12.540 Principles of the Global Positioning System
Lecture 17

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http://geoweb.mit.edu/~tah/12.540

Summary

- Finish propagation medium with discussion of signal characteristics around GPS antennas
  - Basic operation of antenna
  - Ray approximation to effects of multipath
  - Phase center models for GPS ground antennas
  - Phase center models for GPS satellite antennas
  - Use of signal strength (SNR) to assess multipath

Basic antenna operation

- Receiving and transmitting antennas are identical: Time just flows in opposite directions.
- Antenna problems are solved knowing the current distribution **J(x')** in the antenna and using a vector potential

$$\mathbf{A}(\mathbf{x}) = \frac{1}{c}\iiint \mathbf{J}(\mathbf{x}')\frac{e^{ik|x-x'|}}{|\mathbf{x}-\mathbf{x}'|}d^3x'$$

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{E} = \frac{i}{k}\nabla \times \mathbf{B}$$

Basic Antenna theory

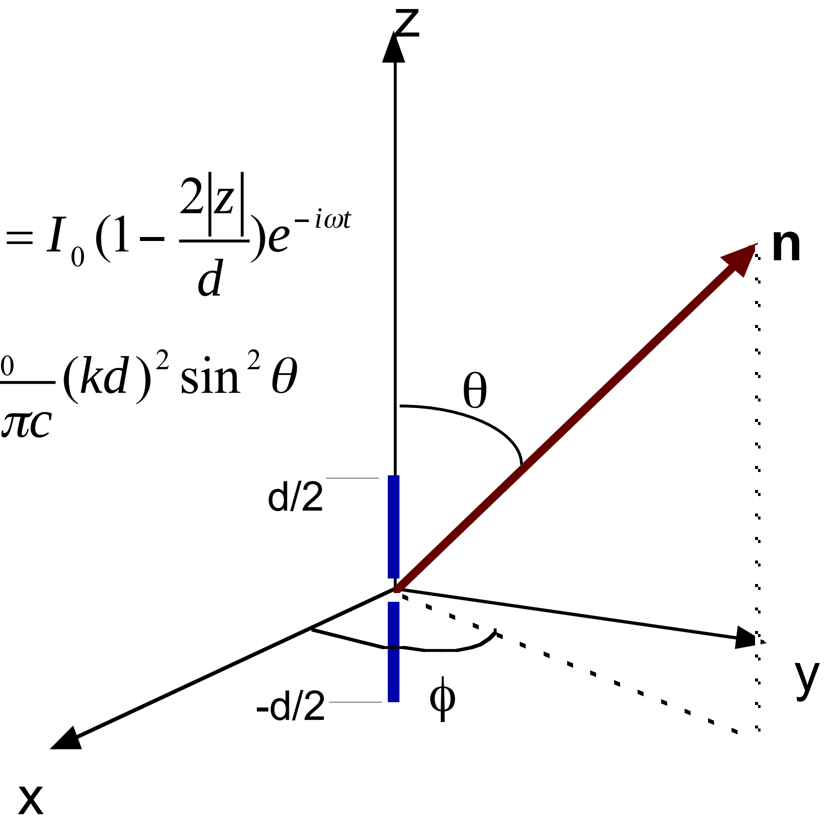
- Basic problem with using these equations is that the propagating EM field induces other currents to flow in the antenna that must be included in the integral.
- Generally three distance ranges are treated with antennas for antenna size d $<<\lambda$
  - The near (static zone) $d<<r<<\lambda$
  - The intermediate (induction) zone $d<<r\sim\lambda$
  - The far radiation zone: $d<<\lambda<<r$

Simplest antenna

- Short center-fed dipole

$$I(z)e^{-i\omega t} = I_0\left(1 - \frac{2|z|}{d}\right)e^{-i\omega t}$$

$$\frac{dP}{d\Omega} = \frac{I_0}{32\pi c}(kd)^2 \sin^2\theta$$



P is the radiated power from the antenna, with current I_0 center fed into antenna

Dipole antenna

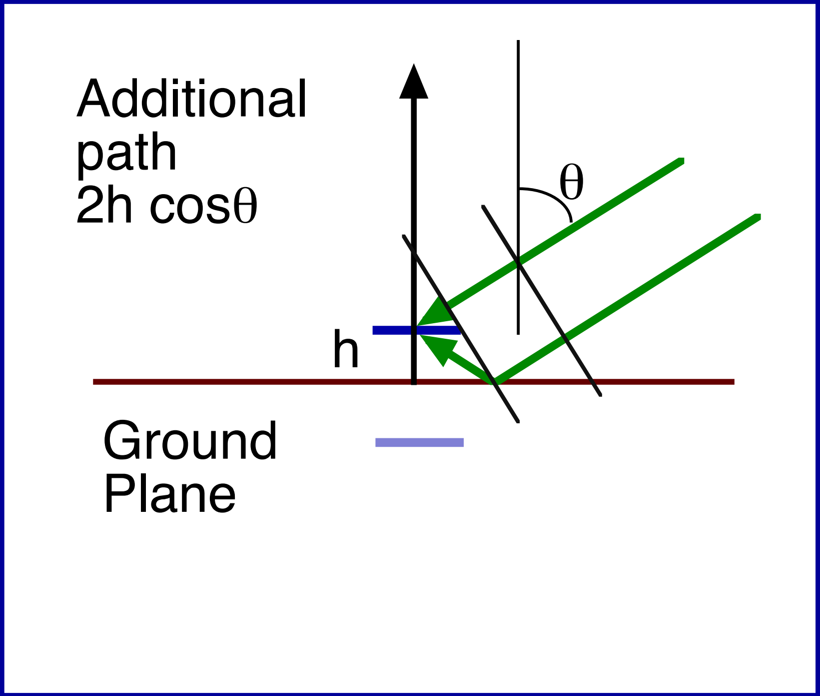
- Notice that no power is transmitted in the direction of the antenna; maximum power is perpendicular to the antenna
- There is no ϕ dependence to the power transmission.
- The received strength follows the same pattern; No gain along the antenna, maximum gain perpendicular to it.
- The first civilian GPS antennas were of this form. But how to mount the antenna?

Dipole antennas

- For GPS, you need to mount the dipole horizontally
- However, a simple dipole mounted this way will see reflections from the ground just as well as the direct signal from the satellite.
- This is called multipath (multiple paths that the signal can travel to get to the antenna)
- How do you solve the ground reflection problem?

Dipole over a ground plane

- To solve reflection from ground problem: You make your own, highly reflective ground.



If the ground plane is infinite, then antenna acts like a point source, in the ground plane below the antenna.

Gain depends on h/λ
In zenith $h=\lambda/4$ give maximum gain

Polarization with dipole

- Since GPS signals are transmitted with right-circular polarization, ideally an antenna should receive RCP radiation
- This can be done with dipoles by having two (horizontal) dipoles perpendicular to each other and adding the output with the correct 90^o phase shift (sets RCP or LCP)
- Macrometer (early MIT GPS receiver) antenna worked this way. (Set height dipole was tricky to get L1 and L2 tracking).

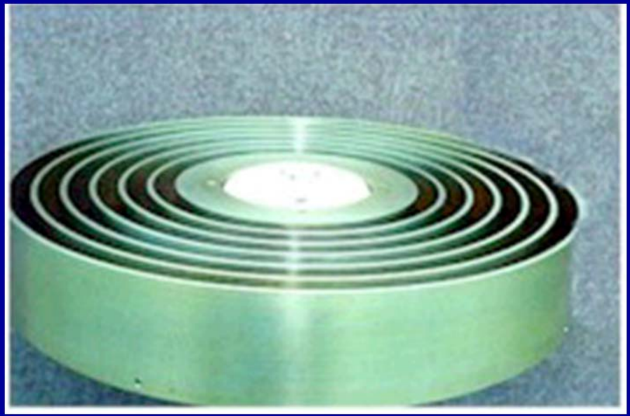
Other antenna styles

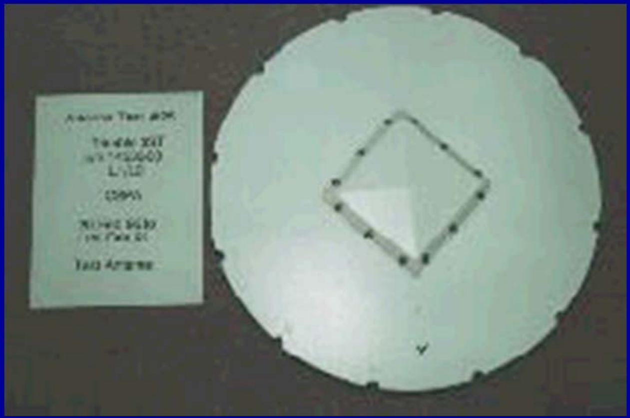
- Other styles of antenna commonly seen in GPS applications:
  - Helical antenna (wire around styrofoam coffee cup is good). Early T14100 antenna was of this design. Some hand-held receivers use this style (Garmin GPS II/III)
  - Microstrip patch antenna. Very common now. Patch mounted close to ground plane embedded in a dielectric.
  - Dorne-Margollian element (4-patchs mounted inside dome) embedded in choke rings. Standard global GPS tracking antenna.

