12.540 Principles of the Global Positioning System
Lecture 04

Prof. Thomas Herring

http://geoweb.mit.edu/~tah/12.540

Review

- So far we have looked at measuring coordinates with conventional methods and using gravity field
- Today lecture:
  - Examine definitions of coordinates
  - Relationships between geometric coordinates
  - Time systems
  - Start looking at satellite orbits

Coordinate types

- Potential field based coordinates:
  - Astronomical latitude and longitude
  - Orthometric heights (heights measured about an equipotential surface, nominally mean-sea-level (MSL)
- Geometric coordinate systems
  - Cartesian XYZ
  - Geodetic latitude, longitude and height

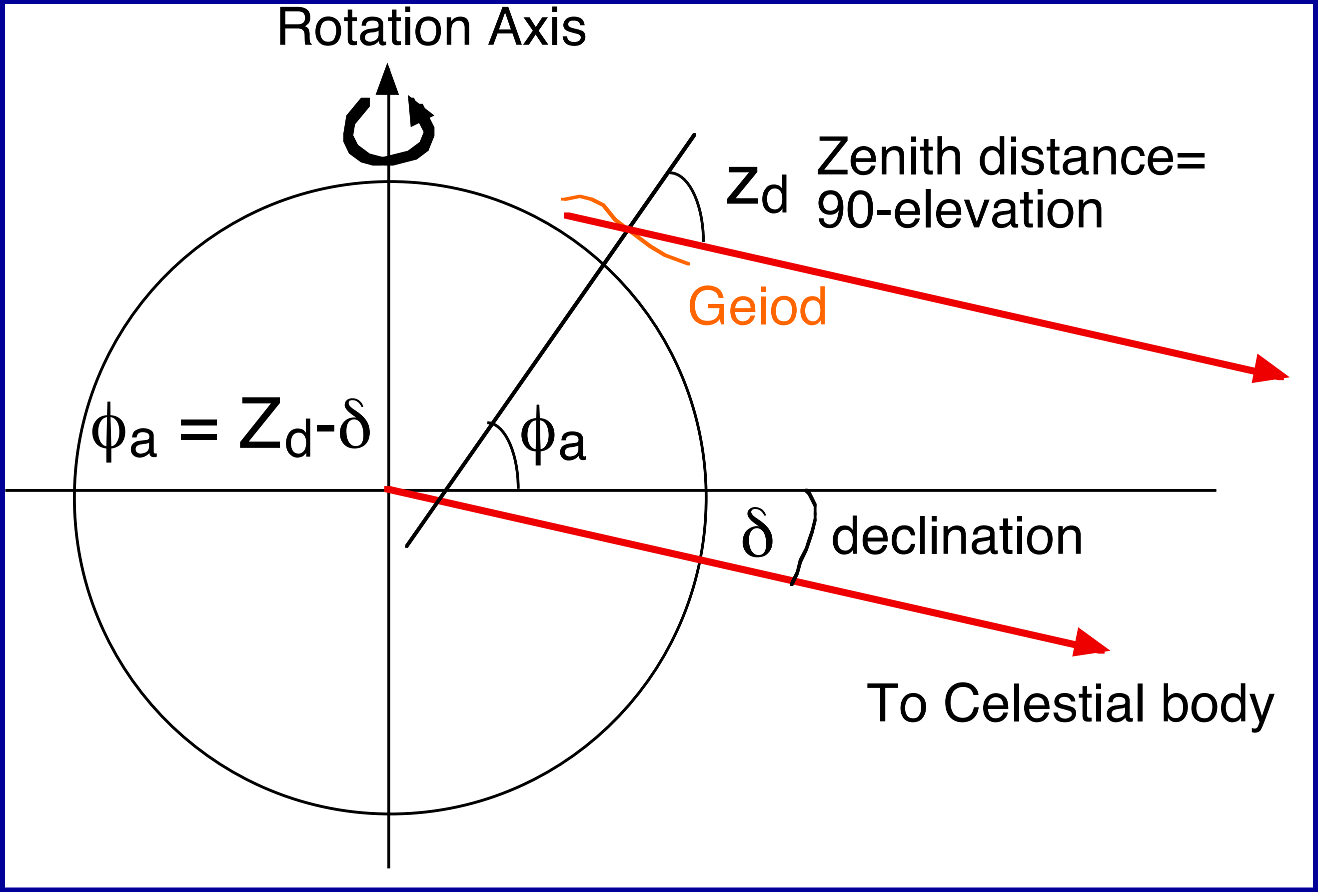
Astronomical coordinates

- Astronomical coordinates give the direction of the normal to the equipotential surface
- Measurements:
  - Latitude: Elevation angle to North Pole (center of star rotation field)
  - Longitude: Time difference between event at Greenwich and locally

Astronomical Latitude

- Normal to equipotential defined by local gravity vector
- Direction to North pole defined by position of rotation axis. However rotation axis moves with respect to crust of Earth!
- Motion monitored by International Earth Rotation Service IERS http://www.iers.org/

Astronomical Latitude



Astronomical Latitude

- By measuring the zenith distance when star is at minimum, yields latitude
- Problems:
  - Rotation axis moves in space, precession nutation. Given by International Astronomical Union (IAU) precession nutation theory
  - Rotation moves relative to crust

