12.540 Principles of the Global Positioning System
Lecture 06

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GPS Observables

- Today's class we start discussing the nature of GPS observables and the methods used to make range and phase measurements
- Start with idea of remotely measuring distances
- Introduce range measurement systems and concepts used in graphically representing electromagnetic signals
- Any questions on homework?

Distance measurement

- What are some of the methods used to measure distance?
- We have talked about:
  - Direct measurement with a "ruler"
  - Inferred distances by measuring angles in triangles
  - Distance measurement using the speed of light (light propagation time)
- GPS methods is related to measuring light propagation time but not directly.

Direct light propagation time

- Distance can be measured directly by sending a pulse and measuring how it takes to travel between two points.
- Most common method is to reflect the signal and the time between when the pulse was transmitted and when the reflected signal returns.
- System used in radar and satellite laser ranging

Direct light propagation delay

- To measure a distance to 1 mm requires timing accuracy of $3x10^{-12}$ seconds (3 picoseconds)
- Timing accuracy needs to be maintained over the "flight time". For satellite at 1000km distance, this is 3 millisecond.
- Clock stability needed 3ps/3ms = 10^{-9}
- A clock with this longtime stability would gain or lose 0.03 seconds in a year (10^{-9}*86400*365)
- (Clock short term and long term stabilities are usually very different -- Characterized by Allan Variance)