GPS Antennas (for precise positioning)

Nearly all antennas are patch antennas (conducting patch mounted in insulating ceramic).

- Rings are called choke-rings (used to suppress multi-path)





Simple Multipath

- A simple approach to treating multipath is with ray-optics. Approach should be valid for reflectors that greater than one wavelength from the antenna.
- It is important to note that all real antennas have gain below the horizon (ie., zero elevation angle) and will therefore see reflections from the ground.

Surface reflections

- The amplitude of a reflected signal from a surface depends on incidence angle and refractive index of medium

E perpendicular to plane of incidence

$$\frac{E_r}{E_i} = \frac{n\cos i - \sqrt{n'^2 - n^2 \sin^2 i}}{n\cos i + \sqrt{n'^2 - n^2 \sin^2 i}}$$

E parallel to plane of incidence

$$\frac{E_r}{E_i} = \frac{n'^2\cos i - n\sqrt{n'^2 - n^2 \sin^2 i}}{n'^2\cos i + n\sqrt{n'^2 - n^2 \sin^2 i}} \qquad n = \sqrt{\mu\varepsilon}$$

Where n' is refractive index of reflecting medium

($\mu' = \mu$)

Normal incidence reflection

- For normal incidence: the two cases reduce to

$$Perpendicular \quad \frac{E_r}{E_i} = \frac{2n}{n'+n}$$

$$Parallel \quad \frac{E_r}{E_i} = \frac{n'-n}{n'+n}$$

- Reflection strength will depend on dielectric constants:
  - Air ε=1; water 80; Dry Sand 3-5; saturated sand 20-30; shale 5-15; silt/clay 5-40; Granite 4-6; Ice 3-4
  - Reflected strength at least 30% of incident signal

Multipath characteristics

- The path length difference between the direct and reflected signal determines the nature of multipath.
- When the reflector is close (d/λ~1) multipath will be slowly varying
- When reflector is distant (d/ λ>>1) multipath will vary rapidly and average to zero quickly.
- A class of multipath is what happens when d/ λ<<1. This characteristic of antenna and is called phase center model (needed when antenna types are mixed in high-precision applications).

Receiving Antenna Phase center models

- The specific characteristics of an antenna need to be calibrated either with:
  - Anechoic chamber measurements (absolute calibration)
  - In-situ relative measurements (one-antenna relative to another)
  - In-situ absolute calibration by antenna rotation
  - In-situ multipath calibration using a directional antenna

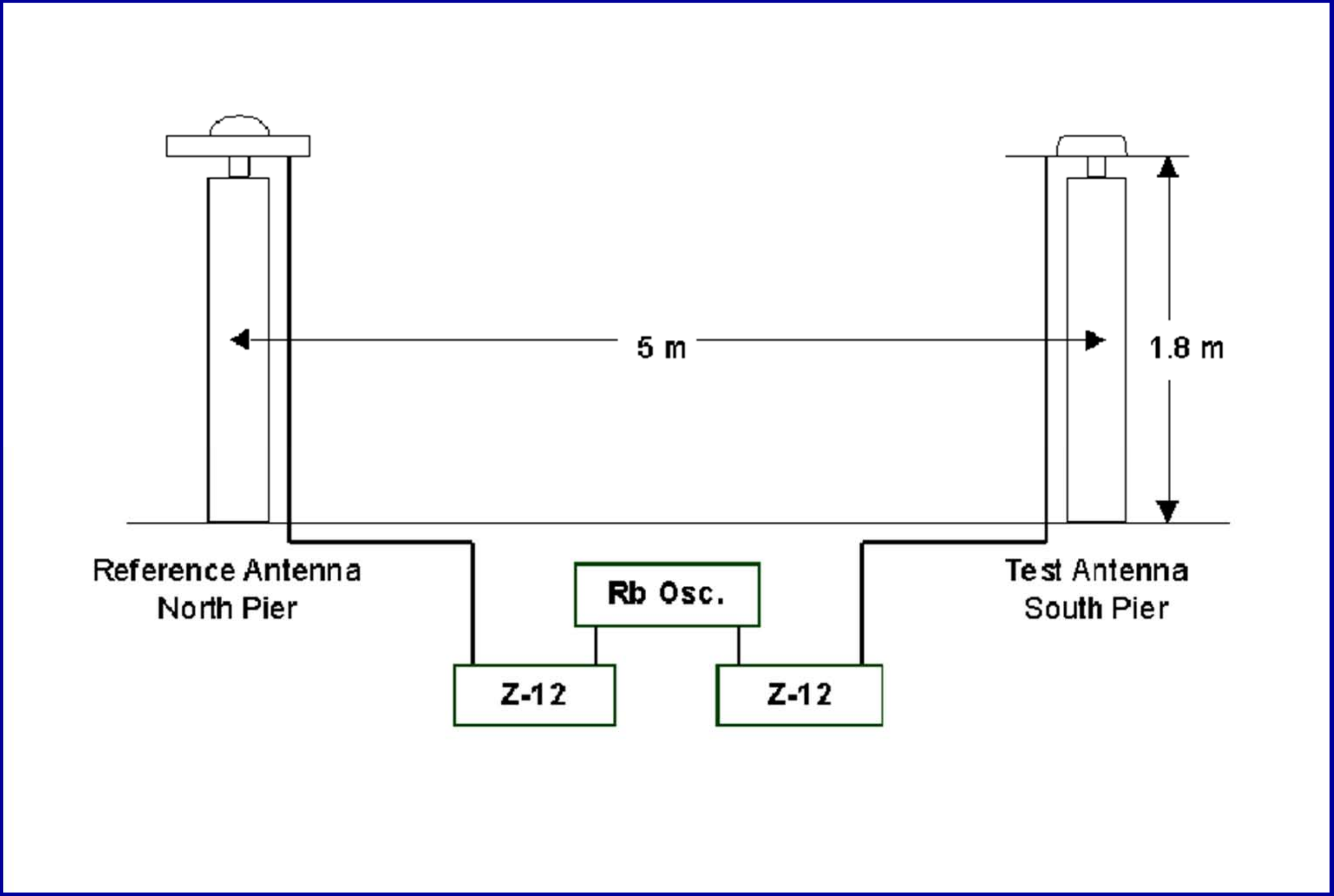
Phase center models

- First phase center models were made using data from a chamber in which L1 and L2 signals were transmitted and antenna rotated to measure phase difference between transmitted and received signal.
- Signal strength also measured so that gain of antenna can be measured (expect it to behave like $\sin^2(\theta)$ but with response for $\theta>90$ (back-plane gain).

Relative phase center models

- If an antenna with 0 phase center variation is available, then phase center of another antenna can be found by making differential measurements between antenna on monuments with known locations.
- National Geodetic Survey (NGS) has largest setup: http://www.ngs.noaa.gov:80/ANTCAL/

NGS Calibration set-up



Typical Calibration results

- Two types of information given:
  - “Phase center Position” relative to physical point on antenna (ARP--normally base of pre-amplifier)
  - Elevation angle dependent deviations of phase:

