

12.540 Principles of the Global  
Positioning System  
Lecture 15

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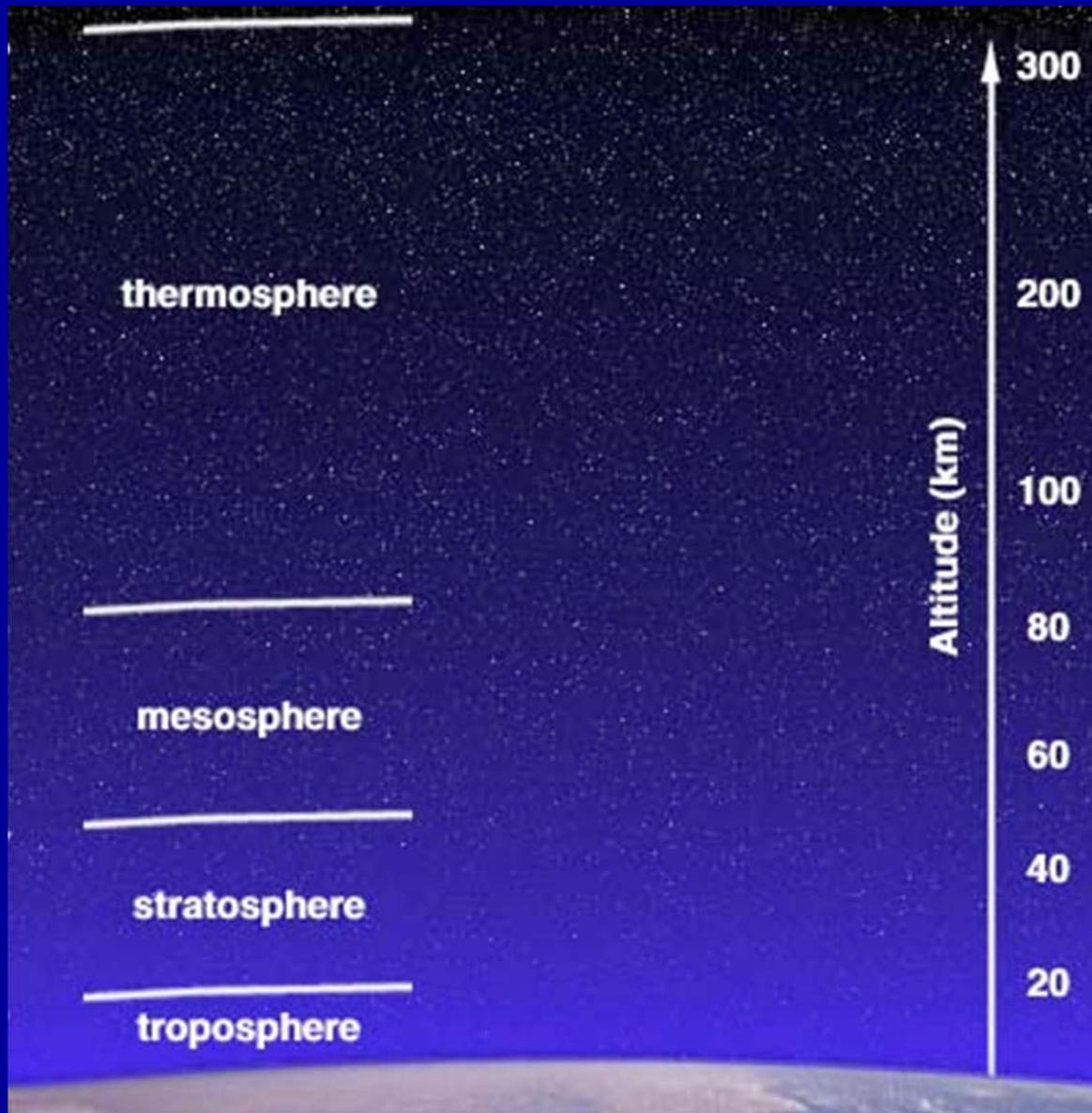
<http://geoweb.mit.edu/~tah/12.540>

