12.540 Principles of the Global Positioning System
Lecture 19

Prof. Thomas Herring

http://geoweb.mit.edu/~tah/12.540

GPS Models and processing

- Summary:
  - Finish up modeling aspects
    - Rank deficiencies
  - Processing methods:
    - Differencing of data
    - Cycle slip detection
    - Bias fixing and cycle slip repair

Rank deficiencies

- Ranks deficiencies are combinations of parameters that can not be separately estimated.

- In GPS, there are several rank deficiencies:
  - UT1, Longitudes of all the stations and the nodes of the satellite orbits, effectively can not be separated.
  - In theory, orbit perturbations by the moon/sun on the GPS orbits could be used to align the orbits in a solar system frame, but effect is too small to be useful (I think: never really tested)
  - Separation is solved by adopting UT1-AT from VLBI, and setting the mean longitude change of stations to ITRF coordinates. Longitude is standard problem because choice of Greenwich as origin is arbitrary.

Rank deficiencies

- Other rank deficiencies:
  - Pole position can not separated from over all rotation of coordinates. Again resolved either by adopting polar motions on one day or on average having zero rotation of the coordinates relative to an initial frame.
  - In principle could be separated by gravity field perturbations but effect is too small.
  - All station and satellite clocks can not be simultaneously estimated. Again there is sensitivity due moon/sun perturbations but these are too small. (Later we will see how differencing data, implicitly eliminates this problem). Solution, if clocks are explicitly estimated, is to adopt one clock as reference or set an average of the clock differences to be zero.
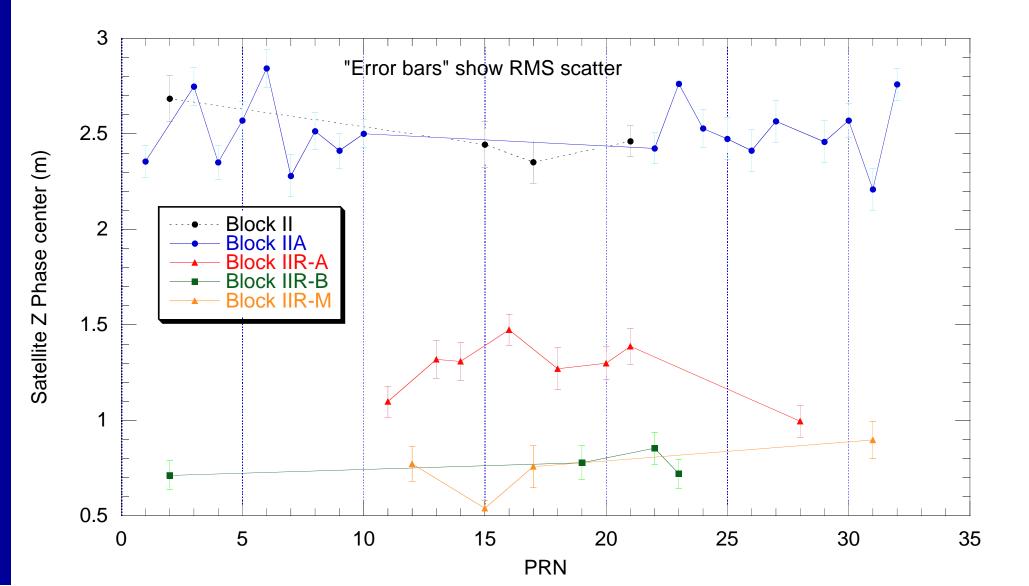
Rank deficiencies

- Velocity rank deficiency:
  - It is not possible to separate "absolute" station motions from secular drift of pole and secular UT1-AT changes. (Remember pole has drifted 10 meters in 100 years--10 cm/yr comparable to plate motions).
  - IERS polar motion is referred to a no-net-rotation geologic frame (Nuvel-1A).