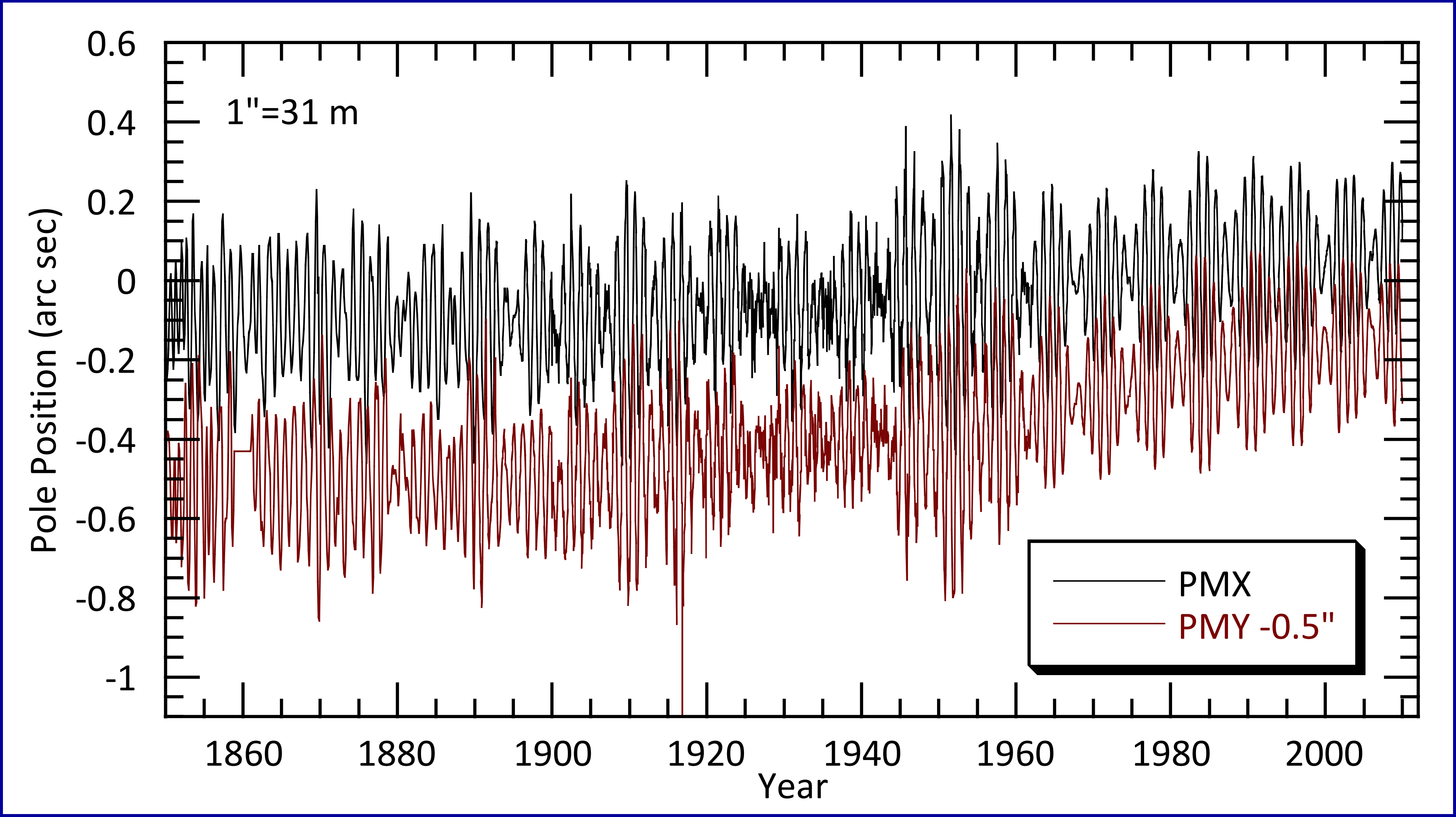
Rotation axis movement

- Precession Nutation computed from Fourier Series of motions
- Largest term 9" with 18.6 year period
- Over 900 terms in series currently (see http://geoweb.mit.edu/~tah/mhb2000/JB000165_online.pdf)
- Declinations of stars given in catalogs
- Some almanacs give positions of "date" meaning precession accounted for

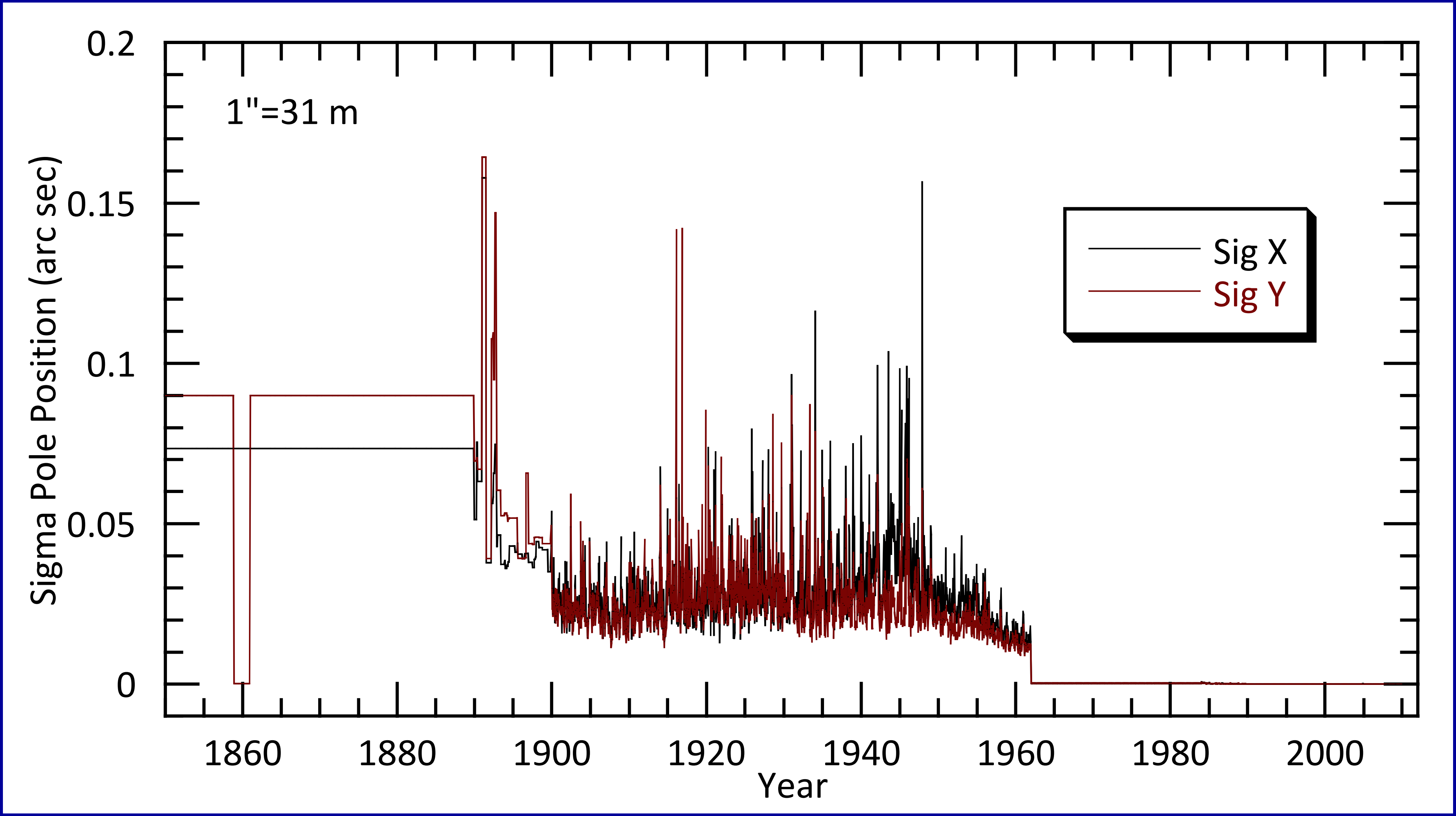
Rotation axis movement

- Movement with respect crust called "polar motion". Largest terms are Chandler wobble (natural resonance period of ellipsoidal body) and annual term due to weather
- Non-predictable: Must be measured and monitored