# Propagation Medium: Neutral atmosphere

- Summary

- Basic structure of the atmosphere: Here we exclude the effects of electrons in the ionosphere (covered next lecture).
- Refractivity of constituents in the atmosphere
- Separation of atmospheric delay into “hydrostatic” and wet components.
- Elevation angle dependence:
  - Azimuthally symmetric mapping functions
  - Gradient formulations
- Effects of atmospheric delays on position estimates

# Basic atmospheric structure



Troposphere is where the temperature stops decreasing in the atmosphere. (~10 km altitude)

# Troposphere

- Lots of examples of web-based documents about the atmosphere: See for example.

<http://www-das.uwyo.edu/~geerts/cwx/notes/chap01/tropo.html>

- Tropopause is where temperature stops decreasing. Generally at pressure levels of about 300 mbar but can be as low as 500 mbar.
- Sometimes term “tropospheric delay” used but this is only about 70% of delay.
- Generally by height of 50-100km all of atmospheric delay accounted for.
- Troposphere is where weather systems occur and aircraft fly on the tropopause.

# Refractivity of air

- Air is made up of specific combination of gases, the most important ones being oxygen and nitrogen.
- Each gas has its own refractive index that depends on pressure and temperature.
- For the main air constituents, the mixing ratio of the constituents is constant and so the refractivity of a packet of air at a specific pressure and temperature can be defined.
- The one exception to this is water vapor which has a very variable mixing ratio.
- Water vapor refractivity also depends on density/temperature due to dipole component.

# Refractivity of air

- The refractivity of moist air is given by:

$$N = k_1 \underbrace{\frac{P_d}{T} Z_d^{-1}}_{\text{Density of dry air}} + k_2 \underbrace{\frac{P_w}{T} Z_d^{-1}}_{\text{Density of water vapor}} + k_3 \underbrace{\frac{P_w}{T^2} Z_d^{-1}}_{\text{Dipole component of water vapor } \rho/T}$$

$$k_1 = 77.60 \pm 0.05 \text{ K/mbar}$$

$$k_2 = 70.4 \pm 2.2 \text{ K/mbar}$$

$$k_3 = (3.730 \pm 0.012) \times 10^5 \text{ K}^2/\text{mbar}$$

- For most constituents, refractivity depends on density (ie., number of air molecules). Water vapor dipole terms depends on temperature as well as density

# Refractivity in terms of density

- We can write the refractivity in terms of density:

$$N = k_1 \frac{R}{M_d} \rho + \left( \frac{k'_2}{T} + \frac{k_3}{T^2} \right) P_w Z_w^{-1}$$

$$k'_2 = k_2 - k_1 M_w / M_d = 22.1 \pm 2.2 \text{ K/mbar}$$

- Density  $\rho$  is the density of the air parcel including water vapor.  $R$  is universal gas constant,  $M_d$  and  $M_w$  are molecular weights.  $Z_w$  is compressibility (deviation from ideal gas law) See Davis, J. L., T. A. Herring, and I.I. Shapiro, Effects of atmospheric modeling errors on determinations of baseline vectors from VLBI, *J. Geophys. Res.*, 96, 643–650, 1991.

# Integration of Refractivity

- To model the atmospheric delay, we express the atmospheric delay as:

$$D = \int_{atm} n(s) ds - \int_{vac} ds \approx m(\varepsilon) \int_Z^{\infty} (n(z) - 1) dz = m(\varepsilon) \int_Z^{\infty} N(z) \times 10^{-6} dz$$

- Where the *atm* path is along the curved propagation path; *vac* is straight vacuum path, *z* is height for station height *Z* and *m*( $\varepsilon$ ) is a mapping function. (Extended later for non-azimuthally symmetric atmosphere)
- The final integral is referred to as the "zenith delay"

# Zenith delay

- The zenith delay is determined by the integration of refractivity vertically.
- The atmosphere is very close to hydrostatic equilibrium meaning that surface pressure is given by the vertical integration of density. Since the first term in refractivity depends only on density, its vertical integration will depend only on surface pressure. This integral is called the “zenith hydrostatic delay (ZHD)”. (Often referred to as “dry delay” but this is incorrect because it has water vapor contribution).