TRM 36569.00+GP 13" Micro Centered with Ground Plane NGS (4) 01/10/12

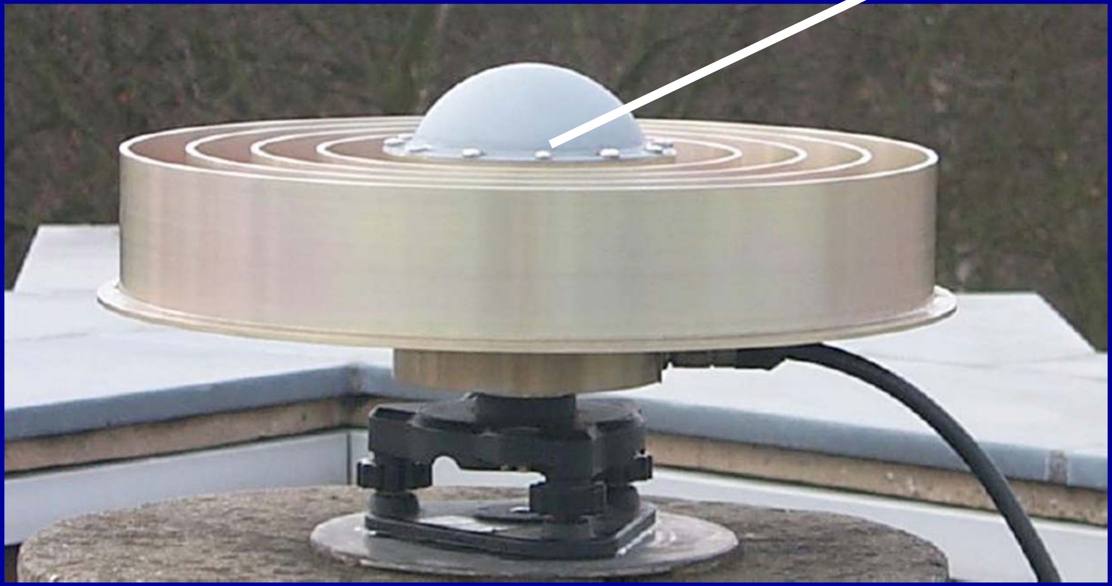
```
         .0        -.3       63.2
     .0     .7   1.4   2.3   3.1   3.9   4.6   5.2   5.6   5.9   5.9   5.6   5.0 ..
        -.9        -.8       44.6
     .0    -.9  -1.5  -1.9  -2.2  -2.4  -2.7  -3.0  -3.3  -3.6  -3.9  -4.0  -3.8 ..
RMS mm (1 sigma)    4 MEASUREMENTS
         .3        1.3         .1
     .0     .1    .1    .1    .1    .2    .2    .2    .2    .1    .1    .1    .1 ..
         .4         .6         .3
     .0     .3    .5    .6    .7    .6    .6    .5    .5    .5    .5    .5    .5 ..
```

General results

- Typical phase variations are quite different at L1 and L2 frequencies and the even larger in the ionospheric free observable (LC)
- Positions can change by 10-cm when phase center models used
- Phase residuals are systematic if wrong antenna type used, but RMS is often less than 10 mm compared to normal noise of ~5 mm
- Where do we get the "zero phase center antenna"?
- The IGS has adopted the Dorne-Margolian Choke ring as standard. What are its phase center variations?

Absolute calibration

- Hannover System:



- http://www.geopp.de/media/docs/AOA_DM_T

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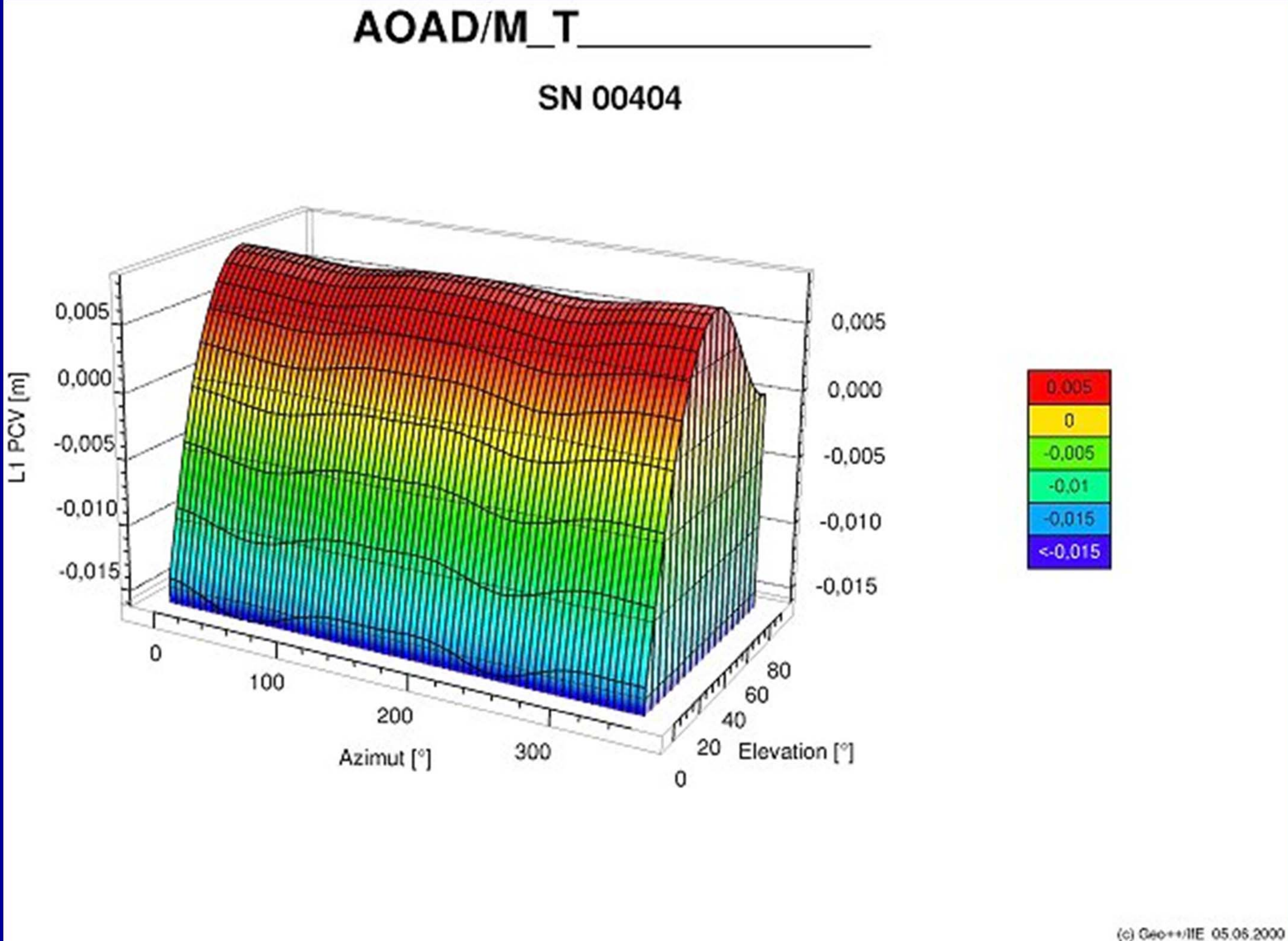

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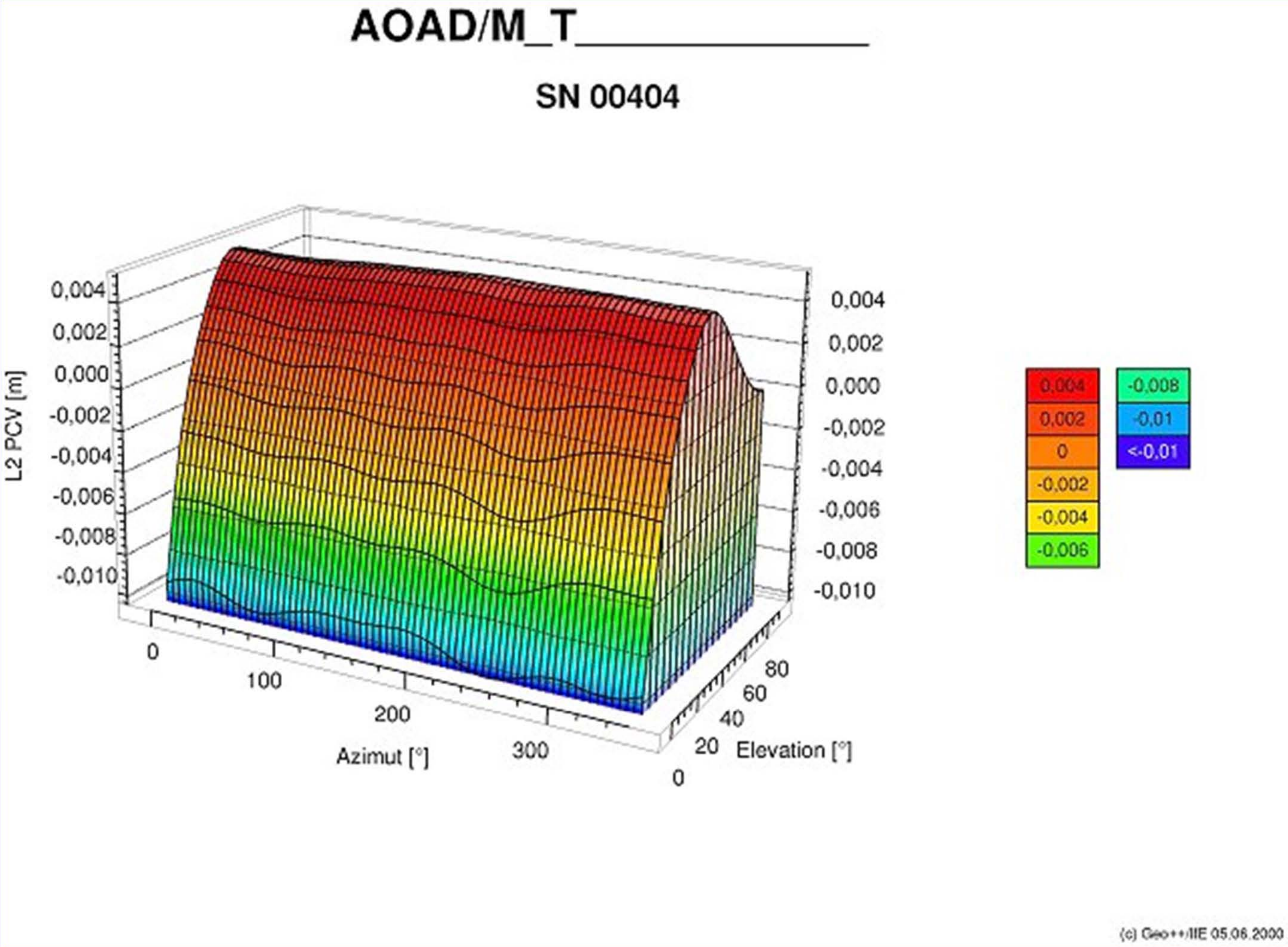