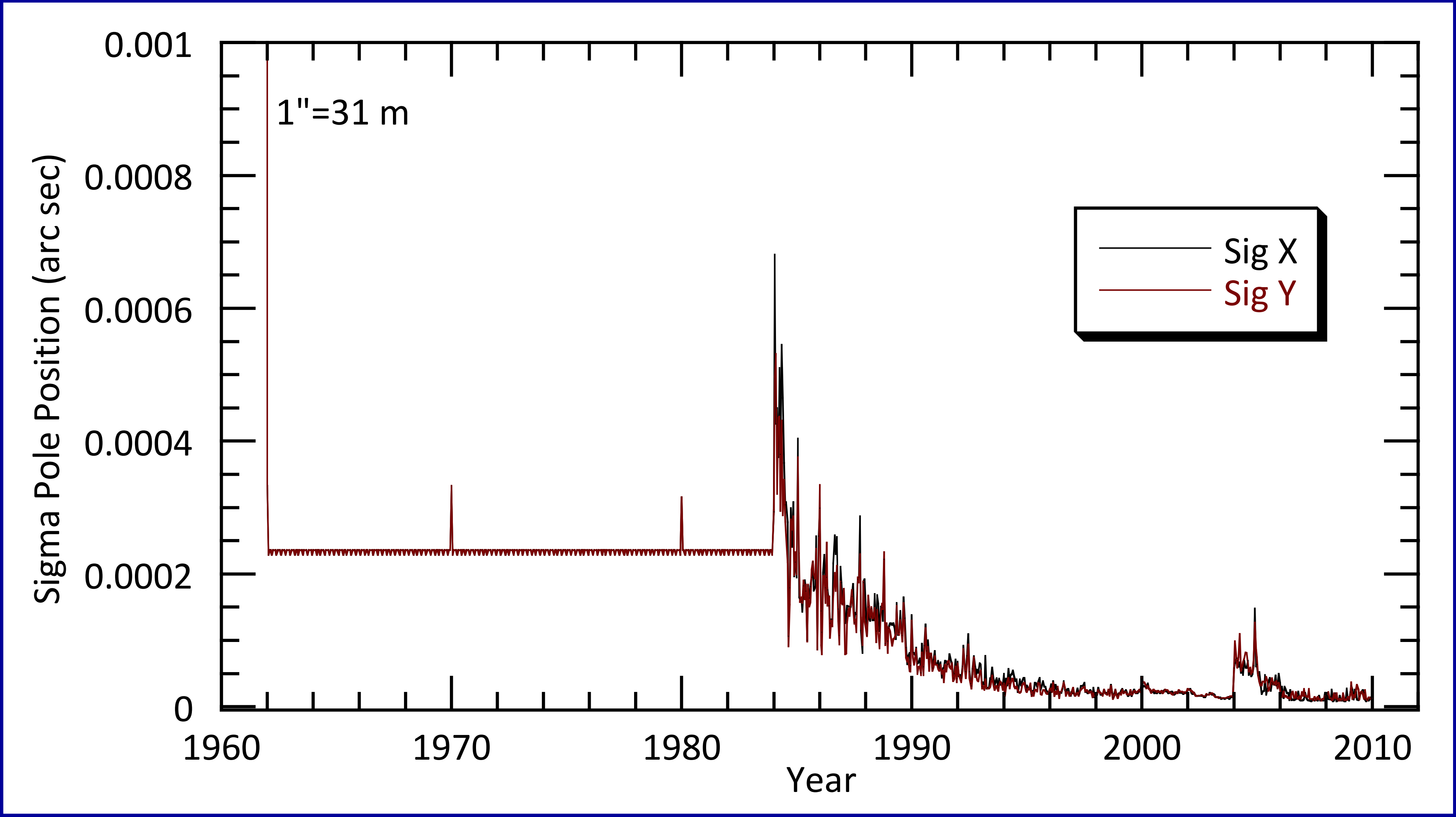
Evolution (IERS C01)

