12.520 Problem Set 7 **Due 12/04/06**

1) (100%) Consider the homogeneous deformation:

$x' = ax + by$ $x = (dx' - by')/(ad - bc)$

$y' = cx + dy$ $y = (ay' - cx')/(ad - bc)$

a) Write expressions for the displacement vector **u** that takes (x,y) -> (x',y'), both in terms of (x,y) and x', y').

b) Write expressions for E_{ij} (Lagrangian strain tensor), e_{ij} (Eulerian strain tensor), Ω_{ij} (Lagrangian rotation tensor), ω_{ij} (Eulerian rotation tensor).

These can be applied incrementally over a time t, $0 \le t \le 1$, by setting

a -> 1 + t(a - 1), b -> bt, c -> ct, d -> 1 + t(d - 1).

c) Write expressions for $e_{ij(t)}$ and $\dot{\varepsilon}_{ij}(t)$, where $\dot{\varepsilon}_{ij}(t)$ is the instantaneous Cauchy strain rate tensor.

Does

$$e_{ij}(t=1) = \int_0^1 \dot{\varepsilon}_{ij}(t)\,dt$$

Why or why not?

Consider the 2 following finite strains:

1) $x' = x + 1.5y$
$y' = y$

2) $x' = 2x$
$y' = y/2$

d) What special strains do they represent?

e) Write $E_{ij}(t=1)$, $e_{ij}(t=1)$, $\Omega_{ij}(t=1)$, $\omega_{ij}(t=1)$, and $\varepsilon_{ij}(t=1)$ for these cases.

f) Calculate the principal axes for $E_{ij}(t=1)$, $\dot{\varepsilon}_{ij}(t=1)$, and $e_{ij}(t=1)$ for both cases.

g) In their final stage, pure and simple shears can be simply related by a rotation. Yet pure shear has 2 equivalent directions, while simple shear has only 1.

Materials such as ice or olivine develop preferred fabrics when subjected to simple shear. They recrystallize under deviatoric *stress* with easy glide at 45° to the maximum compressive *stress*. Since the stresses for pure and simple shear are equivalent, how can pure shear lead to 2 preferred directions, while simple shear leads to only 1?