- There are some other rank deficiencies with nutations and orbits, but the apriori nutation series is very well defined by VLBI
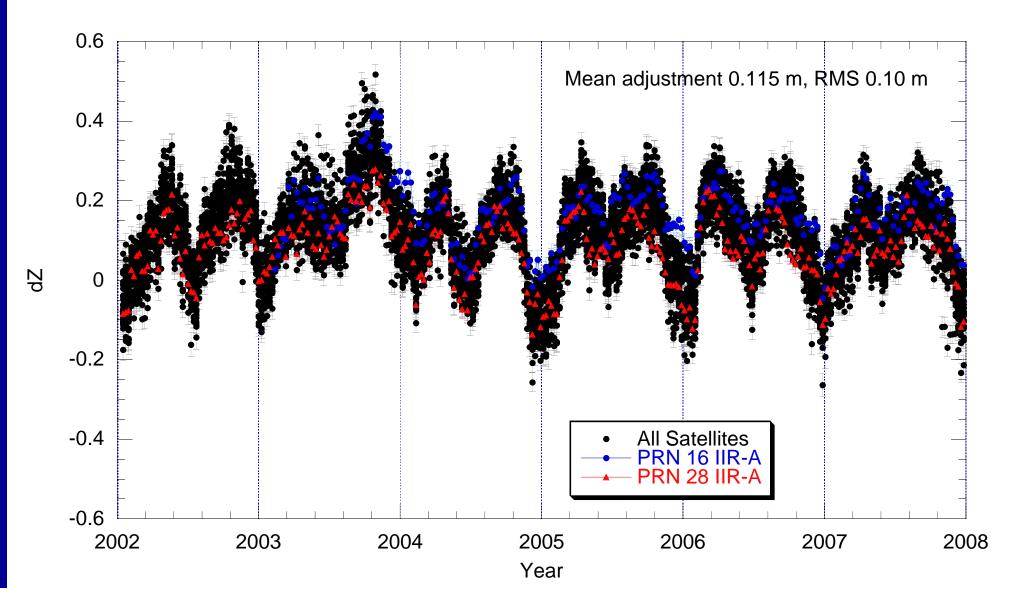
Subtle rank deficiencies

- Phase center patterns for satellites and ground receivers can not separately determined using just GPS antennas.
- Because the satellites point towards the center of the Earth; a given elevation angle at a GPS receiver can be mapped to an off-bore-sight angle on the satellite and two effects can not separated.
- Interestingly, if the GPS satellites could be "rocked" (so no longer pointing at the center of the Earth), the two effects could be separated.
- Even with low precision satellite phase center positions can be estimated assuming "point" antenna
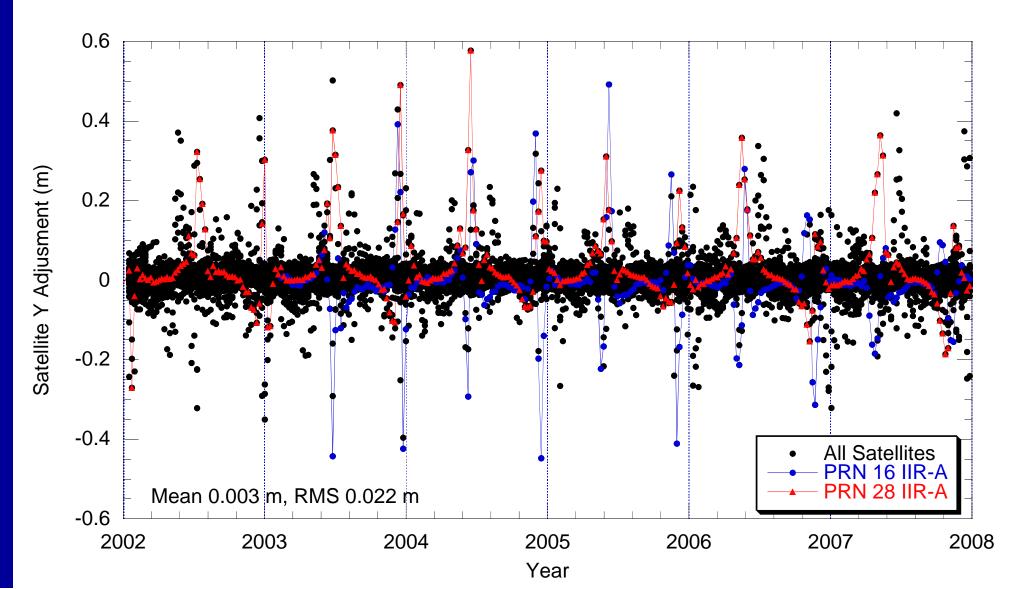
Estimated Satellite Z-offsets