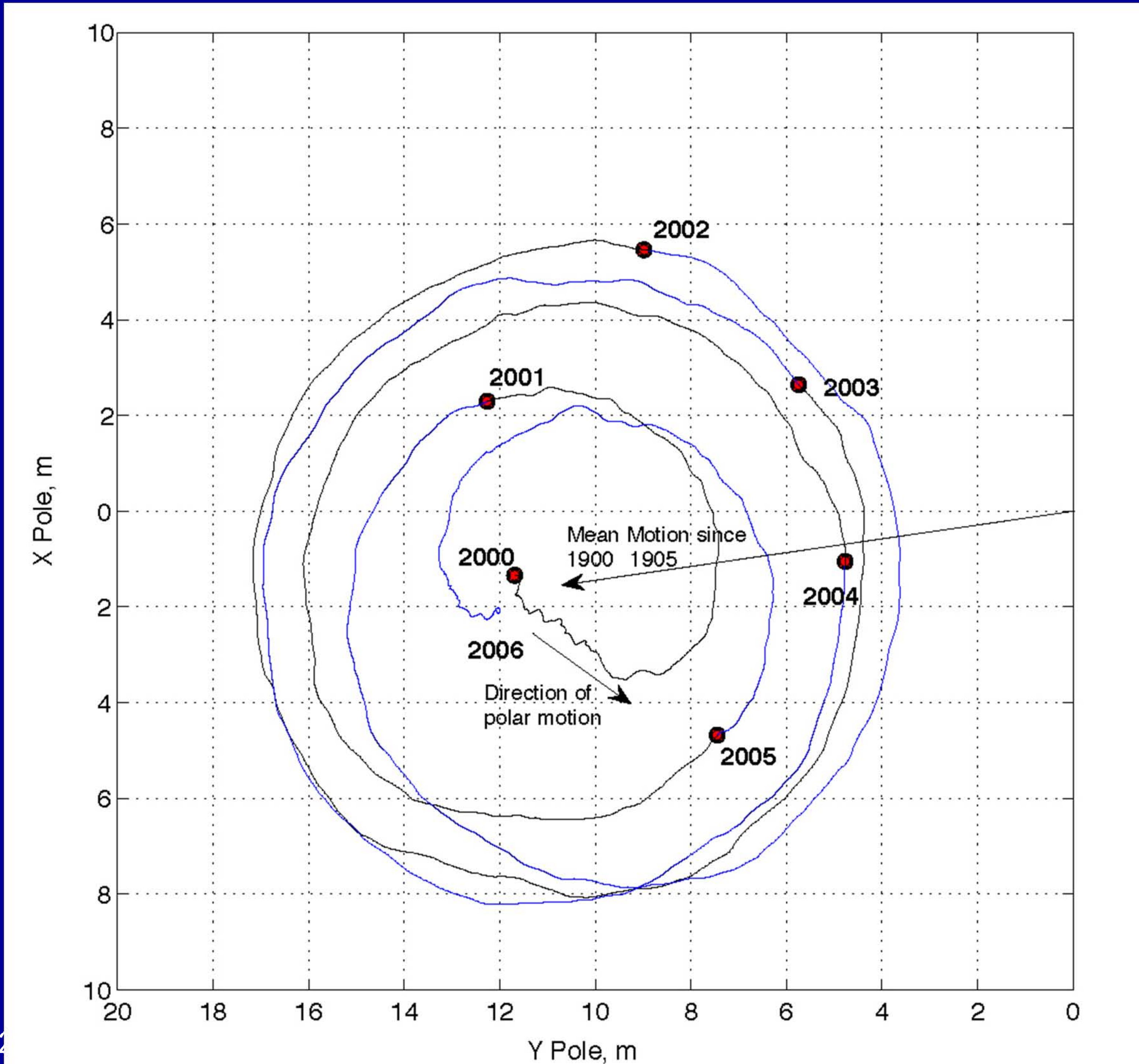


Evolution of uncertainty



Recent Uncertainties (IERS C01)




X Pole, m
Y Pole, m
10
8
6
4
2
0
2
4
6
8
10
20
18
16
14
12
10
8
6
4
2
0
2000
2001
2002
2003
2004
2005
2006
Mean Motion since
1900 1905
Direction of
polar motion

Astronomical Longitude

- Based on time difference between event in Greenwich and local occurrence
- Greenwich sidereal time (GST) gives time relative to fixed stars

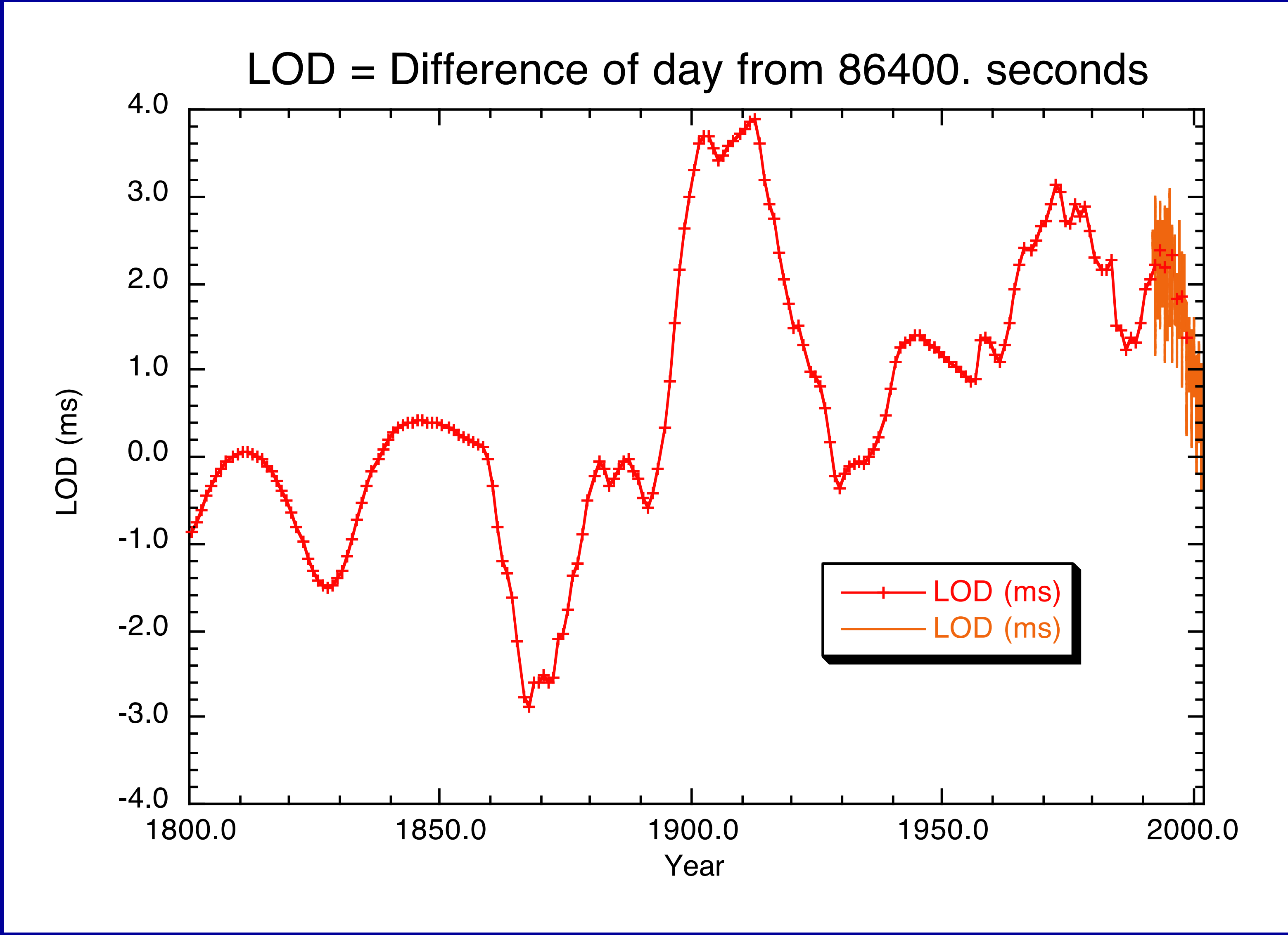
$$GST = 1.0027379093\,UT1 + \underbrace{\vartheta_0}_{\text{GMST}} + \underbrace{\Delta\psi\cos\varepsilon}_{\text{Precession}}$$

$$\vartheta_0 = 24110.54841 + 8640184.812866 \underbrace{T}_{\text{Julian Centuries}} + 0.093104T^2 - 6.2\times10^{-6}T^3$$