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Absolute calibrations

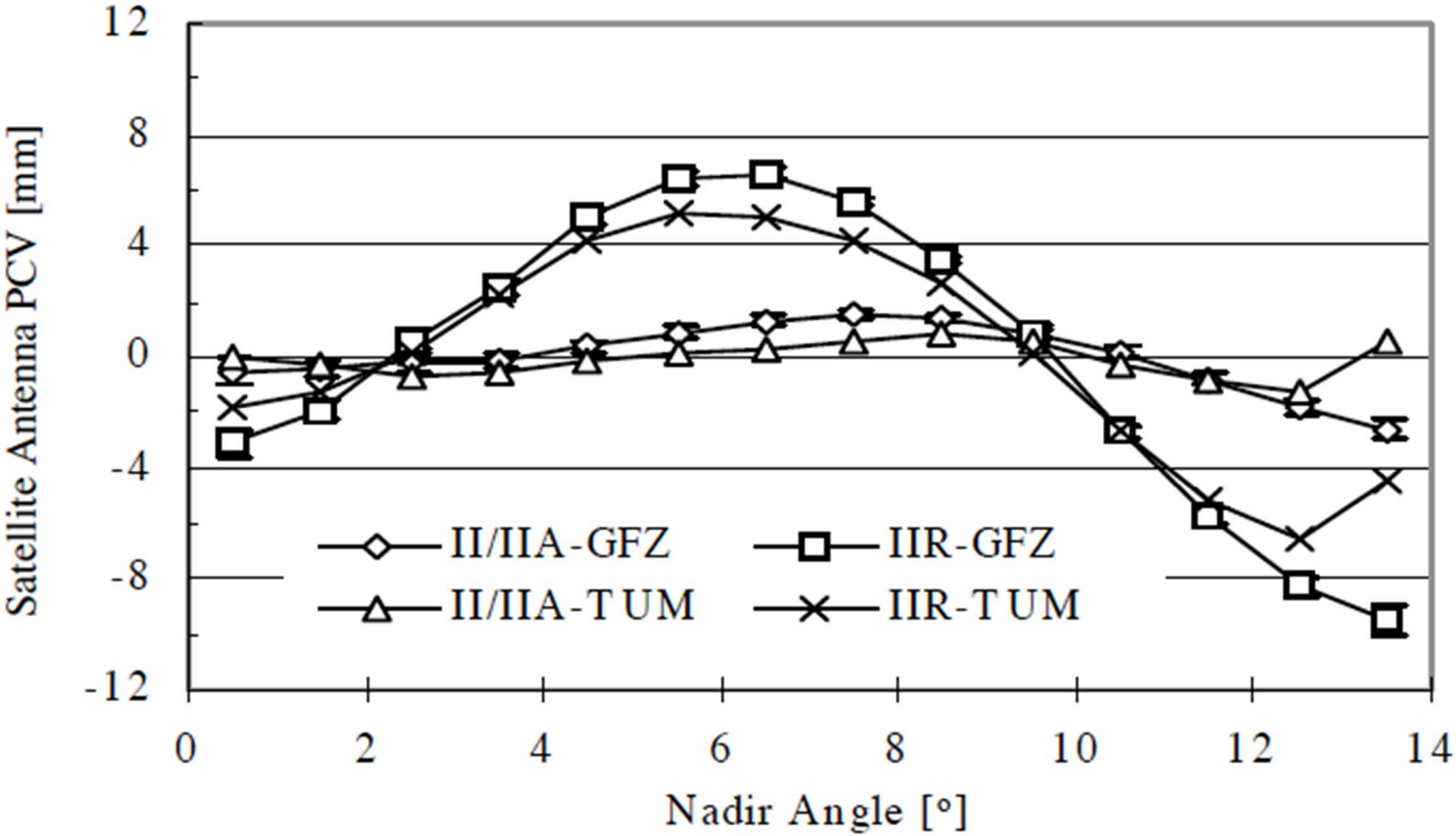
- The Hanover results are similar to anechoic chamber results although there are problems with this type of measurement: As the antenna is rotated, ground reflections have higher gain.
- Major problem at the moment: 10-cm height changes (14-ppb scale change) in global GPS when absolute models are used.
- Could be at satellite? Where are phase centers on satellites?

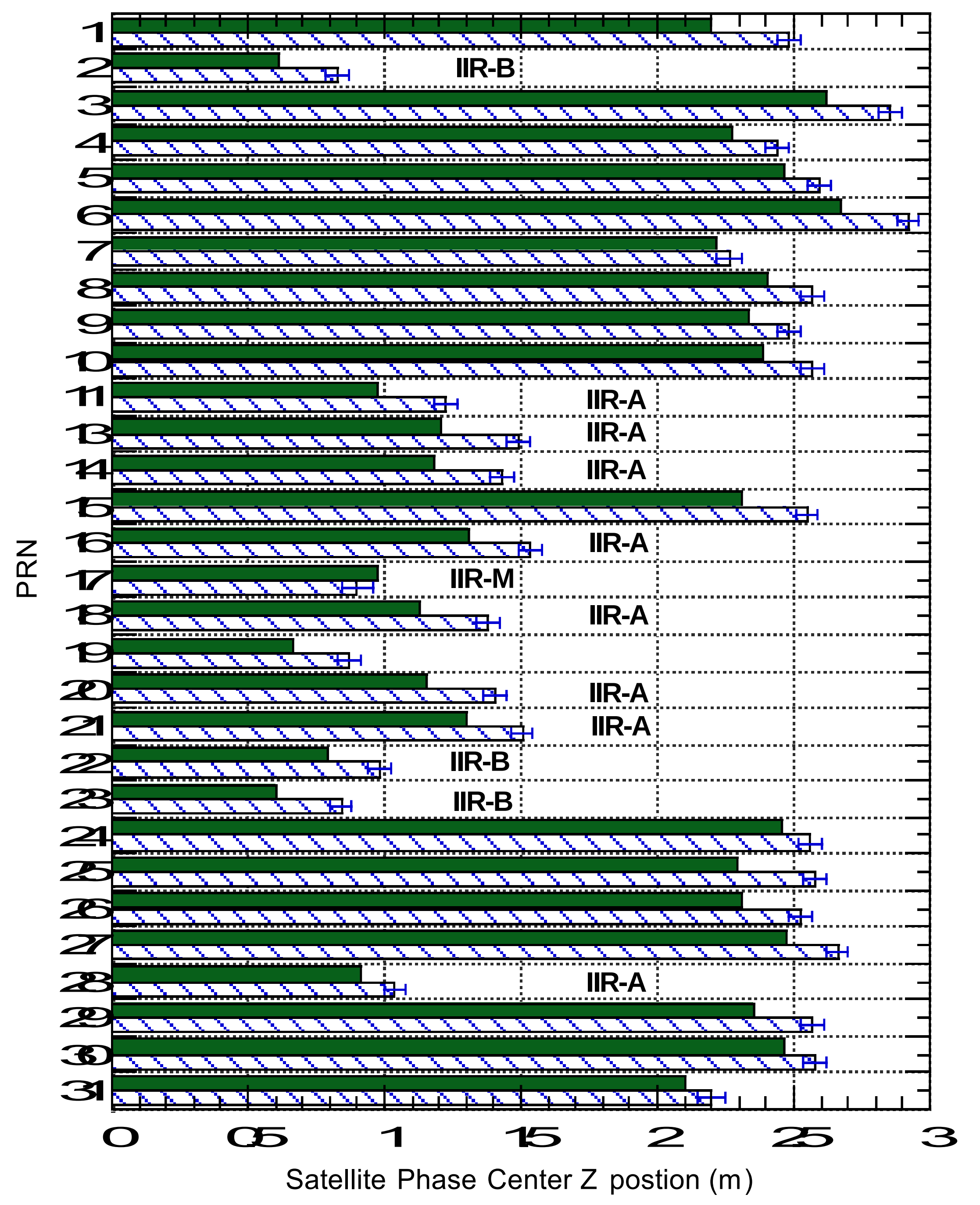
Satellite phase centers

- Satellites transmit from an array
- Figure at left gives some idea of size.
- For current GPS precisions, we need phase center to a few centimeters
- See NGS ANTCAL site
- Currently adopted positions of phase centers could be in error by over 1 m. (Block IIR satellites are definitely wrong by this amount)

Images courtesy of NOAA.