# Zenith hydrostatic delay

- The Zenith hydrostatic delay is given by:

$$ZHD = 10^{-6} k_1 \frac{R}{M_d} g_m^{-1} P_s \approx 0.00228 \text{ m/mbar}$$

- Where  $g_m$  is mean value of gravity in column of air (Davis et al. 1991)  
 $g_m = 9.8062(1 - 0.00265 \cos(2\phi) - 3.1 \times 10^{-7}(0.9Z + 7300)) \text{ ms}^{-2}$
- $P_s$  is total surface pressure (again water vapor contribution included)
- Since  $P_s$  is 1013 mbar at mean sea level; typical ZHD = 2.3 meters

# Zenith wet delay

- The water vapor delay (second term in refractivity) is not so easily integrated because of distribution of water vapor with height.
- Surface measurements of water vapor pressure (deduced from relative humidity) are not very effective because it can be dry at surface and moist above and visa versa.
- Only effective method is to sense the whole column of water vapor. Can be done with water vapor radiometer (WVR) which infers water vapor delay from thermal emission from water vapor molecules and some laser profiling methods (LIDAR). Both methods are very expensive (200K\$ per site)

# Zenith wet delay

- In meteorology, the term “Precipitable water” (PW) is used. This is the integral of water vapor density with height and equals the depth of water if all the water vapor precipitated as rain (amount measured on rain gauge).
- If the mean temperature of atmosphere is known, PW can be related to Zenith Wet Delay (ZWD) (See next page)

# PW and ZWD

- Relationship:

$$ZWD = 10^{-6} \frac{R}{M_w} (k'_2 + k_3 / T_m) PW$$
$$T_m = \frac{\int P_w / T dz}{\int P_w / T^2 dz}$$

- The factor for conversion is ~6.7 mm delay/mm PW
- This relationship is the basis of ground based GPS meteorology where GPS data are used to determine water vapor content of atmosphere.
- ZWD is usually between 0-30cm.

# Mapping functions

- Zenith delays discussed so far; how to relate to measurements not at zenith
- Problem has been studied since 1970' s.
- In simplest form, for a plain atmosphere, elevation angle dependence would behave as  $1/\sin(\text{elev})$ . (At the horizon,  $\text{elev}=0$  and this form goes to infinity.
- For a spherically symmetric atmosphere, the  $1/\sin(\text{elev})$  term is “tempered” by curvature effects.
- Most complete form is “continued fraction representation” (Davis et al., 1991).

# Continued fraction mapping function

- Basic form of mapping function was deduced by Marini (1972) and matches the behavior of the atmosphere at near-zenith and low elevation angles. Form is:

$$m(\varepsilon) = \frac{1}{\sin(\varepsilon) + \frac{a}{\sin(\varepsilon) + \frac{b}{\sin(\varepsilon) + \frac{c}{\sin(\varepsilon) + \dots}}}}$$

# Truncated version

- When the mapping function is truncated to the finite number of terms then the form is:

$$m(\varepsilon) = \frac{1 + \frac{a}{1 + \frac{b}{1 + c}}}{\sin(\varepsilon) + \frac{a}{\sin(\varepsilon) + \frac{b}{\sin(\varepsilon) + c}}}$$

when  $\varepsilon = 90$ ;  $m(e) = 1$

Davis et al. 1991 solved problem by using tan for second sin argument.