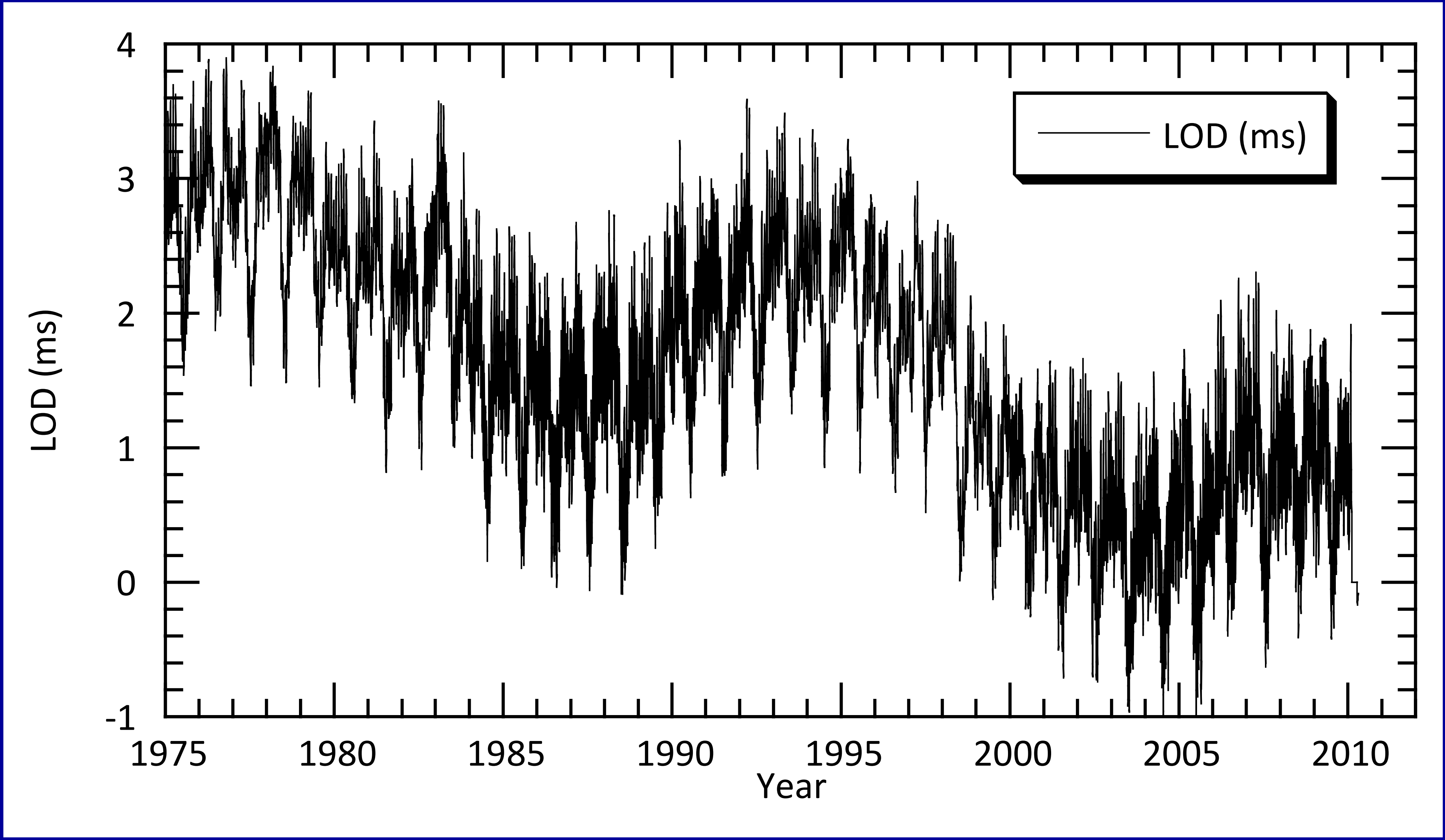
Universal Time

- UT1: Time given by rotation of Earth. Noon is "mean" sun crossing meridian at Greenwich
- UTC: UT Coordinated. Atomic time but with leap seconds to keep aligned with UT1
- UT1-UTC must be measured

Length of day (LOD)