Satellite antenna phase center model



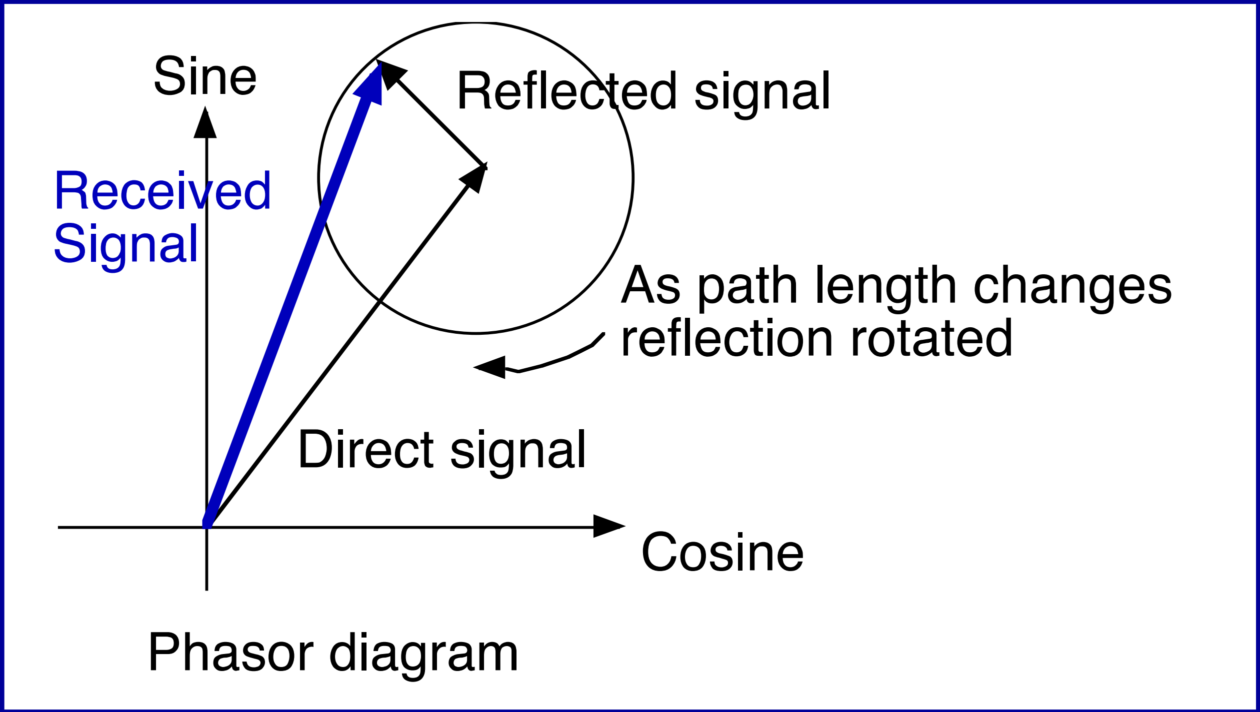


Z-offset by PRN

- Estimates of the Z-offset (distance towards the center of the Earth) by satellite PRN and type.
- Even within on generation there is variation.

Use of Signal-to-noise ratio (SNR)

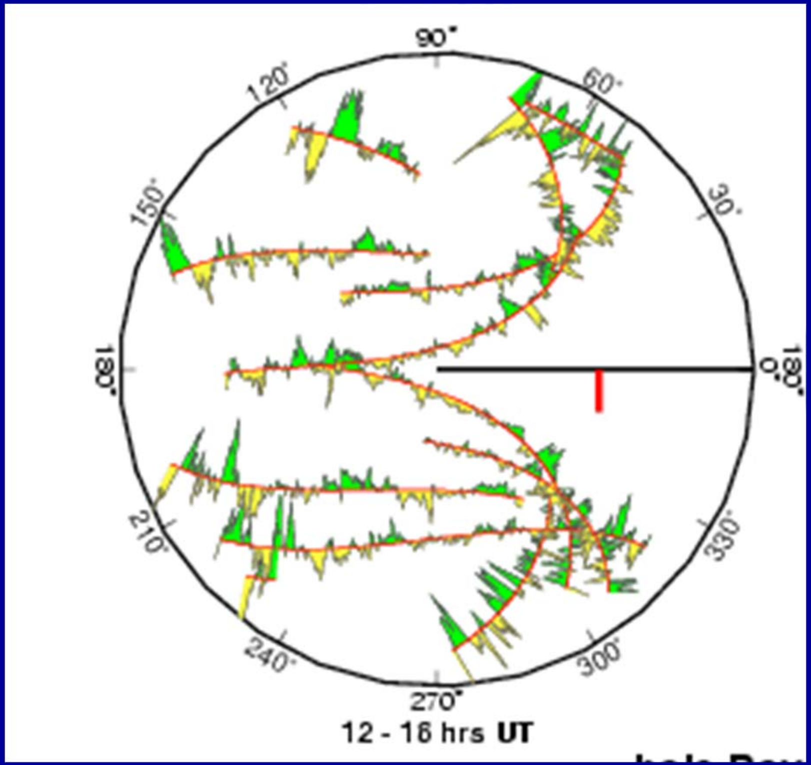
- One method of characterizing multipath at a site is SNR analysis.



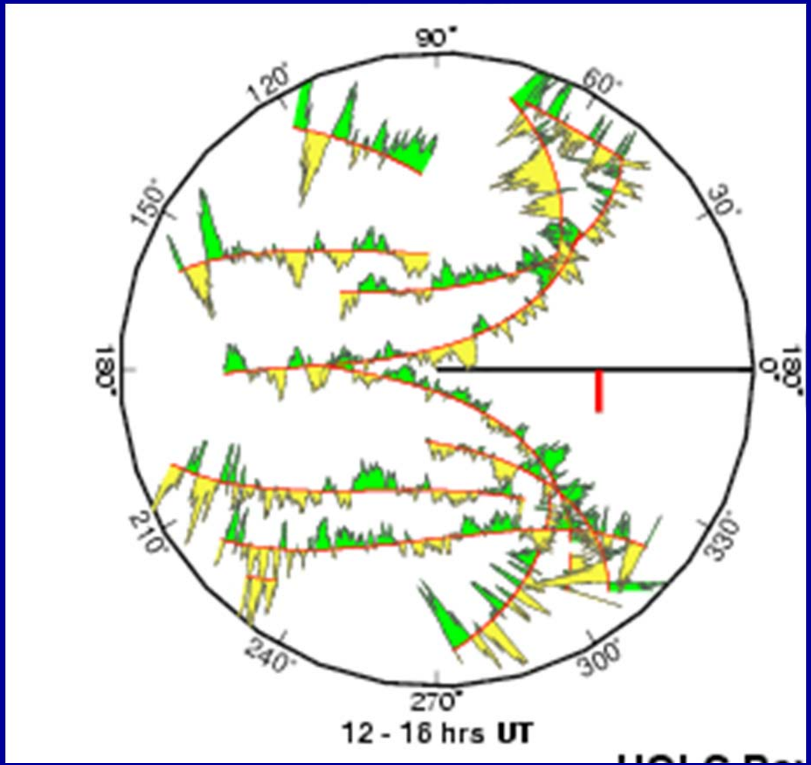
SNR analysis

- Changing path length difference between direct and reflected signals causes oscillating signal amplitude and phase (90^{o} out-of-phase)
- Analysis of signal strength variations can allow prediction of phase errors (but ambiguous in sign).
- Implementation: http://geoweb.mit.edu/~tah/snrprog/

Example HOLC California (LC)