# Mapping functions

- Basic problem with forming a mapping function is determining the coefficient  $a, b, c$  etc for specific weather conditions.
- There are different parameterizations:
  - Niell mapping function uses  $a, b, c$  that are latitude, height and time of year dependent
  - MTT (MIT Temperature) model uses temperature as proxy for atmospheric conditions.
  - Recent Niell work uses height of 500mbar surface (needs to be determined from assimilation models).
  - Vienna Mapping function (VMF) uses ray-tracing through 3-D atmospheric assimilation models to generate coefficients for mapping function every 6 hrs on global grid  
<http://ggosatm.hg.tuwien.ac.at/DELAY/>

# Coefficients in mapping function

- The typical values for the coefficients are
- Hydrostatic:
  - $a=1.232e-3$ ,  $b=3.16e-3$ ;  $c=71.2e-3$
- Wet delay
  - $a=0.583e-3$ ;  $b=1.402e-3$ ;  $c=45.85e-3$
- Since coefficients are smaller for wet delay, this mapping function increases more rapidly at low elevation angles.
- At 0 degrees, hydrostatic mapping function is ~36. Total delay ~82 meter

# Mapping functions

- The basic form of the continued fraction fit raytracing through radiosonde temperature, pressure and humidity profiles to a few millimeters at 3deg elevation angle.
- Basic problem is parameterizing  $a, b, c$  in terms of observable meteorological parameters.
- Comparison on mapping functions:

<http://gauss.gge.unb.ca/papers.pdf/igs97tropo.pdf>

# Gradients

- In recent years; more emphasis put on deviation of atmospheric delays from azimuthal symmetry. See: Chen, G. and T. A. Herring, Effects of atmospheric azimuthal asymmetry of the analysis of space geodetic data, *J. Geophys. Res.*, 102, 20,489–20,502, 1997.
- These effects are much smaller (usually <30mm) but do effect modern GPS/VLBI measurements.
- There is a mean NS gradient that is latitude dependent and probably due to equator to pole temperature gradient.
- Parameterized as  $\cos(\text{azimuth})$  and  $\sin(\text{azimuth})$  terms with a “tilted” atmosphere model ( $1/(\sin(\varepsilon)+0.032)$ )

# Effects of atmospheric delays

- Effects of the atmospheric delay can be approximately assessed using a simple WLS model of the form:

$$y = [1 \quad \cos(\varepsilon) \quad m(\varepsilon)] \begin{bmatrix} \Delta clk \\ \Delta H \\ \Delta ZHD \end{bmatrix} \quad m(\varepsilon) \approx 1 / \sin(\varepsilon)$$

- Simulated data  $y$  (e.g. error in mapping function) can be used to see effects on clock estimate ( $\Delta clk$ ), Height ( $\Delta h$ ), and atmospheric delay ( $\Delta zhd$ )
- If  $m(\varepsilon)$  is removed from partials, then effects in zenith delay error on height can be estimated.

