10^{14}\ s^{-1}$, $A'_{disl-um}=5.11\times10^{22}\ s^{-1}$, and $A'_{diff-lm}=4.22\times10^{14}\ s^{-1}$.

Play around with: lower mantle dislocation creep activation coefficient, viscosity magnitude, transition stress, strength of slab…

“Acceptable” class of models has lower mantle dominated by diffusion creep.

Calculating strain/LPO

- Use strain as a proxy for development of LPO, and track strains only in regions dominated by dislocation creep.
- Calculate Lagrangian finite strain (as a post-processing step) by time-integrating the DGT (deformation gradient tensor) for individual strain tracers.
- When a tracer leaves the dislocation creep regime, diffusion creep destroys LPO.

So, the main features of the model results are…

- Deformation in slab is dominated by dislocation creep in a mantle otherwise dominated by diffusion creep…
- Directly above CMB: high-magnitude, laterally-directed strain. Consistent feature of nearly all models.
- Details of strain field are time-dependent and can be quite complicated.

Integration with mineral physics data…

- The authors consider (Mg,Fe)O to be a more likely candidate than (Mg,Fe)-pvskite. [(Mg,Fe)O is weaker phase & has higher intrinsic anisotropy.]
- Steady-state (strains > 400-500%) LPO develops in (Mg,Fe)O; horizontal shear will result in $V_{SH} > V_{SV}$.

Conclusions

- Slabs are characterized by high stress, resulting in deformation dominated by dislocation creep.
- Complicated strain fields result, but one consistent feature is a large degree of laterally directed strain directly above the CMB.
- Combined with mineral physics experiments on (Mg,Fe)O, predict $V_{SH} > V_{SV}$ anisotropy (consistent with seismological results for paleoslab regions).
- LPO is a likely candidate mechanism for D'' anisotropy in slab regions.
- Although other processes may contribute to the formation of anisotropy, they are not required, and solid-state processes within a homogenous material may suffice.