12.540 Principles of the Global Positioning System Lecture 02

Prof. Thomas Herring

http://geoweb.mit.edu/~tah/12.540

Coordinate Systems

- Today we cover:
  - Definition of coordinates
  - Conventional "realization" of coordinates
  - Modern realizations using spaced based geodetic systems (such as GPS).

Coordinate system definition

- To define a coordinate system you need to define:
  - Its origin (3 component)
  - Its orientation (3 components, usually the direction cosines of one axis and one component of another axes, and definition of handed-ness)
  - Its scale (units)

Coordinate system definition

- In all 7 quantities are needed to uniquely specify the frame.
- In practice these quantities are determined as the relationship between two different frames
- How do we measure coordinates
- How do we define the frames

Measuring coordinates

- Direct measurement (OK for graph paper)
- Triangulation: Snell 1600s: Measure angles of triangles and one-distance in base triangle
- Distance measured with calibrated “chain” or steel band (about 100 meters long)
- “Baseline” was about 1 km long
- Triangles can build from small to larges ones.
- Technique used until 1950s.

Measuring coordinates

- Small errors in the initial length measurement, would scale the whole network
- Because of the Earth is “nearly” flat, measuring angles in horizontal plane only allows “horizontal coordinates” to be determined.
- Another technique is needed for heights.

Measuring coordinates

- In 1950s, electronic distance measurement (EDM) became available (out growth of radar)
- Used light travel times to measure distance (strictly, travel times of modulation on either radio, light or near-infrared signals)

Measuring coordinates

- Advent of EDM allowed direct measurements of sides of triangles
- Since all distances measured less prone to scale errors.
- However, still only good for horizontal coordinates

Accuracies

- Angles can be measured to about 1 arc second ($5x10^{-6}$ radians)
- EDM measures distances to $1x10^{-6}$ (1 part-per-million, ppm)
- Atmospheric refraction 300 ppm
- Atmospheric bending can be 60" (more effect on vertical angles)

Height coordinates

- Two major techniques:
  - Measurement of vertical angles (atmospheric refraction)
  - "Leveling" measurement of height differences over short distances (<50 meters).
  - Level lines were used to transfer height information from one location to another.