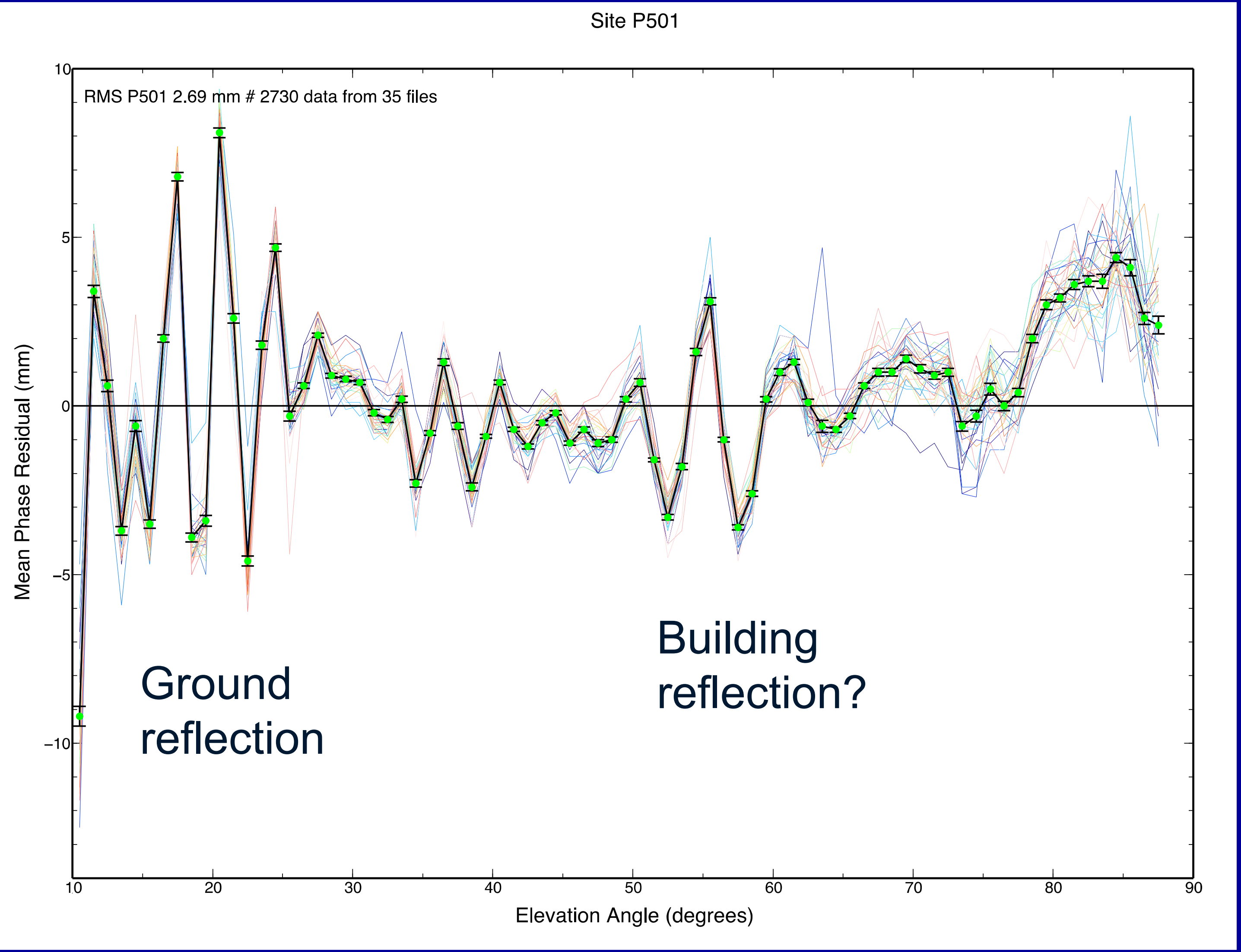
Theoretical from SNR



Measured Phase residuals

Example P501

- Example of ground reflection and building



Color are different days; and symbols with error bars are mean.

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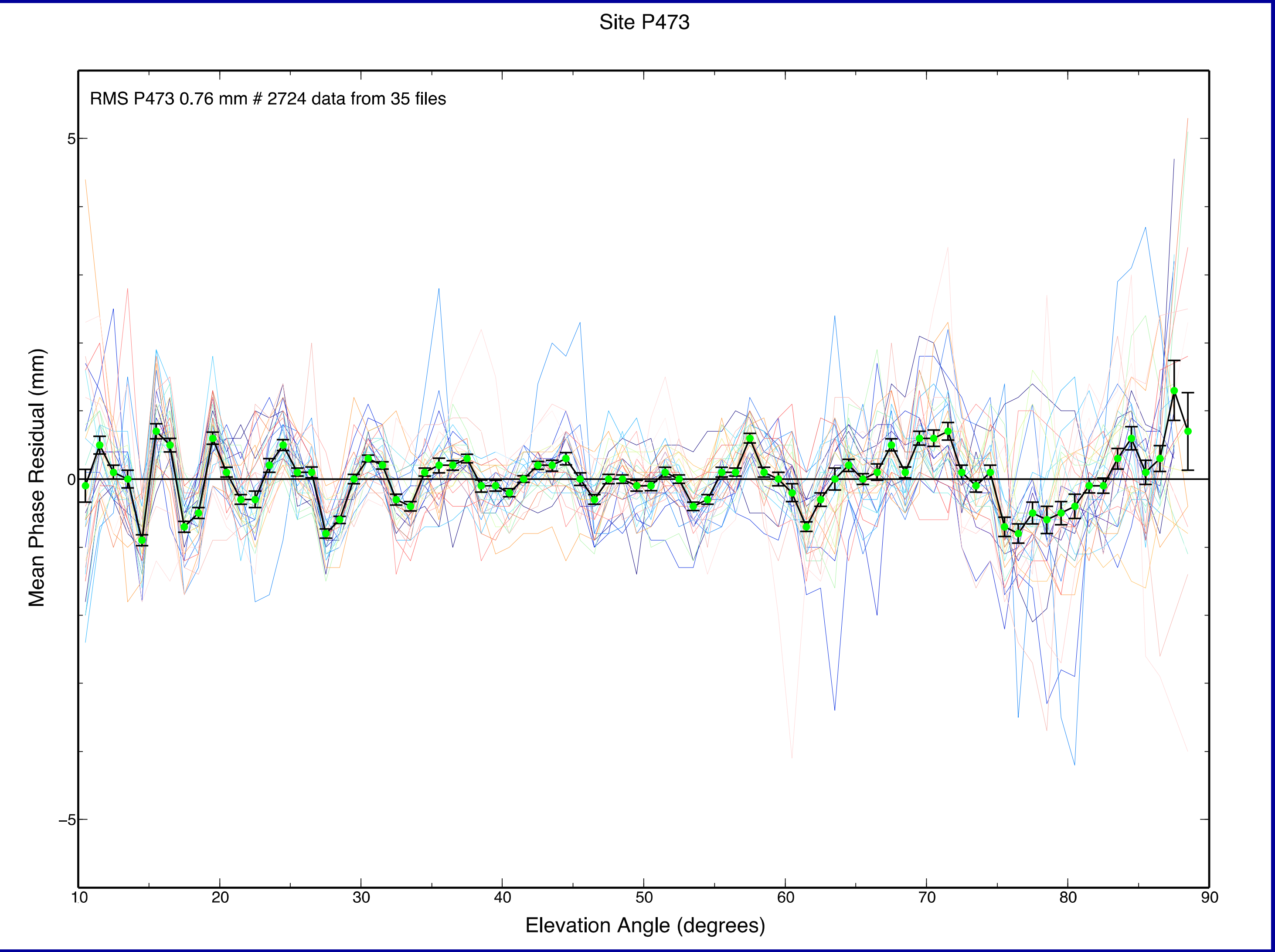
- Example with little ground reflection