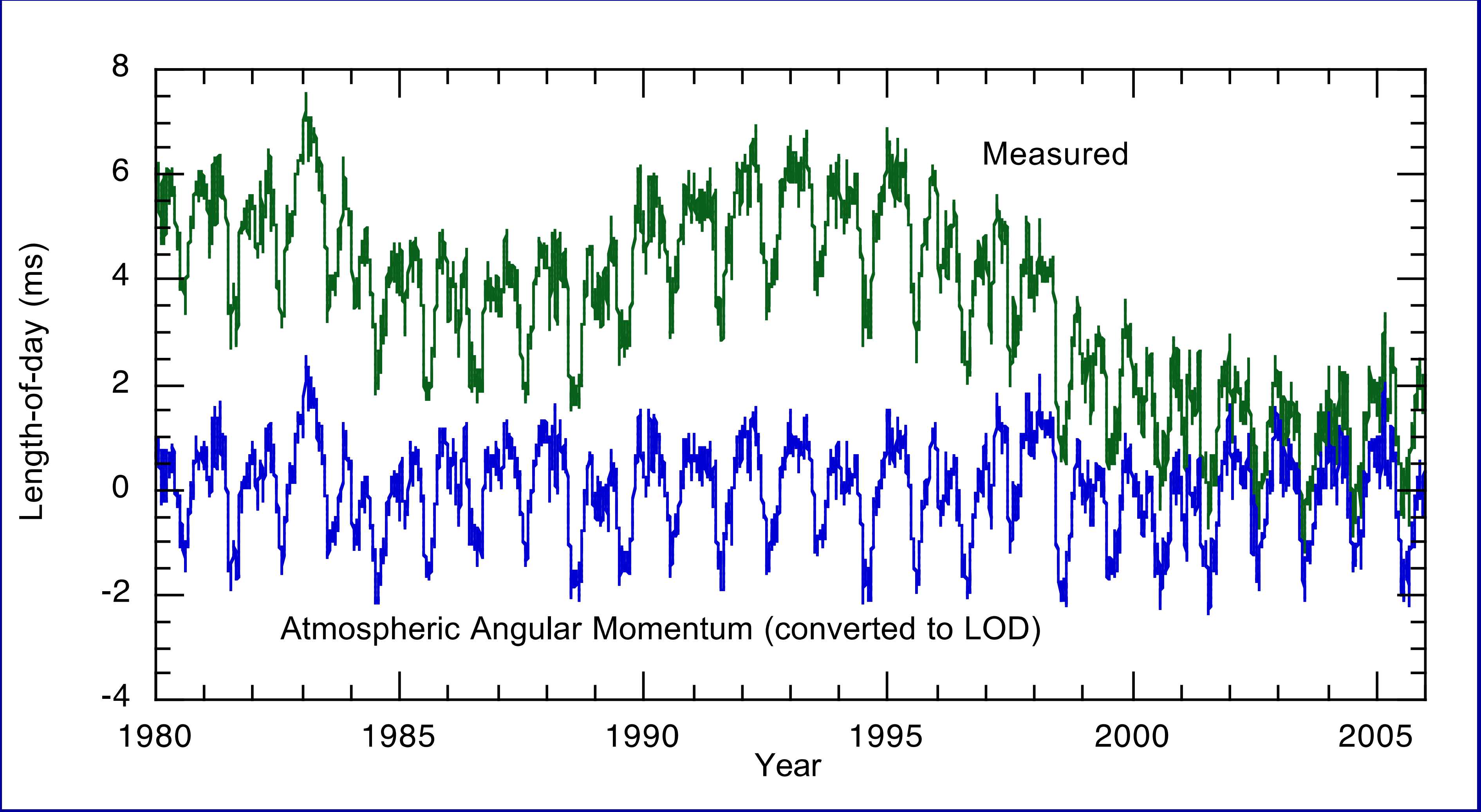


Recent LOD



LOD (ms)
4
3
2
1
0
-1
1975
1980
1985
1990
1995
2000
2005
2010
Year
LOD (ms)

LOD compared to Atmospheric Angular Momentum



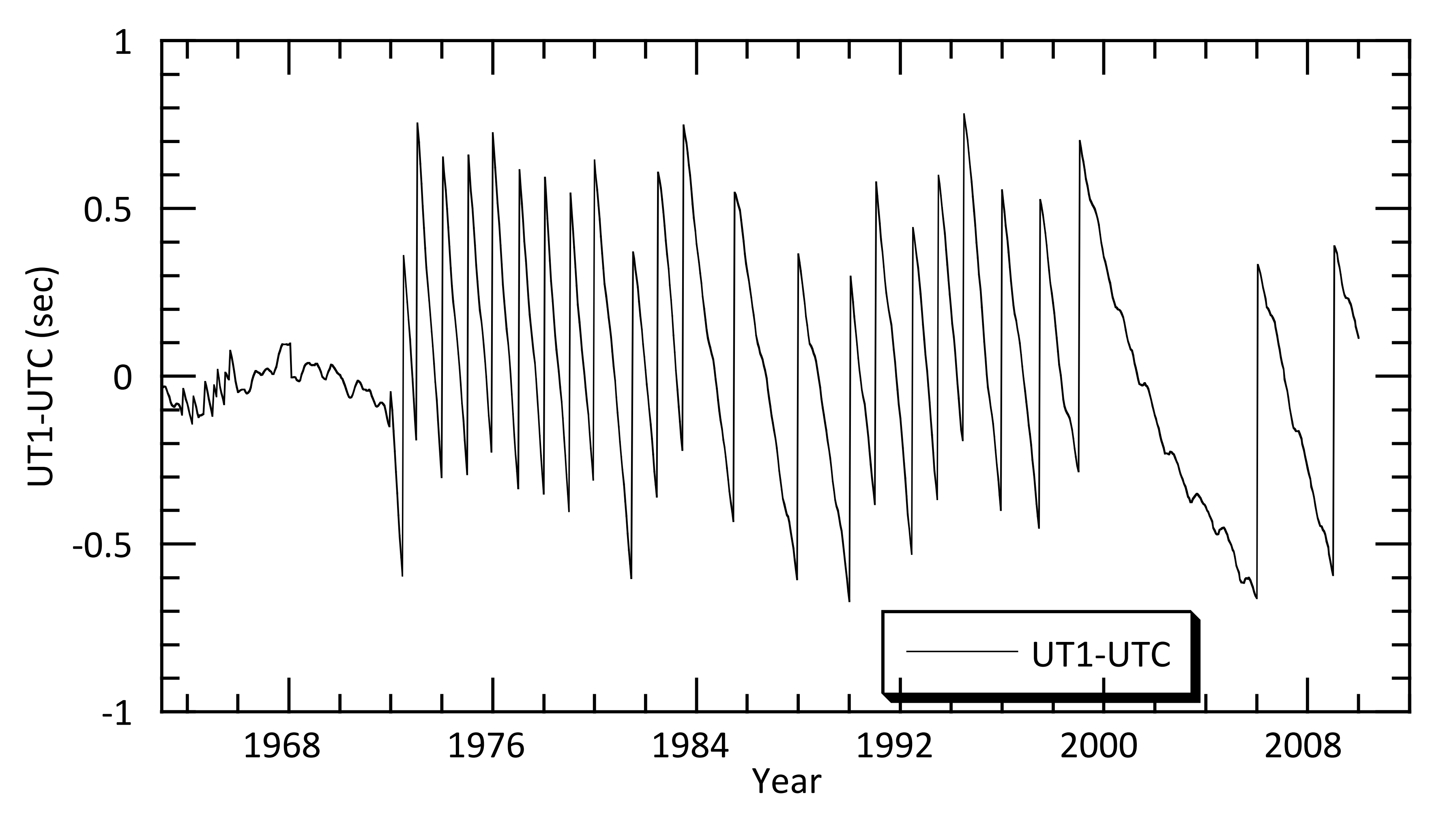
LOD to UT1

- Integral of LOD is UT1 (or visa-versa)
- If average LOD is 2 ms, then 1 second difference between UT1 and atomic time develops in 500 days
- Leap second added to UTC at those times.