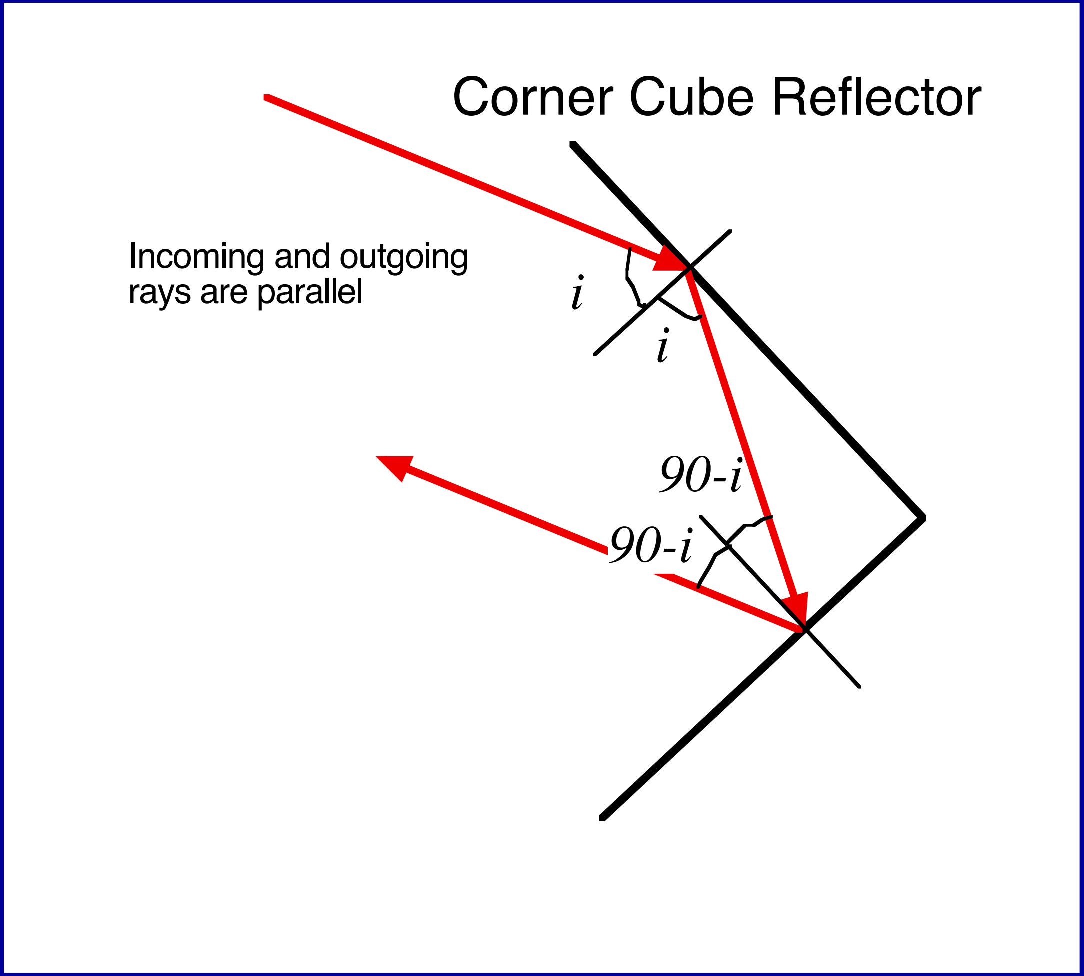
Direct light propagation measurement

- The noise in measuring the time will be proportional the duration of the pulse
- For mm-level measurements, need a pulse of the duration equivalent of a few millimeters.
- Pulse strength also enters (you need to be able to detect the return pulse).
- In general, direct time measurement needs expensive equipment.
- A laser system capable of mm-level ranging to satellites costs ~$1M

Reflecting the signal back

- With optical (laser) systems you want to reflect signal back: a plain mirror won't do this unless perfectly normal to ray.
- Use a "corner cube" reflector. In 2-D shown on next page
- For satellites, need to "spoil" the cube (i.e., corner not exactly 90 degrees because station not where it was when signal transmitted)

Corner cube reflector



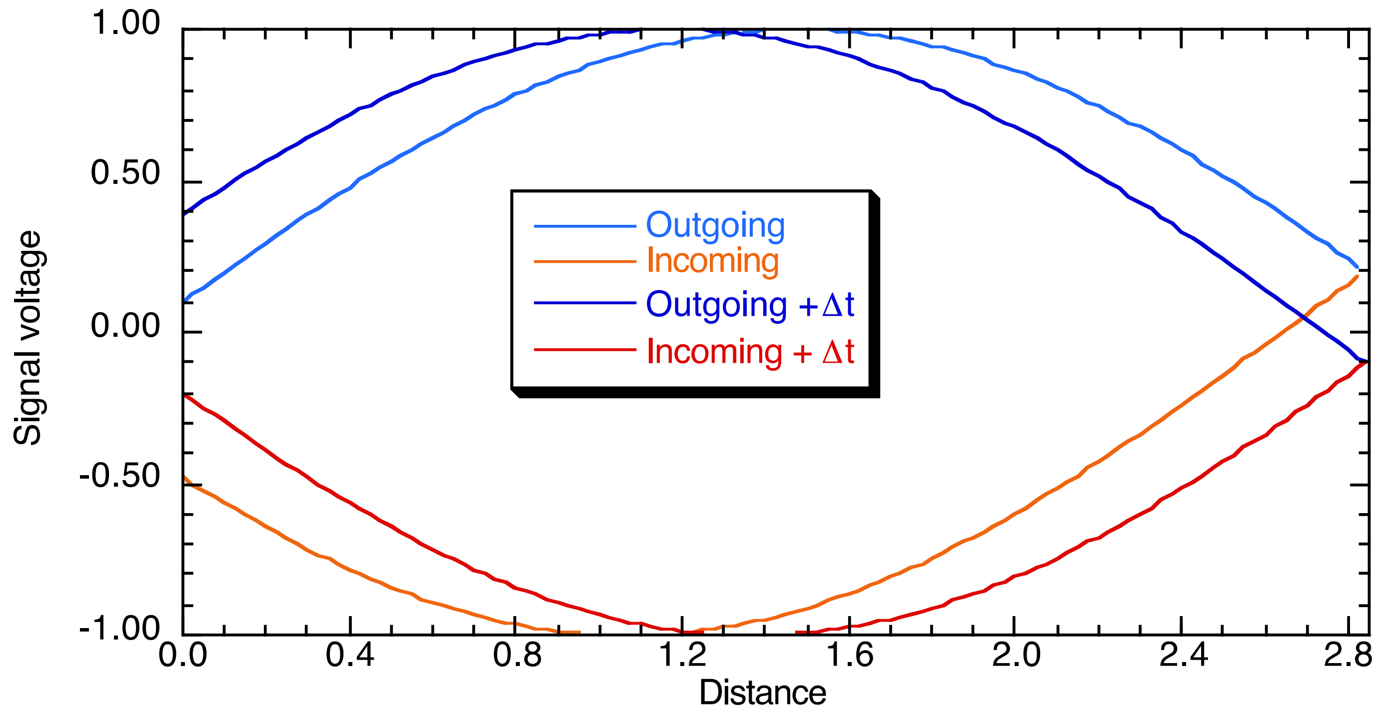
The return angle is twice the corner angle

For 90 degree corner, return is 180 degrees.