Time series Z-estimates

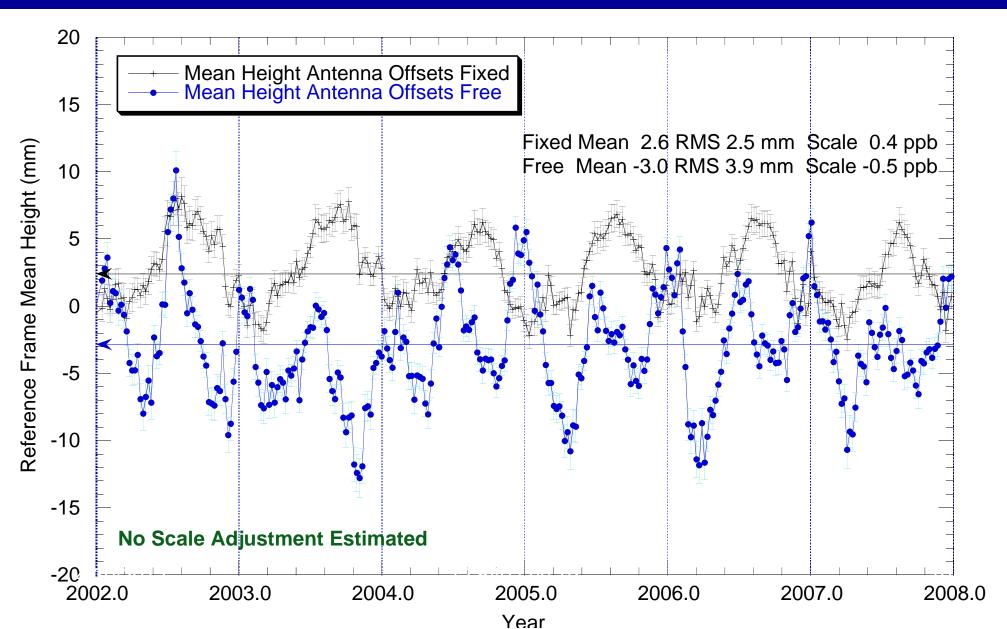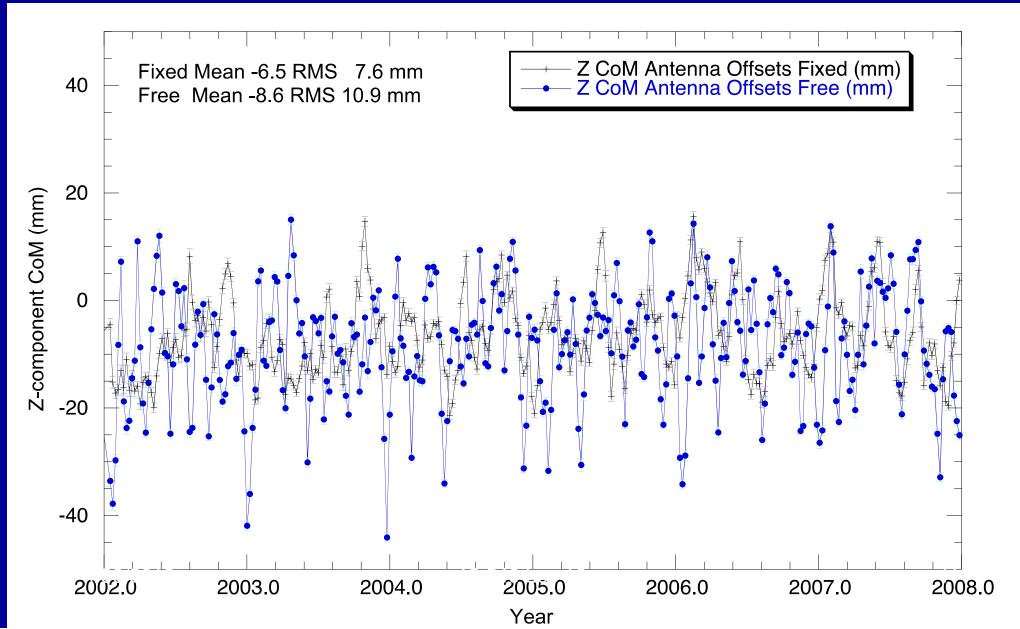Common pattern has only recently been noted and is not yet understood.

Time series Y-adjustment

Mean 0.003 m, RMS 0.022 m

Legend:
- All Satellites (black)
- PRN 16 IIR-A (blue)
- PRN 28 IIR-A (red)

X-axis: Year (2002–2008)
Y-axis: Satellite Y Adjustment (m)

Effects on Terrestrial Scale

Fixed Mean 2.6 RMS 2.5 mm Scale 0.4 ppb
Free Mean -3.0 RMS 3.9 mm Scale -0.5 ppb

Legend:
— Mean Height Antenna Offsets Fixed
— Mean Height Antenna Offsets Free

No Scale Adjustment Estimated

Y-axis: Reference Frame Mean Height (mm), from -20 to 20
X-axis: Year, from 2002.0 to 2008.0

Effect on Z-Center of Mass

Summary of phase center

- The effects of ground antenna phase center model only satellite phase center estimates are large (~3.6 meters)
- Block II/IIA definitely different from Block IIR and some indication of differences between satellites within the same type (differences are a few centimeters)
- Radial orbit changes are small (<1 cm on average). Interestingly better agreement of loose solution with constrained when satellite PC estimated (10 cm differences),

Scale effects

- With current models
  - Scale nominal 0.4 ppb (mean height difference 2.6 mm)
  - When phase center locations at GPS satellites is estimated: Scale correction is -0.5 ppb (mean height difference -3.0 mm).

- These two scale effects correspond to mean Z change in satellite phase center of 0.12 m.

