

**12.540 Homework #2****Due Wednesday, April 14, 2010****Question 1: Non-linear estimation problem**

The data below are from a model of data of the form:

$$y(t) = A\cos(2\pi t/15 + \phi) + Y_0 + N(t)$$

where A, the amplitude,  $\phi$  the phase, and  $Y_0$  are unknown parameters to be estimated.  $N(t)$  is Gaussian noise with standard deviations given by the +- column (all constant in this case).

Data:

Time	y(t)	+-
0.0	14.02	1.00
1.0	9.60	1.00
2.0	2.98	1.00
3.0	0.61	1.00
4.0	-3.68	1.00
5.0	-7.19	1.00
6.0	-10.64	1.00
7.0	-9.83	1.00
8.0	-7.26	1.00
9.0	-5.24	1.00
10.0	2.38	1.00
11.0	6.14	1.00
12.0	9.68	1.00
13.0	12.23	1.00
14.0	14.27	1.00
15.0	12.51	1.00
16.0	9.28	1.00
17.0	5.23	1.00
18.0	1.93	1.00
19.0	-3.30	1.00
20.0	-6.41	1.00
21.0	-7.43	1.00
22.0	-9.58	1.00
23.0	-6.59	1.00
24.0	-3.76	1.00
25.0	2.61	1.00
26.0	7.37	1.00
27.0	10.79	1.00
28.0	11.75	1.00
29.0	13.82	1.00

(a) Estimate A,  $\phi$ , and  $Y_0$  using a non-linear weighted least-squares estimator and give the values and their standard deviations.

(b) Reformulate the problem to be of the form:

$$y(t) = A_c \cos(2\pi t/15) + A_s \sin(2\pi t/15) + Y_0 + N(t)$$

and estimate  $A_c$ ,  $A_s$ , and  $Y_0$  using linear weighted least-squares estimator and give the estimates and their standard deviations. The  $A_c$  and  $A_s$  values are quadrature components.

(c) Use the  $A_c$  and  $A_s$  values and their variance-covariance matrix from part (b)

to compute the amplitude and phase and their variance-covariance matrix. Compare the estimated with those from part (a).

(d) These data are uniformly spaced in time and constant standard deviations. Determine the quadrature components from an FFT of the data. Compare the results with part (b). Why are the results the same?

**Question 2:** Linear estimation problem with regular and sequential estimation. Using the data set listed below:

(a) Fit a linear regression ( $y=ax+b$ ) to the data using standard weighted least squares

(b) Compute the chi-squared per degree of freedom of the postfit residuals. Is this value consistent with the variations expected from random variations?

(c) Divide the data into two parts (first and last 5 data points) and use sequential estimation to determine a and b.

(d) Find the weighted mean y (the weighted mean is the weighted least squares solution to the equation  $y = b$ ). Compare this estimate of b with the estimates of the a' when x is shifted such that estimates of a' and b' are uncorrelated (i.e., the solution to  $y=a'(x-x_0)+b'$  where  $x_0$  is selected such that the estimates of a' and b' are uncorrelated).

**Data:**

X	Y	+-
1	5.759	1.050
2	-2.389	1.080
3	-10.512	0.975
4	-18.220	0.974
5	-19.745	1.075
6	-24.429	1.227
7	-32.521	1.025
8	-36.411	0.817
9	-43.655	0.989
10	-50.882	0.842

**Question 3:** Example of using differenced data

Using the data below:

(a) Estimate A given the model that  $y(t) = A*\sin(X)+C(t)$  where C(t) is a randomly varying function of time.

(b) Instead of estimating C(t) at each time, use a differencing method to eliminate C(t) from the estimation. (Hint: Use propagation of variances to determine the covariance matrix of the differenced data.

(c) Determine A by treating C(t) as correlated data noise (i.e., the C(t) noise is that same at each time and therefore highly correlated at each time but independent between times).

(d) Compare the results from the "brute force", differencing, and data noise approaches (Hint: When solved correctly they all generate the same result).

**Data:**

T	X	Y	+ -
1	1.1000	44.1946	1.00
1	0.3000	36.8838	1.00
1	-0.1900	32.0616	1.00
2	1.2000	-0.1419	1.00
2	0.1000	-5.9336	1.00
2	-0.1600	-9.1461	1.00
3	1.3000	-7.8164	1.00
3	-0.1000	-19.3260	1.00
3	-0.1100	-21.5875	1.00
4	1.4000	35.3201	1.00
4	-0.3000	24.3602	1.00
4	-0.0400	26.2854	1.00
5	1.5000	-34.5531	1.00
5	-0.5000	-47.7803	1.00
5	0.0500	-41.8605	1.00
6	1.6000	0.6797	1.00
6	-0.7000	-13.2281	1.00
6	0.1600	-6.1792	1.00
7	1.7000	1.1492	1.00
7	-0.9000	-17.6524	1.00
7	0.2900	-8.0827	1.00
8	1.8000	25.4898	1.00
8	-1.1000	6.2613	1.00
8	0.4400	16.9865	1.00
9	1.9000	16.7815	1.00
9	-1.3000	-4.4243	1.00
9	0.6100	11.4170	1.00
10	2.0000	-14.0549	1.00
10	-1.5000	-37.0554	1.00
10	0.8000	-14.9231	1.00

MIT OpenCourseWare  
<http://ocw.mit.edu>

12.540 Principles of the Global Positioning System  
Spring 2012

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.