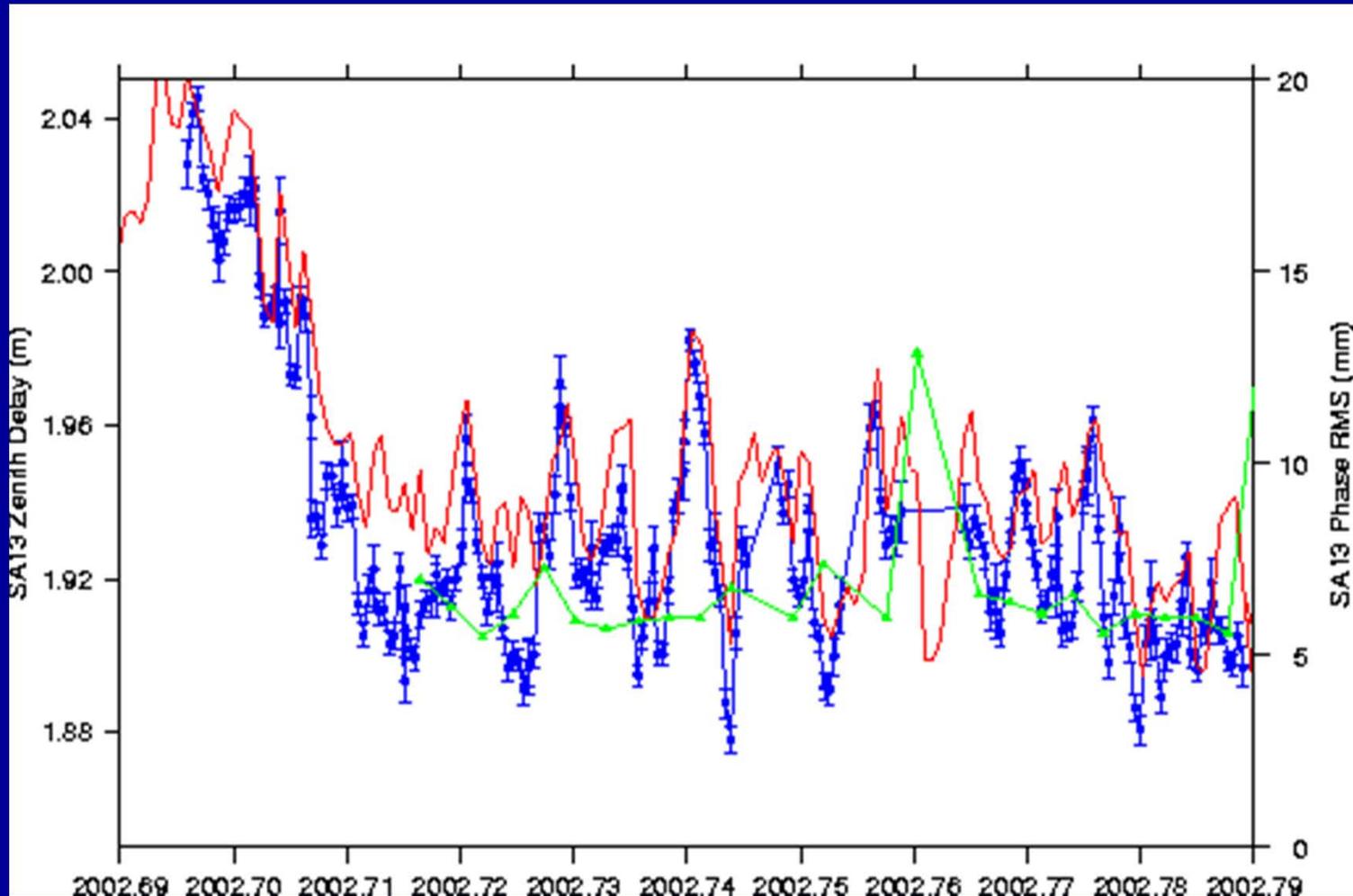
# Effects of atmospheric delay

- If atmospheric zenith delay not estimated, then when data is used to 10 degree elevation angle, error in height is  $\sim 2.5$  times zenith atmospheric delay error (see Herring, T. A., Precision of vertical position estimates from very-long-baseline interferometry, *J. Geophys. Res.*, 91, 9177–9182, 1986.
- A simple matlab program can reproduce these results
- Herring Kalman filter paper also discusses effects of process noise value in height estimate uncertainty.

# Parameterization of atmospheric delay

- Given the sensitivity of GPS position estimates to atmospheric delay, and that external calibration of the delay is only good to a few centimeters; atmospheric zenith delays and often gradients are estimated high-precision GPS analyses.
- Parameterization is either Kalman filter or coefficients of piece-wise linear functions (GAMIT)

# Example using NCEP analysis field



Blue is GPS estimates of delay, red is NCEP calculation

# Newer Mapping functions

- There has been recent work on mapping functions and zenith delay estimates.
- The Vienna Mapping Function and associated products developed at the Technical University of Vienna.
- Web site includes ECMWF derived values and the empirical global mapping function and global pressure temperature models (GMF and GPT).

<http://ggosatm.hg.tuwien.ac.at/DELAY/>

# Effects of errors in a priori Zenith delays

- Figure from Tregoning and Herring, 2006 showing effects of error in hydrostatic zenith delay estimated using a wet mapping function.