UT1-UTC

•Jumps are leap seconds, longest gap 1999-2006. Historically had occurred at 12-18 month intervals

•Prior to 1970, UTC rate was changed to match UT1



Transformation from Inertial Space to Terrestrial Frame

- To account for the variations in Earth rotation parameters, as standard matrix rotation is made

$$\underbrace{x_i}_{\text{Inertial}} = \underbrace{P}_{\text{Precession}} \underbrace{N}_{\text{Nutation}} \underbrace{S}_{\text{Spin}} \underbrace{W}_{\text{Polar Motion}} \underbrace{x_t}_{\text{Terrestrial}}$$

Geodetic coordinates

- Easiest global system is Cartesian XYZ but not common outside scientific use
- Conversion to geodetic Lat, Long and Height

$$X = (N + h)\cos\phi\cos\lambda$$

$$Y = (N + h)\cos\phi\sin\lambda$$

$$Z = (\frac{b^2}{a^2}N + h)\sin\phi$$

$$N = \frac{a^2}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}}$$

Geodetic coordinates

- WGS84 Ellipsoid:
  – a=6378137 m, b=6356752.314 m
  – f=1/298.2572221 (=[a-b]/a)
- The inverse problem is usually solved iteratively, checking the convergence of the height with each iteration.
- (See Chapters 3 &10, Hofmann-Wellenhof)

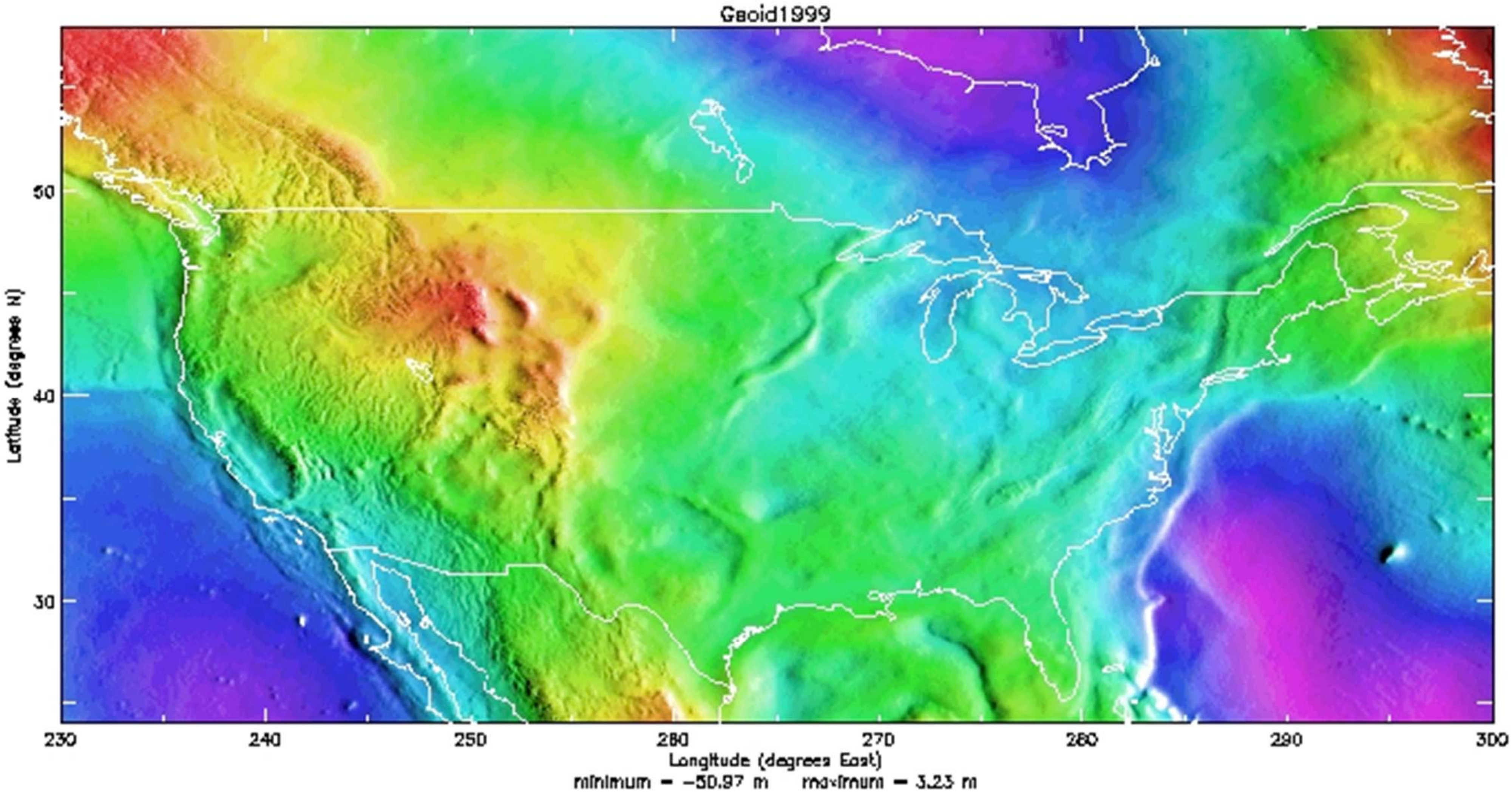
Heights

- Conventionally heights are measured above an equipotential surface corresponding approximately to mean sea level (MSL) called the geoid
- Ellipsoidal heights (from GPS XYZ) are measured above the ellipsoid
- The difference is called the geoid height