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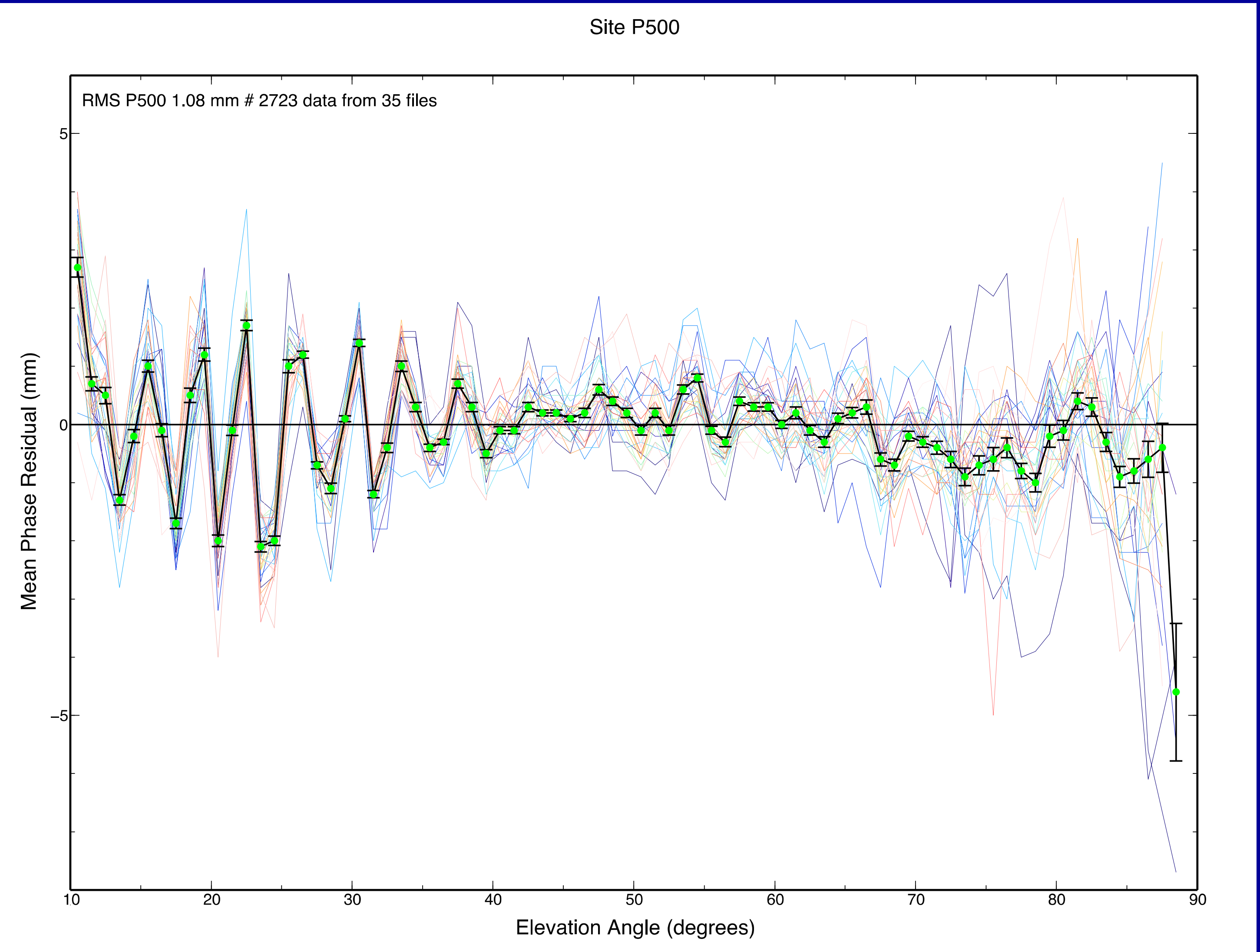
- Large ground reflection; flat surface



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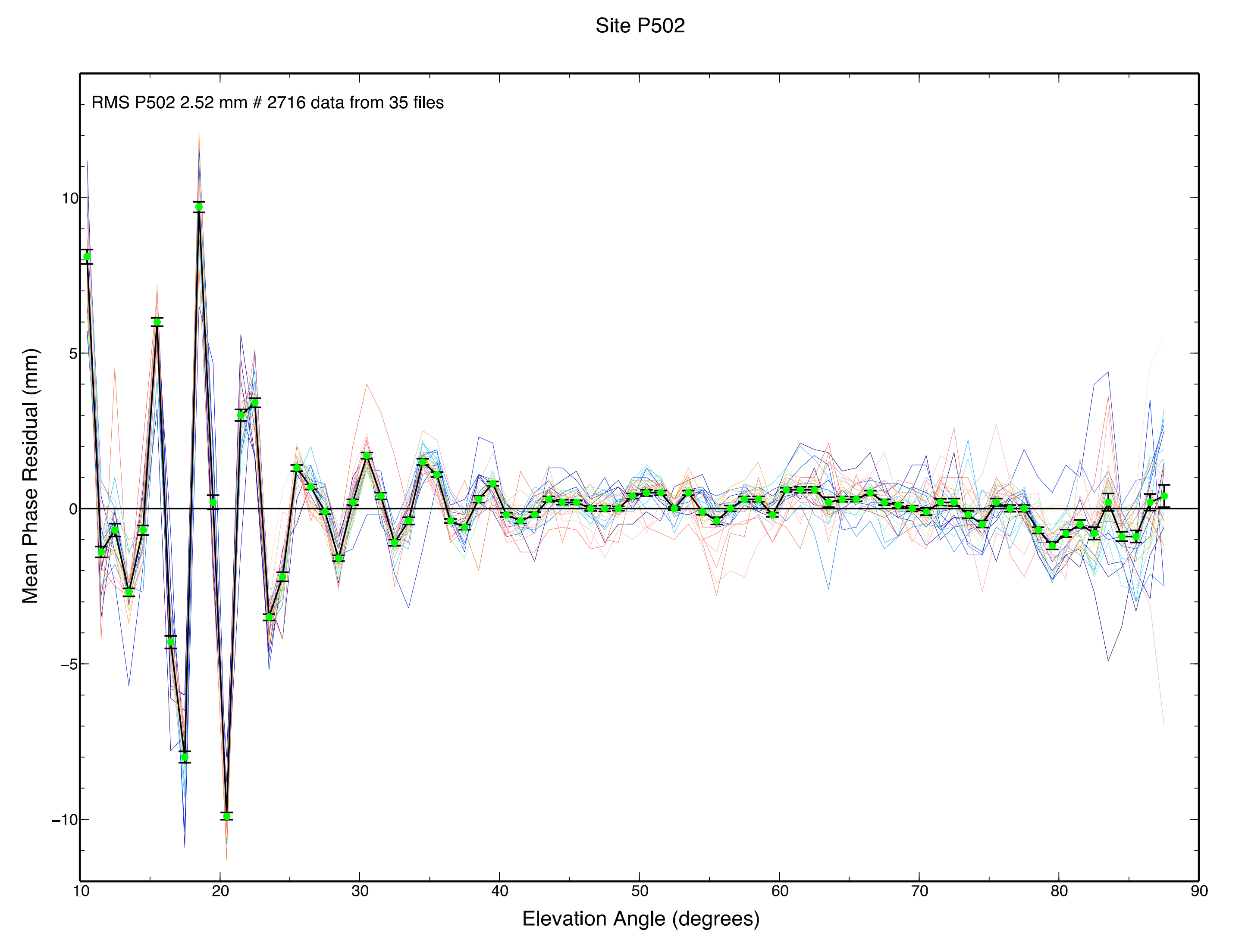
- Strong Ground reflection