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# Summary

- Atmospheric delays are one of the limiting error sources in GPS
- Delays are nearly always estimated:
  - At low elevation angles there can be problems with mapping functions
  - Spatial inhomogeneity of atmospheric delay still unsolved problem even with gradient estimates. Some GPS stations (e.g., GRNX, P549) have asymmetric position estimate residuals.
  - Estimated delays can be used for weather forecasting if latency  $< 2$  hrs.
- Class web page has links to some sites that deal with GPS atmospheric delay estimates.
- Example on next slide shows some unknown effects of atmospheric delays on GPS position estimates (“raining” (one-sided) position residuals)

# P549 Position residuals

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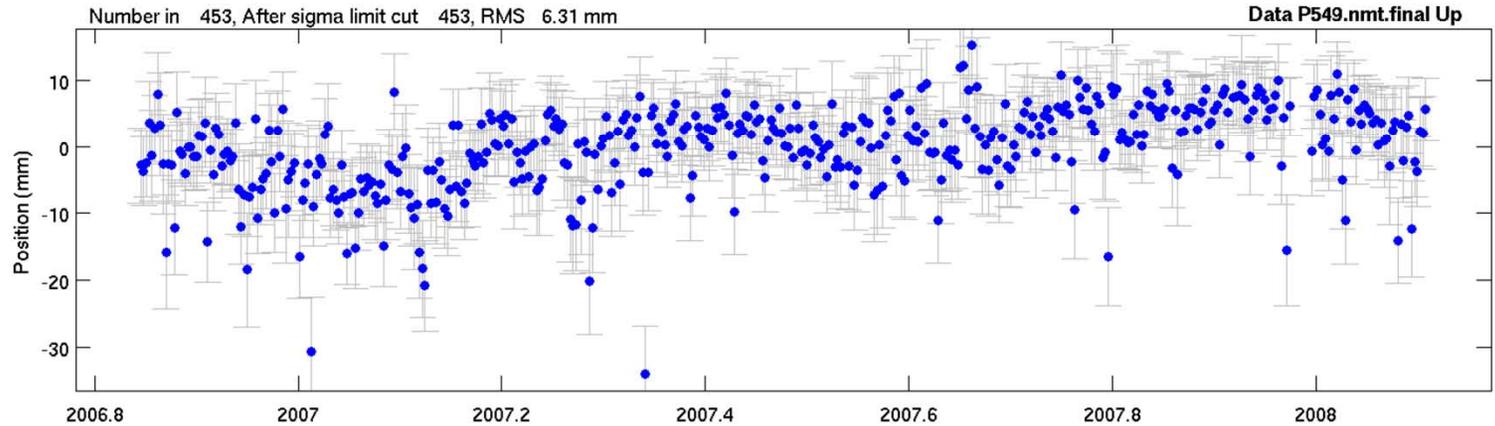