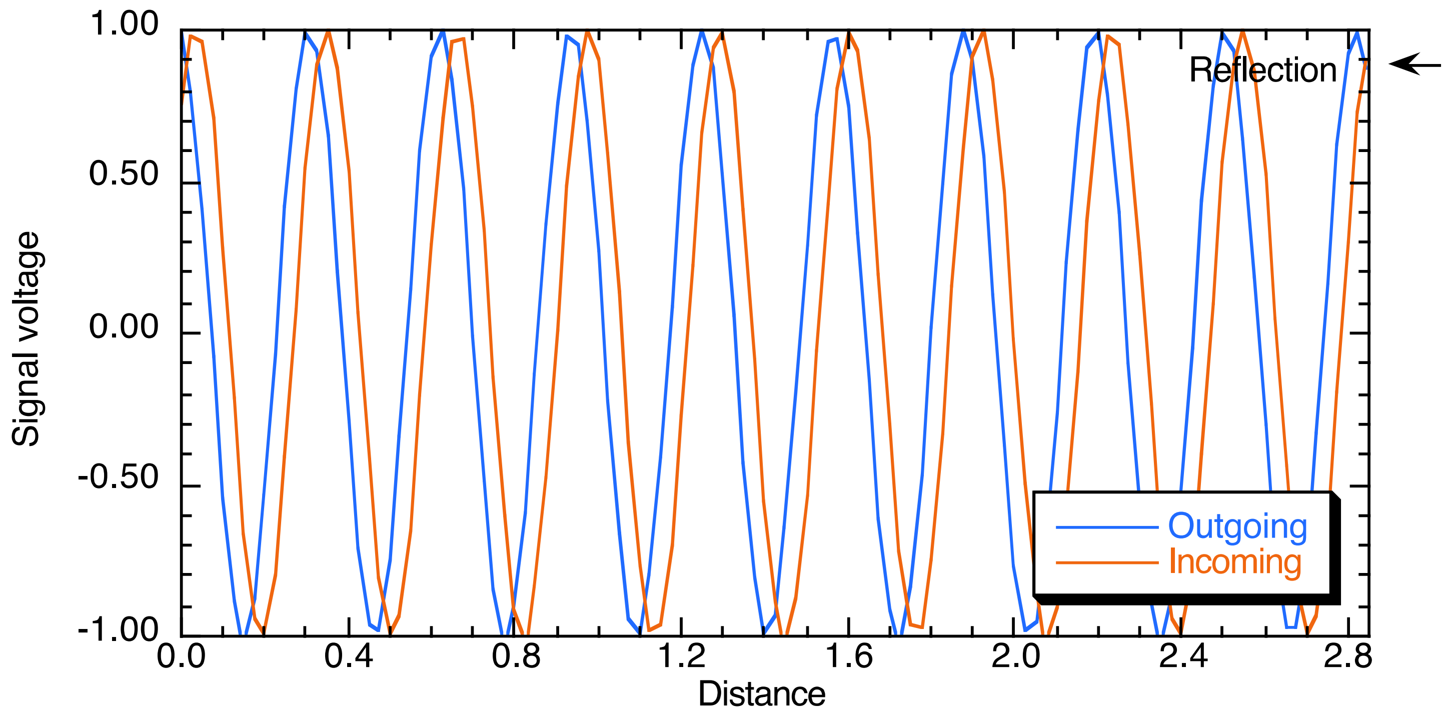
Alternative way to measure distance

- Instead of generating a short pulse and measuring round trip propagation time (also requires return pulse be detected), you can measure phase difference between outgoing and incoming continuous wave
- Schematic shown on next page
- Basic method used by interferometer

Difference measurement (stays constant with time and depends on distance)



Higher frequency. Phase difference still says something about distance but how to know number of cycles?