Geiod Heights

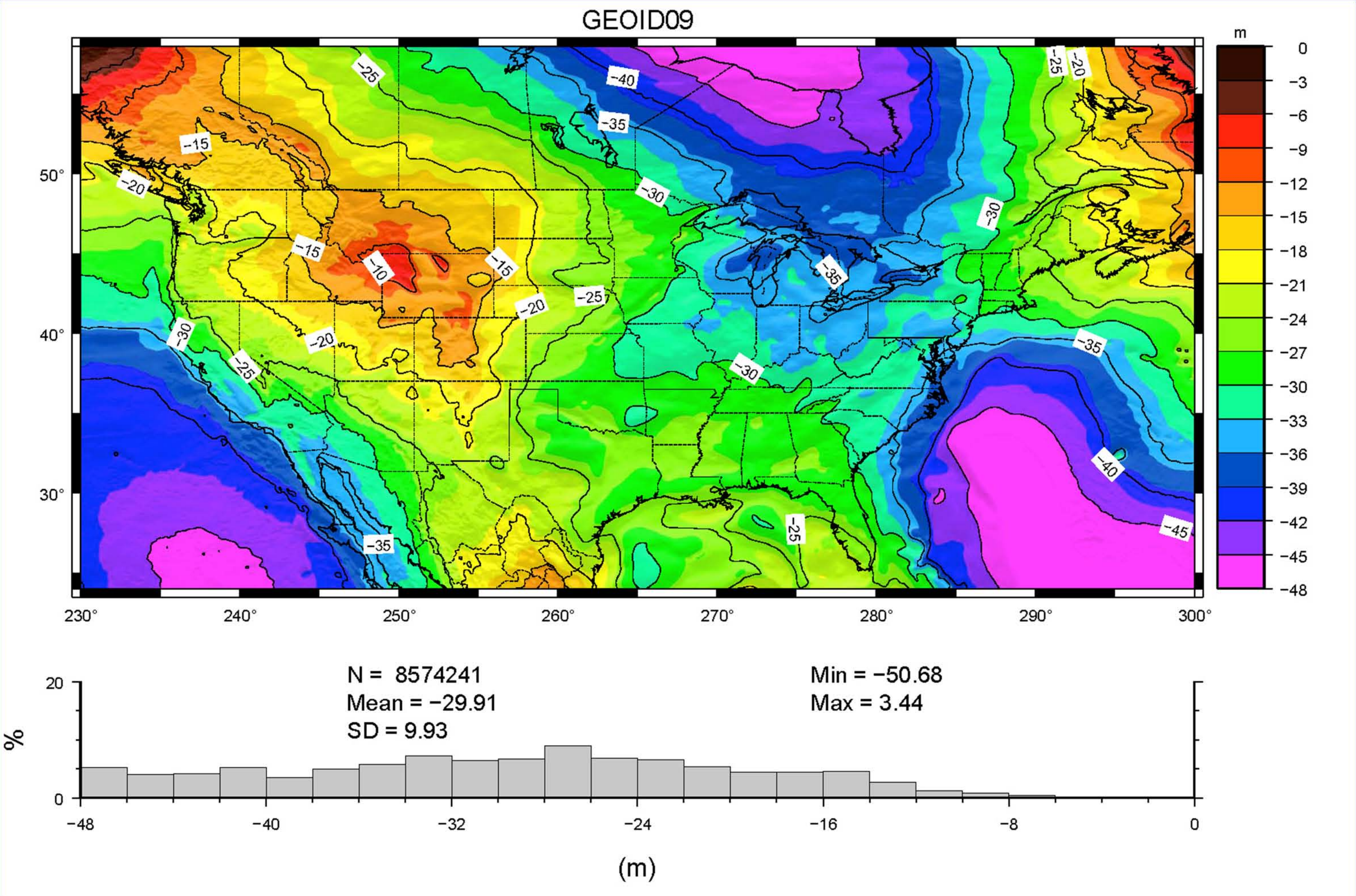
- National geodetic survey maintains a web site that allows geiod heights to be computed (based on US grid)
- http://www.ngs.noaa.gov/cgi-bin/GEOID_STUFF/geoid99_prompt1.prl
- New Boston geiod height is -27.688 m

NGS Geoid 99 http://www.ngs.noaa.gov/GEOID/GEOID99/



http://www.ngs.noaa.gov/GEOID/images/2009/geoid09conus.jpg

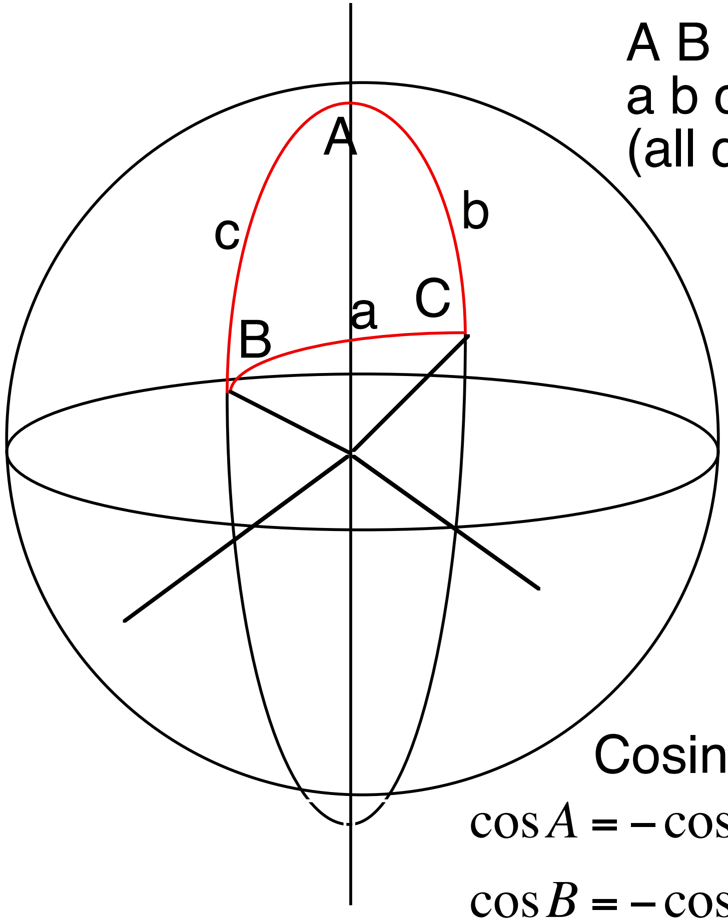
NGS GEIOD09



Spherical Trigonometry

- Computations on a sphere are done with spherical trigonometry. Only two rules are really needed: Sine and cosine rules.
- Lots of web pages on this topic (plus software)
- http://mathworld.wolfram.com/SphericalTrigonometry.html is a good explanatory site

Basic Formulas



A B C are angles
a b c are sides
(all quanties are angles)

Sine Rule

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

Cosine Rule sides

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos b = \cos c \cos c + \sin c \sin a \cos B$$

$$\cos c = \cos b \cos a + \sin a \sin b \cos C$$

Cosine Rule angles

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$

$$\cos B = -\cos A \cos C + \sin A \sin C \cos b$$

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c$$

Basic applications

- If b and c are co-latitudes, A is longitude difference, a is arc length between points (multiply angle in radians by radius to get distance), B and C are azimuths (bearings)
- If b is co-latitude and c is co-latitude of vector to satellite, then a is zenith distance (90-elevation of satellite) and B is azimuth to satellite
- (Colatitudes and longitudes computed from ΔXYZ by simple trigonometry)

Summary of Coordinates

- While strictly these days we could realize coordinates by center of mass and moments of inertia, systems are realized by alignment with previous systems
- Both center of mass (1-2cm) and moments of inertia (10 m) change relative to figure
- Center of mass is used based on satellite systems
- When comparing to previous systems be cautious of potential field, frame origin and orientation, and ellipsoid being used.