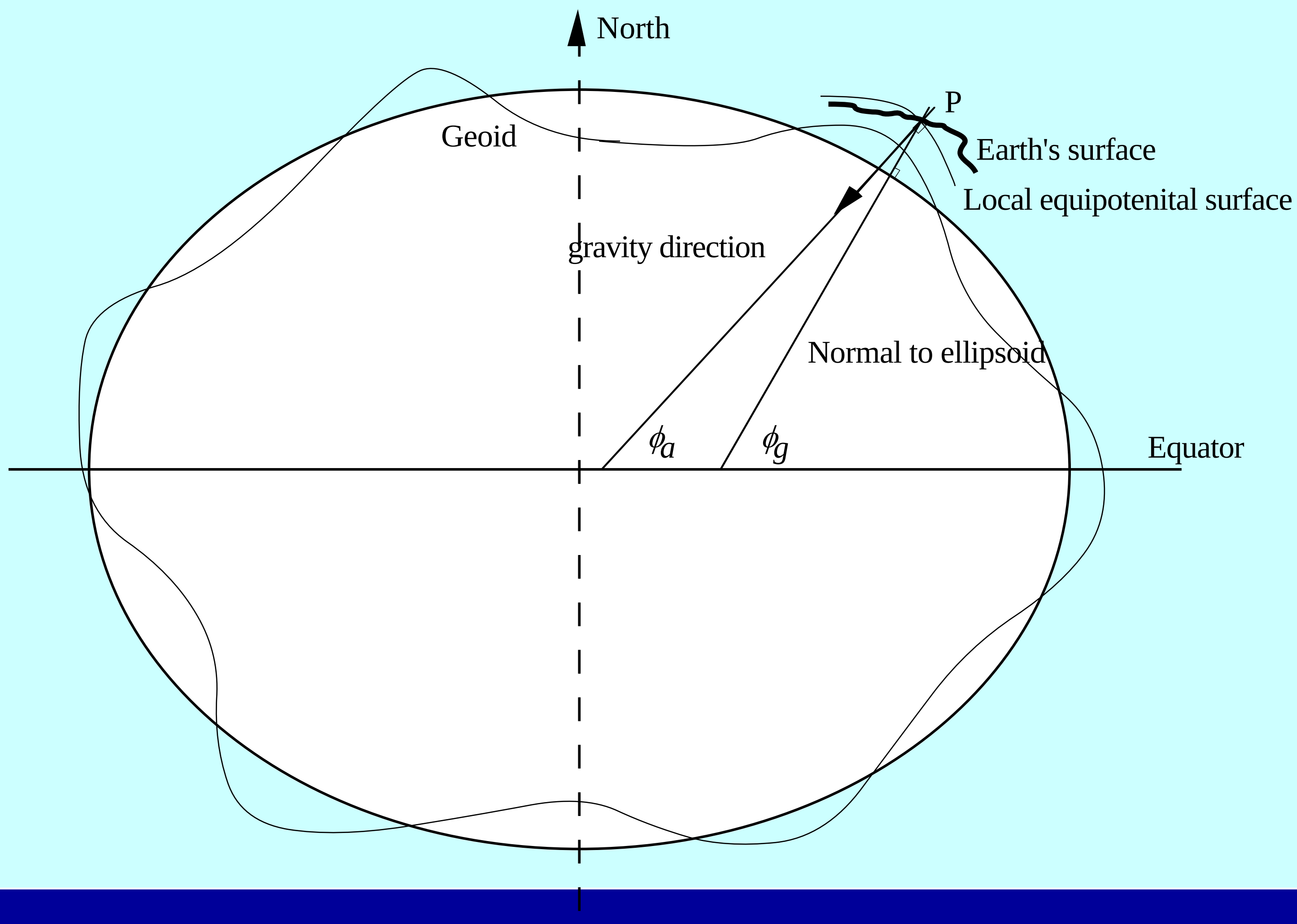
Other methods

- Maps were made with “plotting tables” (small telescope and angular distance measurements-angle subtended by a known distance
- Aerial photogrammetry coordinates inferred from positions in photographs. Method used for most maps