Mathematics behind this

- In an isotropic medium a propagating electromagnetic wave can be written as:

$$\vec{E}(t,\mathbf{x}) = E_0 e^{-i(\omega t - 2\pi \mathbf{k}.\mathbf{x})} = E_0 e^{-i2\pi(ft - \mathbf{k}.\mathbf{x})}$$

- Where E is the vector electric field, t is time, **x** is position and **k** is wave-vector (unit vector in direction of propagation divided by wavelength λ = velocity of light/frequency
  ω is frequency in radians/second, *f* is frequency in cycles/second.

Basic mathematics

- When an antenna is placed in the electric field (antenna in this case can be as simple as a piece of wire), the E-field induces a voltage difference between parts of the antenna that can be measured and amplified
- For static receiver and antenna, the voltage V is

$$V(t) = GE_0 e^{i2\pi\mathbf{k}.\mathbf{x}_0} e^{-i\omega t} = GE_0 e^{i2\pi\mathbf{k}.\mathbf{x}_0} e^{-i2\pi ft}$$

- G is gain of antenna. The phase of signal is $2\pi\mathbf{x_0.k}$

Basic Mathematics

- The use of complex notation in EM theory is common. The interpretation is that the real part of the complex signal is what is measured
- To recover the phase, we multiple the returned signal by the outgoing signal (beating the two signals together)
- Take the outgoing signal to be $V_o\cos(2\pi ft)$
- You also generate a $\pi/2$ lagged version $V_o\sin(2\pi ft)$
- These are called quadrature channels and they are multiplied by the returning signal

Basic Mathematics

- Using trigonometric identities:

$$\mathrm{Re}(e^{-ia}\cos b) = \cos a \cos b = 1/2\left[\cos(a+b)+\cos(a-b)\right]$$
$$\mathrm{Im}(e^{-ia}\cos b) = \sin a \cos b = 1/2\left[\sin(a+b)-\sin(a-b)\right]$$

- Using these relationships we can derive the output obtained by multiplying by cos and sin versions of the outgoing signal are

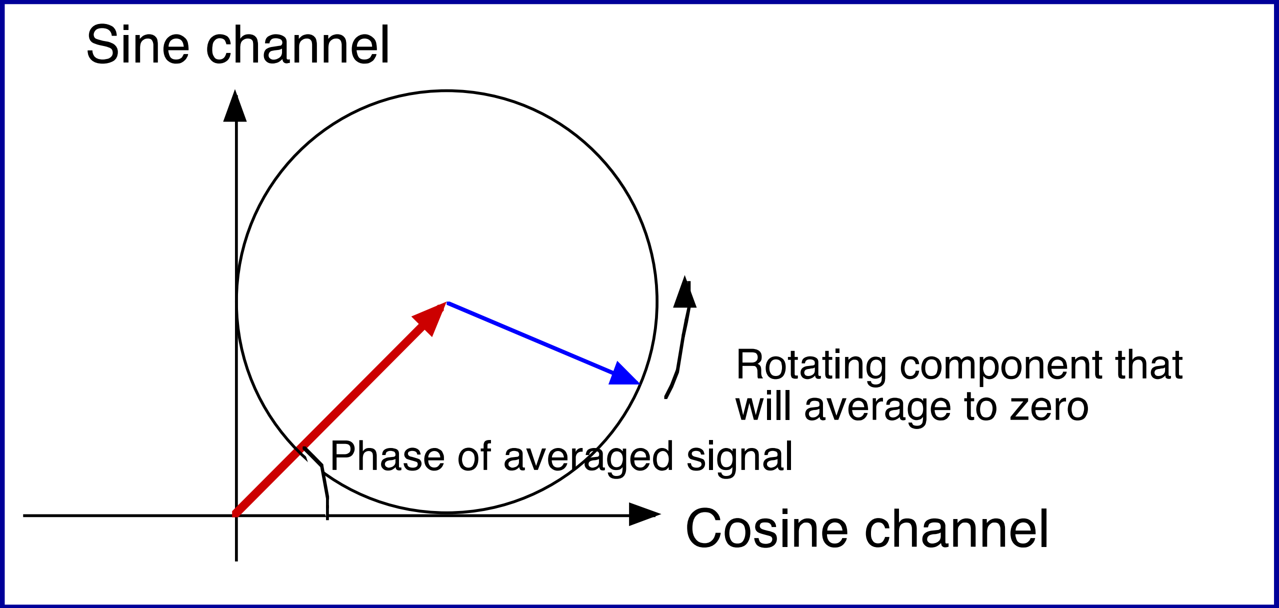
$$V(t)\cos 2\pi ft = 1/2\, GE_0[\cos 2\pi \mathbf{k}.\mathbf{x}_0 + \cos(2\pi \mathbf{k}.\mathbf{x}_0 + 4\pi ft)]$$
$$V(t)\sin 2\pi ft = 1/2\, GE_0[\sin 2\pi \mathbf{k}.\mathbf{x}_0 + \sin(2\pi \mathbf{k}.\mathbf{x}_0 + 4\pi ft)]$$