- ITRF 2008 is being finalized in summer 2009 and should allow better resolution of scale.

Processing methods

- The clock and local oscillator phase variations are the biggest deviations in the model of GPS phase and range data.
- These terms can be explicitly handled by estimation of clock variations (but if done brut-force in least squares is a very large estimation problem). Can be attacked with sequential LSQ or a Kalman filter.
- When multiple sites see the same satellite, the satellite clocks can also be estimated, but at every epoch of measurement, one clock needs to be fixed, or an ensemble average of clocks set to have zero mean adjustment.

Differencing

- An alternative approach to explicit estimation is differencing data.
- Single differences: two forms:
  - Difference measurements from two sites that see the same satellite. Eliminates error due to satellite clock.
  - Difference measurements from two satellites at the one site: Eliminates the ground receiver clock.
- Double differences:
  - By differencing a pair a single differences, but the ground and satellites clocks are eliminated.
  - The local oscillator phases also cancel except the differences in the number of cycles of phase between the combination of two satellites and two stations. This difference should be an integer.

Differencing

- There are subtle problems with the exact times that measurements are made with differencing.
- In the receivers, the measurements of range and phase to all the satellites can be made at exactly the same (within electronics noise)
- But signals measured at the same time receivers separated by large distances must have been transmitted from the satellite at different times due to the light propagation time..

Light propagation time and differencing

- This effect can lead to 20 ms differences in the transmission times. When SA was on and satellite clocks had frequency drifts of ~1Hz, this lead to errors of 0.02 cycles (~4mm). Not such a problem anymore and even with SA was not severe.

- Non-synchronized receiver sampling can cause problems. Normally receivers stay with in 1 ms of GPS time (by resetting their clock counters). Older receivers could be off by up to 80 ms:

  Feigl, K. L, R. W. King, T. A. Herring, and M. Rotchacher, A scheme for reducing the effect of selective availability on precise geodetic measurements from the Global Positioning System, *Geophys. Res. Lett.*, 1289–1292, 1991.

Cycle slip detection

- When processing phase, cycles slips are a potential problem. You can look at this in HW2 data set. The L1 and L2 phase values are in the L1 and L2 slots in the rinex file. The have a large offset from the range values (initial ambiguity, which in double differences should be a integer value)

- When the receiver looses lock (typically range will be missing but not always), a cycle slip occurs and this must be re-fixed to an integer or left as an unknown parameter

Cycle slip detection

- When o-minus-c is computed in one-ways for phase, variations are dominated by clocks in receiver.

- Multiple techniques are used to detect cycle slips:
  - Ln phase - Ln range (n=1,2). Removes geometry but affected by ionospheric delay (opposite sign on phase and range) and noise in range measurements
  - L1 phase - L2 phase. Some times called wide-lane. Affected by ion-delay but is common detector
  - Double difference phase residuals: On short baselines, removes ionosphere and if good apriori positions are known, should be a smooth function of time. Often used to estimate number of cycles in sip and resolve to integer value.
  - Melbourne-Wubbena wide lane (ML-WL) (see over)

MW wide lane

- Very useful combination of data that is often used in kinematic GPS where receiver coordinates are changing.

- The MW WL should equal number of cycles of phase between between L1 and L2 and is calculated, effectively, be computing the expected L1 and L2 phase difference from the pseudorange data.

MW Wide lane

- From the equations for range and phase with the phase offsets for cycle offsets you can derive:

$$MW - WL = N_1 - N_2 = \phi_{L2} - \phi_{L1} + (P_1 + P_2)\frac{f_{L1} - f_{L2}}{f_{L1} + f_{L2}}$$

- The MW-WL should be constant if there are no cycle slips. When the phase and range values are double differences, N_2-N_1 should be integer.
- The factor that scales range is ~0.1 and so range noise is reduced.
- Average values of the MW-WL are used to esimate L1/L2 phase difference independent of ion-delay and geometry changes.

Ambiguity resolution

- The MW-WL is often used to get N1-N2 and then N1 is estimated, as non-integer value, from the least-squares fit to the phase data.
- If the sigma of the N1 estimate is small, and the estimate is close to an integer then it can be resolved to an integer values. There are various methods for deciding if an N1 estimate or a group of N1 estimates can be fixed to integers (e.g., LAMBDA method)
- Fixing ambiguities, improves the sigma of the east position estimate by typically a factor of two and makes it similar to the North sigma. Bias fixing has little effect on North and Up sigma except for short sessions.
- Recent analysis also suggest systematic error propagate into solutions less effectively when ambiguities are resolved to integers.

Summary

- Today's lecture examined:
- rank deficiencies
- Differencing of data to eliminate clock errors
- Cycle slip detection and bias fixing (also called ambiguity resolution).

12.540 Principles of the Global Positioning System

Spring 2012