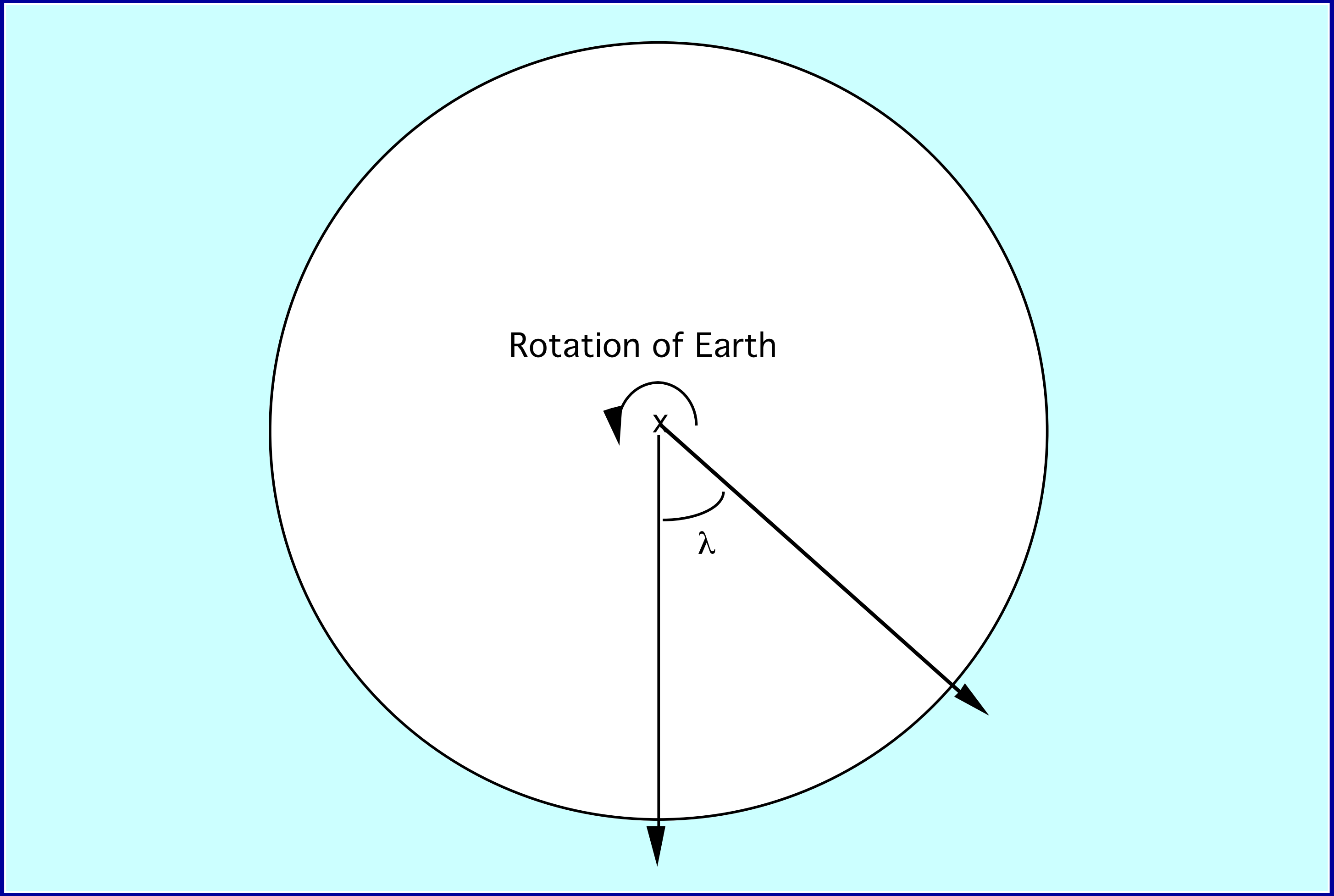
Other methods

- What is latitude and longitude
- Based on spherical model what quantities might be measured
- How does the rotation of the Earth appear when you look at the stars?
- Concept of astronomical coordinates

Geodetic coordinates: Latitude



Longitude



Longitude measured by time difference of astronomical events

Astronomical coordinates

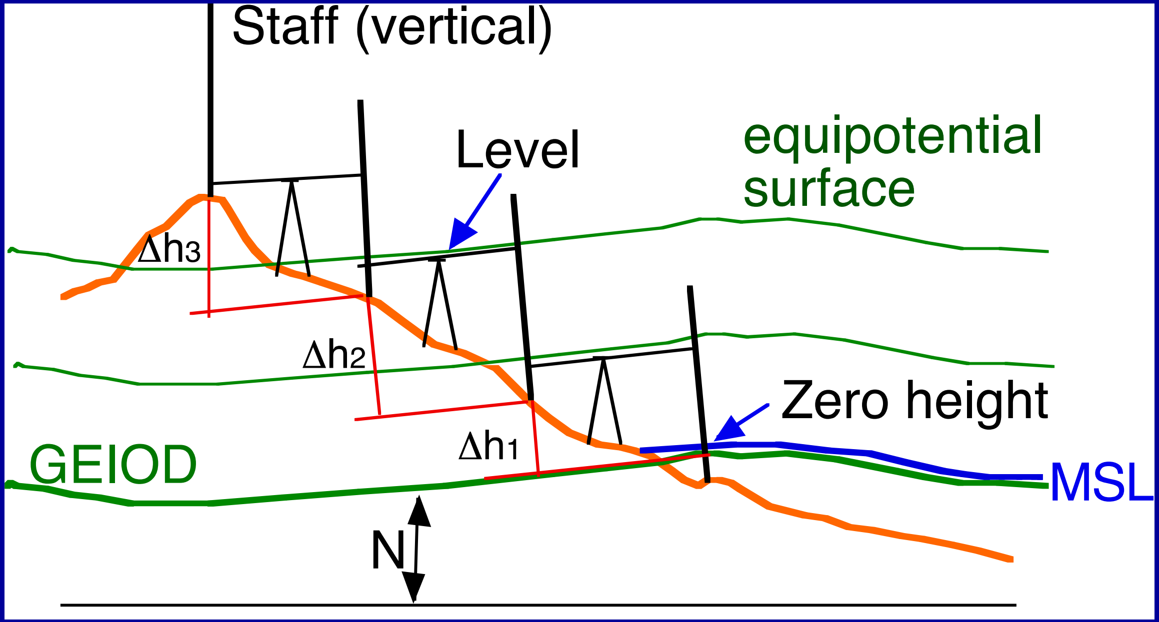
- Return to later but on the global scale these provide another method of determining coordinates
- They also involve the Earth's gravity field
- Enters intrinsically in triangulation and trilateration through the planes angles are measured in

Height determination

- Height measurements historically are very labor intensive
- The figure on the next page shows how the technique called leveling is used to determine heights.
- In a country there is a primary leveling network, and other heights are determined relative to this network.
- The primary needs to have a monument spacing of about 50 km.

Leveling

- The process of leveling is to measure height differences and to sum these to get the heights of other points.



Orthometric height of hill is $\Delta h_1+\Delta h_2+\Delta h_3$

N is Geoid Height. Line at bottom is ellipsoid

Leveling

- Using the instrument called a level, the heights on the staffs are read and the difference in the values is the height differences.
- The height differences are summed to get the height of the final point.
- For the primary control network: the separation of the staffs is between 25-50 meters.
- This type of chain of measurements must be stepped across the whole country (i.e., move across the country in 50 meter steps: Takes decades and was done).