Basic mathematics

- Notice the $4\pi ft$ term: this is twice the frequency of the original signal and by averaging the product over a period long compared to 1/f, this will average to zero
- The remaining terms are the cosine and sine of the phase
- This is an example of the “modulation theorem” of Fourier transforms

Phasor Diagrams

- These cosine and sine output are often represented in EM theory by phasor diagrams
- In this case it would look like:



Phase measurement of distance

- Phase difference between outgoing and incoming reflected tells something about distance
- If distance is less than 1 wavelength then unique answer
- But if more than 1 wavelength, then we need to number of integer cycles (return later to this for GPS).
- For surveying instruments that make this type of measurement, make phase difference measurements at multiple frequencies. (Often done with modulation on optical carrier signal).

Resolving ambiguities

- The range accuracy will be low for long-wavelength modulation: Rule of thumb: Phase can be measured to about 1% of wavelength
- For EDM: Use multiple wavelengths each shorter using longer wavelength to resolve integer cycles (example next slide)
- Using this method EDM can measure 10's of km with millimeter precision

Ambiguity example

- A typical example would be: Measure distances to 10 km using wavelengths of 20 km, 1 km, 200 m, 10 m, 0.5 m
- True distance 11 785.351 m

Wavelength	Cycles	Resolved	Distance
20 km	0.59	0.59	11800
1 km	0.79	11.79	11790
200 m	0.93	58.93	11786
10 m	0.54	1178.54	11785.4
0.5m	0.70	23570.70	11785.350

EDM basics and GPS

- For optical systems where reflection is from a mirror, this method works well
- For microwave, a simple reflector is difficult (radar). Most systems are active with the “reflector” receiving the signal and re-transmitting it (transceiver)
- Satellite needs to know about ground systems
- Some systems work this way (e.g., DORIS) but it limits the number of ground stations
- GPS uses another method: One-way pseudorange measurement with bi-phase modulation (explained later)

GPS Methods

- Basic problem with conventional methods:
- Pulsed systems:
  - "Idle" time in transmission (not transmitting during gaps between pulses called "duty cycle"
  - Pulses need to be spaced enough to avoid ambiguity in which pulse is being received (There are ways around this)
- Phase modulation systems:
  - Active interaction between ground and satellite that limits number of users