Site will be monitored to see how it changes as ground conditions change



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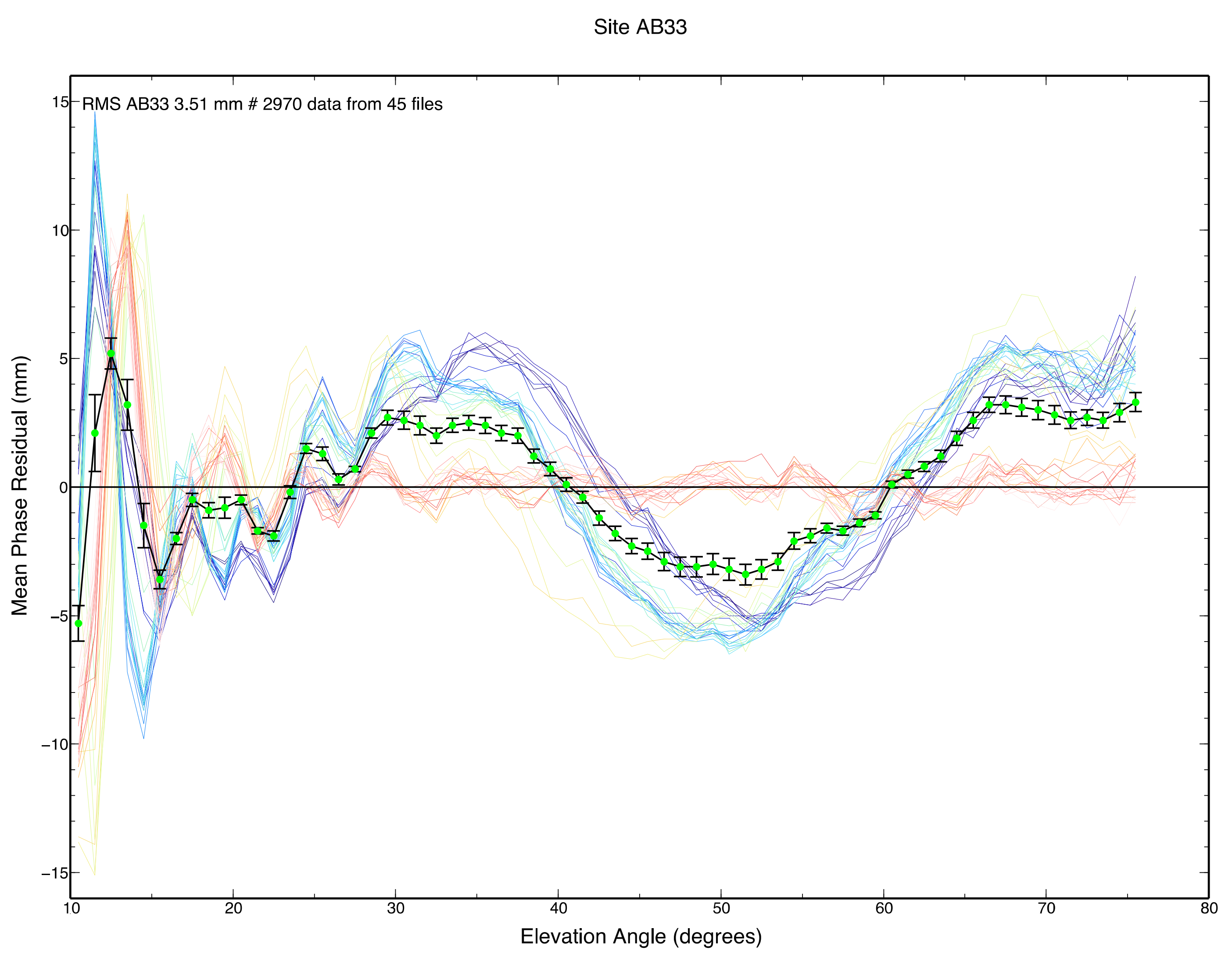
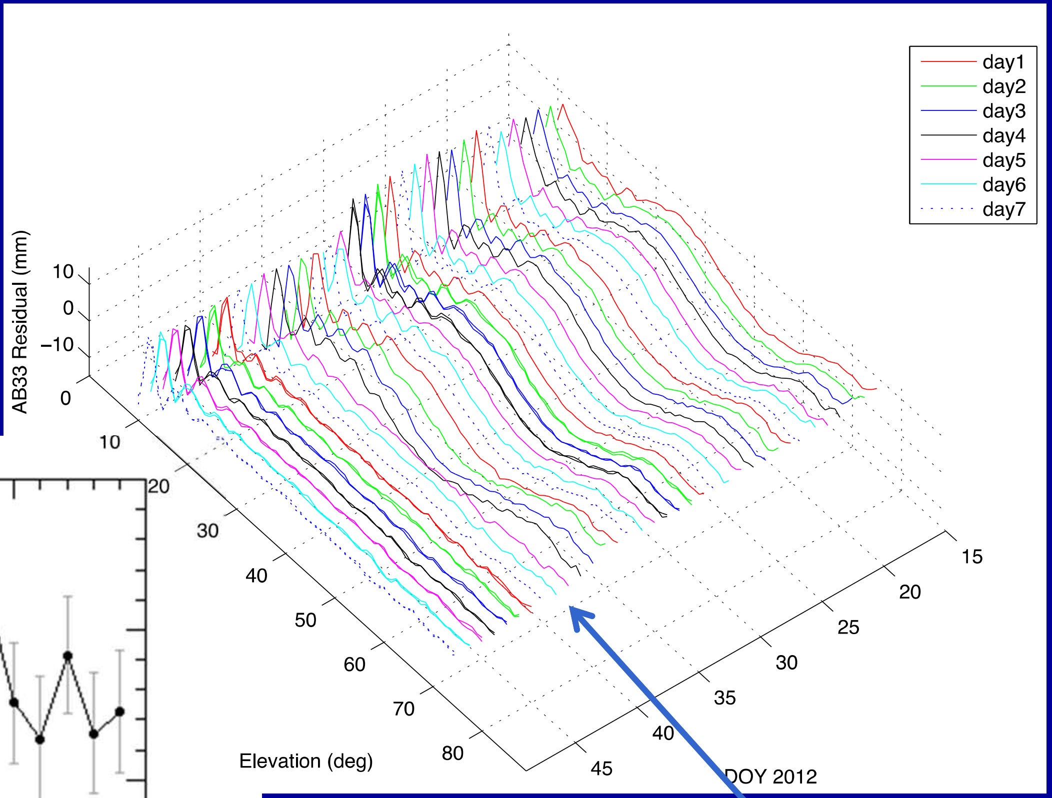
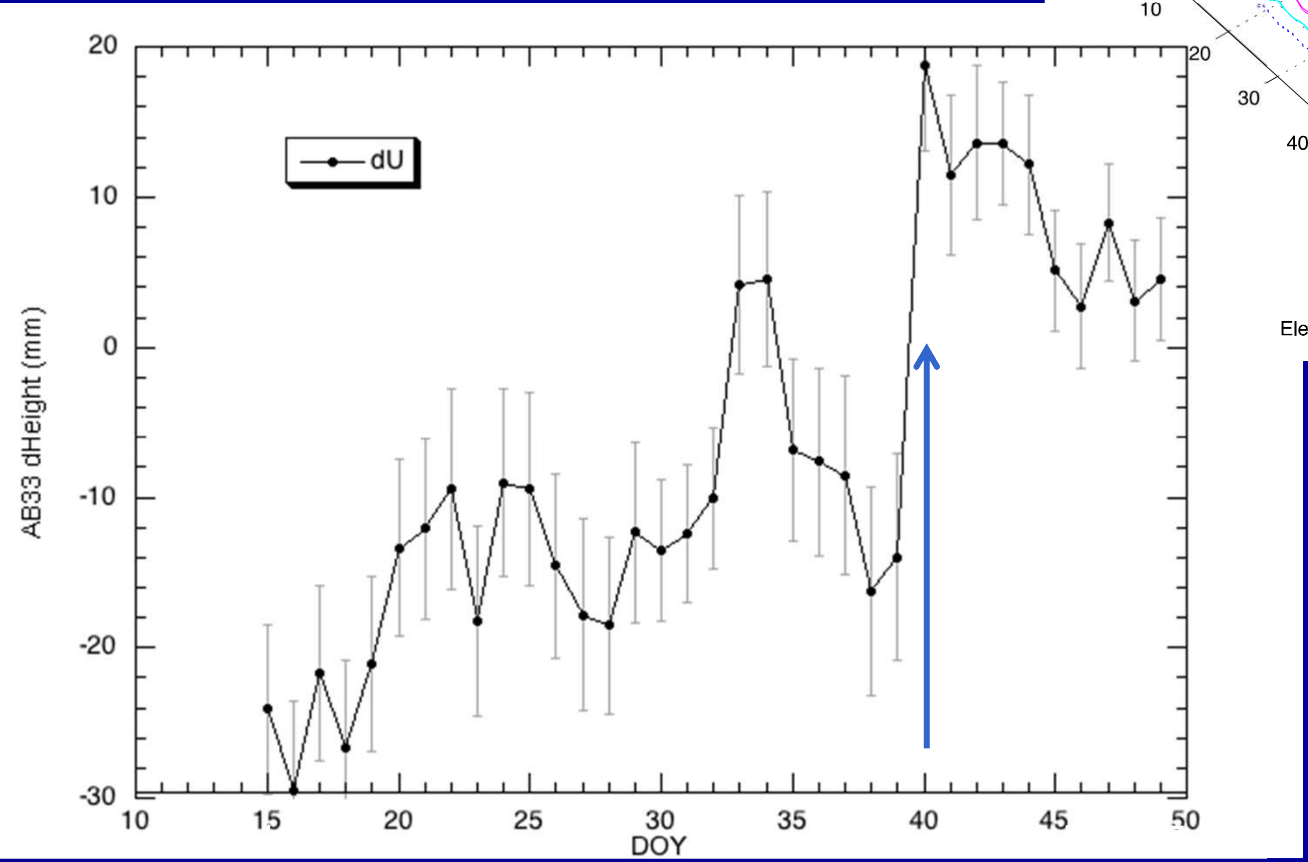


- Time variable signals. Alaskan site with snow accumulation and melting (believed)
- Colors span a 30-day interval. Site height changes when residuals change

Time series and changes

Height changes correspond to residual changes





Residuals and heights have large change at day 40.

Summary

- Measurements at mm level require careful evaluation of multipath and near-antenna scattering
- Phase center variations can be many centimeters
- Probably largest problem is vegetation near antennas since it changes with time and allows transmission of GPS signals.