Leveling problems

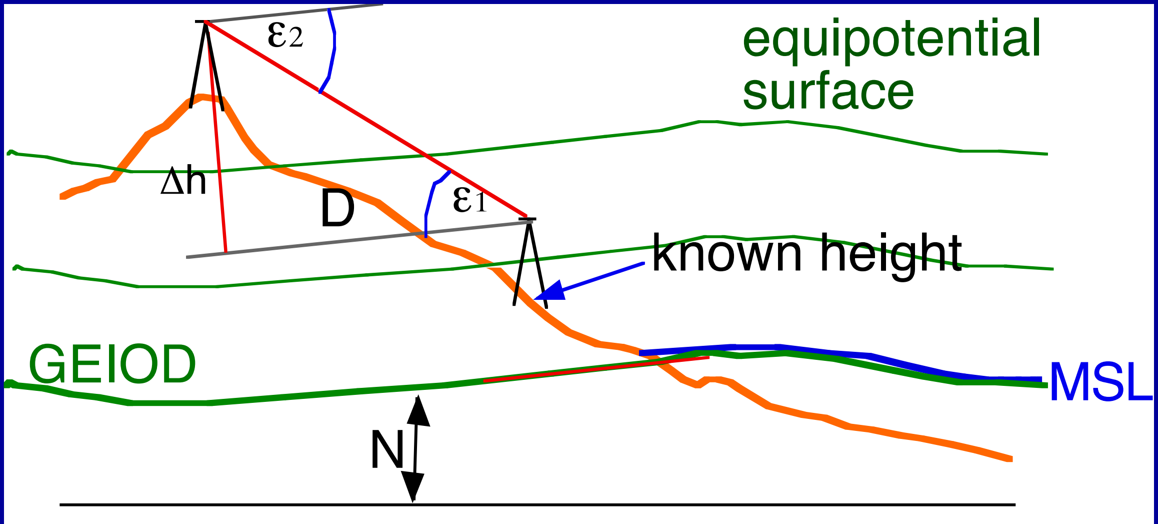
- Because heights are determined by summing differences, system very prone to systematic errors; small biases in the height differences due to atmospheric bending, shadows on the graduations and many other types of problem
- Instrument accuracy is very good for first-order leveling: Height differences can be measured to tens of microns.
- Accuracy is thought to about 1 mm-per-square-root-km for first order leveling.
- Changes in the shapes of the equipotential surface with height above MSL also cause problems.
- The difference between ellipsoidal height and Orthometric height is the Geoid height

Trigonometric Leveling

- When trying to go the tops of mountains, standard leveling does not work well. (Image trying to do this to the summit of Mt. Everest).
- For high peaks: A triangulation method is used call trigonometric leveling.
- Schematic is shown on the next slide
- This is not as accurate as spirit leveling because of atmospheric bending.

Trigonometric Leveling schematic

- Method for trigonometric leveling. Method requires that distance D in known and the elevation angles are measured. Trigonometry is used to compute Δh



Trigonometric Leveling

- In ideal cases, elevation angles at both ends are measured at the same time. This helps cancel atmospheric refraction errors.
- The distance D can be many tens of kilometers.
  In the case of Mt. Everest, D was over 100 km (the survey team was not even in the same country; they were in India and mountain is in Nepal).
- D is determined either by triangulation or after 1950 by electronic distance measurement (EDM) discussed later
- The heights of the instruments, called theodolites, above the ground point must be measured. Note: this instrument height measurement was not needed for leveling.

Web sites about geodetic measurements

- http://sco.wisc.edu/surveying/networks.php Geodetic control for Wisconsin
- Try search “trilateration network” search. Finding maps of networks is now difficult (replaced with GPS networks)
- http://www.ngs.noaa.gov/ is web page of National Geodetic Survey which coordinates national coordinate systems

Earth's Gravity field

- All gravity fields satisfy Laplace's equation in free space or material of density ρ. If V is the gravitational potential then

$$\nabla^2 V = 0$$

$$\nabla^2 V = 4\pi G\rho$$

Solution to gravity potential

- The homogeneous form of this equation is a “classic” partial differential equation.
- In spherical coordinates solved by separation of variables, r=radius, λ=longitude and θ=co-latitude

$$V(r,\theta,\lambda) = R(r)g(\theta)h(\lambda)$$

Summary

- Examined conventional methods of measuring coordinates
- Triangulation, trilateration and leveling
- Astronomical positioning uses external bodies and the direction of gravity field
- Continue with the use of the gravity field.

12.540 Principles of the Global Positioning System
Spring 2012