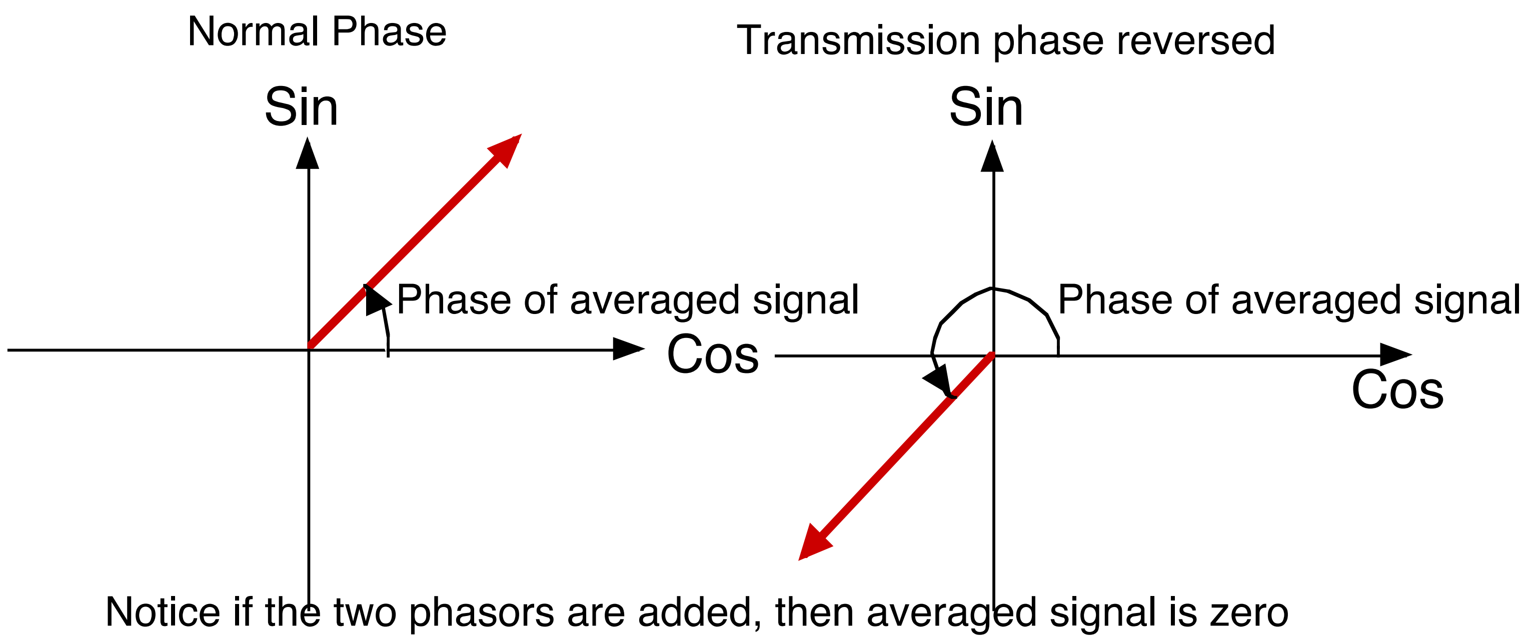
GPS Scheme

- GPS is like a pulsed ranging system except to avoid "dead time" it effectively transmits negative pulses
- To minimize range ambiguities it transmits positive and negative pulse in a known but pseudorandom sequence.
- How do you transmit as negative pulse?
- Change the phase of the outgoing signal by π thus reversing its sign -- Called bi-phase modulation
- The rate at which the sign is changed is called the "Chip rate"

GPS scheme

- To see how this works, use phasor diagrams
- Assume we multiply the incoming signal by a frequency that:
  - exactly matches the GPS frequency;
  - the sign changes occur at intervals long compared to the GPS carrier frequency
  - we average the high-frequency component
  - Phase difference between GPS and receiver is not changing
- Schematic of phasor diagrams shown next

Phasor diagrams for GPS tracking



GPS tracking

- With the sign reversals in the GPS signal, if simple tracking is used, then the signal averages to zero and satellite can not be detected
- Signal strength of GPS transmission is set such that with omni-directional antenna, signal is less than typical radio frequency noise in band – spread spectrum transmission
- Times of phase reversals must be known to track with omni-directional antenna
- Pattern of reversals is pseudorandom and each satellite has is own code.

GPS PRN

- The code is generated from a number between 1-37 (only values 1-32 are used on satellites, remainder are used for ground applications)
- This is the pseudo-random-number (PRN) for each satellite
- The 37 codes used are “orthogonal” over the chip rate interval of the code, i.e., when two codes are multiplied together you get zero.

GPS Codes

- The coding scheme is such that you can write multiple codes on the same carrier and track the signal even if one of the codes is not known
- The overall sign of the code can be changed to allow data to transmitted on the signal as well
- In the next class we look at these details

Summary of Lecture 6

- Examine the methods used to measure range with propagating EM waves
- Pulsed systems and phase systems
- GPS is a merger of the two methods
- Modulation theorem and phasor diagrams allow graphical interpretation of the results.

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