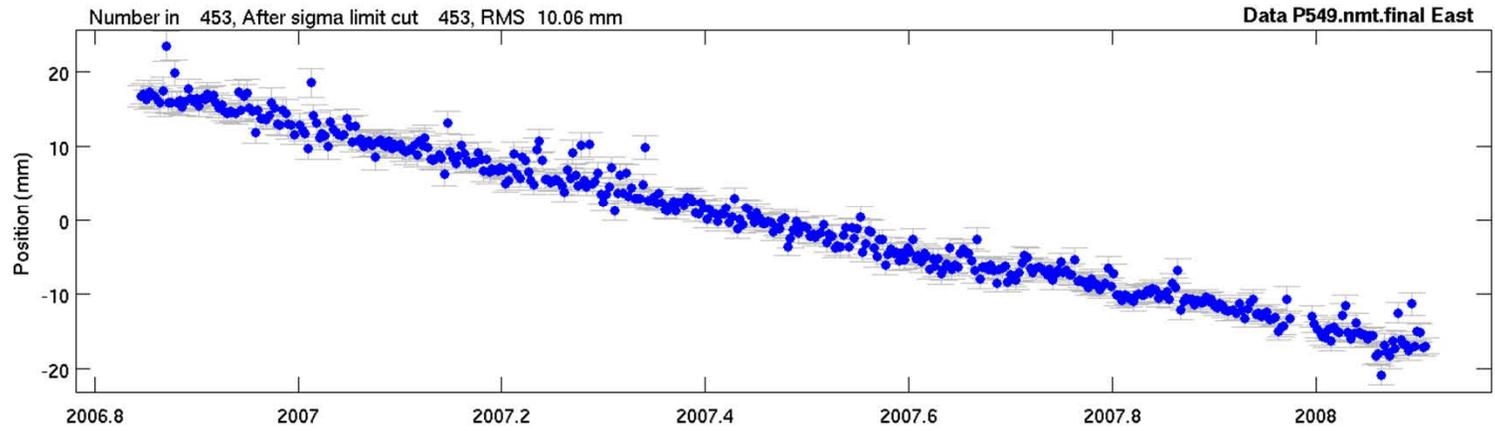
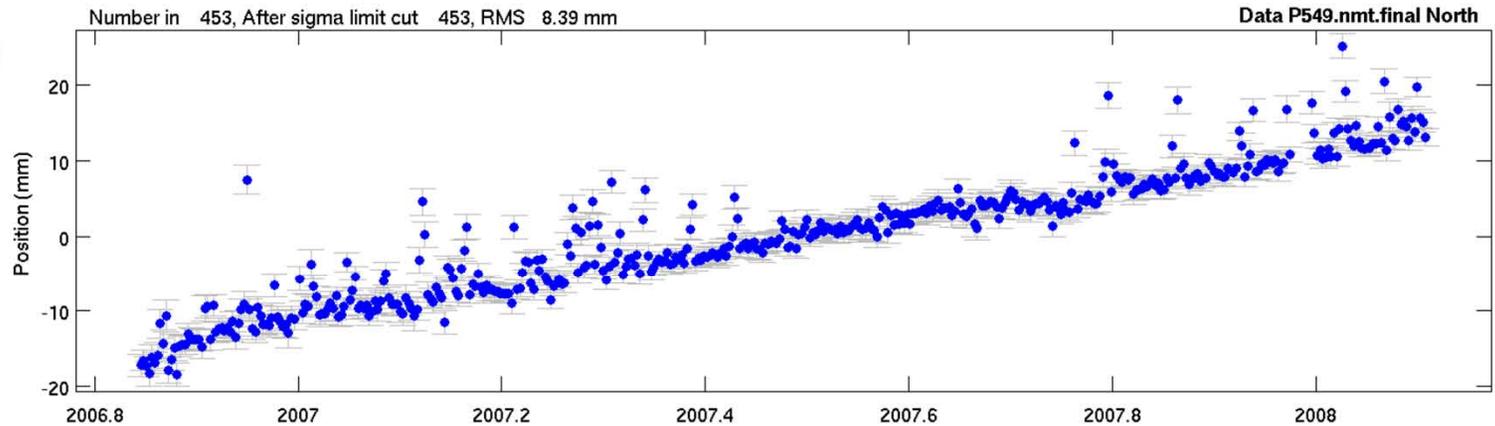
- HOLM.cwu.rapid
- HOLM.nmt.final
- HOLM.nmt.rapid
- HOLM.pbo.final
- HOLM.pbo.rapid
- P259.bsl.final
- P259.bsl.rapid
- P259.cwu.final
- P259.cwu.rapid
- P259.nmt.final
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- AC17.pbo.final

- Load
- Append
- Difference
- Detrend
- Edit
- Block Edit
- Break
- Exponent
- Log
- Edit Breaks
- Zoom
- Span
- Save

- Outliers (n-sigma)
- 4
- Max Sigma (mm)
- 1000.0
- Keep Breaks
- Brk on Appnd
- Report Edits
- Delete Breaks
- None

- Parameter Set
- Linear Only Re...
- +Annual
- +Semiannual
- Exp Rates
- RealSigma
- DisplayFit
- Average
- 30
- Hide Error Bars

- Name of Edit File
- tsview.renames
- Output ENS File
- sutsview.ens
- Root for Detail
- ts\_
- ENS file
- ../sut
- GLOBK Eq file



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