

12.520: Problem set 1
Due 9/22/06

1) Write out symbolically the relationship between the traction vector acting on a surface, T_i , the normal vector describing the orientation of that surface, n_j , and the local stress tensor, σ_{ji} , using a) vector-matrix notation, b) the summation symbol Σ , c) Einstein summation notation, and d) full explicit evaluation, writing out each term. (10%)

2) Turcotte & Schubert, 2-14 (2-12 old edition)(10%) (Recall that T&S use a different sign convention for compressive stress).

3) For practice in Einstein summation, evaluate the expressions $\delta_{ij} \delta_{ij}$ and δ_{ii} (i.e., calculate numerical values), both for a three-dimensional coordinate system ($i, j = 1-3$) and a 4-D system ($i, j = 1-4$). Briefly explain what you are doing. (10%)

4) (40%) For the stress tensor

$$\sigma_{ij} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

i) Find the principal stresses and directions. (Be sure that your principal coordinate system is right-handed.)

ii) Calculate the three Invariants for the this stress tensor and for the associated principal stress tensor and compare their values.

iii) Find the deviatoric stress tensor σ_{ij}^{dev}

iv) Find the principal stresses and directions of the deviatoric stress tensor. Find the invariants of the deviatoric stress tensor.

v) What are the relations of the principal stresses, directions, shears, invariants, etc. for these two tensors (the complete and deviatoric tensors investigated above)? Can you say anything in general about these relations for an arbitrary stress tensor and its deviator?

5) (15%) If there are no body forces, demonstrate whether equilibrium exists for stresses:

$$\sigma_{xx} = 3x^2 + 4xy - 8y^2$$

$$\sigma_{yy} = 2x^2 + xy + 3y^2$$

$$\sigma_{xy} = 1/2 x^2 - 6xy - 2y^2$$

$$\sigma_{zz} = \sigma_{xz} = \sigma_{yz} = 0$$

What are the normal and shear tractions on a plane perpendicular to the y axis at $x = y = 1$?

6) (15%) We can define the "lithostatic stress" at a depth d below the surface of rock of density ρ as:

$$\boldsymbol{\sigma}_\ell = \begin{bmatrix} -\rho g d & 0 & 0 \\ 0 & -\rho g d & 0 \\ 0 & 0 & -\rho g d \end{bmatrix}$$

Suppose, in addition to this lithostatic stress, there is a small tectonic stress τ superimposed, as given below. Write both the deviatoric stress $\boldsymbol{\sigma}^{dev}$ and the nonlithostatic stress $\boldsymbol{\sigma}^{nl}$ for the following stress tensors:

$$\text{i) } \boldsymbol{\sigma} = \begin{bmatrix} -\rho g d - \sigma & 0 & 0 \\ 0 & -\rho g d & 0 \\ 0 & 0 & -\rho g d \end{bmatrix};$$

$$\text{ii) } \boldsymbol{\sigma} = \begin{bmatrix} -\rho g d - \tau & 0 & 0 \\ 0 & -\rho g d & 0 \\ 0 & 0 & -\rho g d + \tau \end{bmatrix}$$

$$\text{iii) } \boldsymbol{\sigma} = \begin{bmatrix} -\rho g d & \tau & 0 \\ \tau & -\rho g d & 0 \\ 0 & 0 & -\rho g d \end{bmatrix}$$

6) (15%) It is possible to use the concept of nonlithostatic stress to make some interesting conclusions regarding the state of stress that arises from isostatically compensated structures (e.g., continents vs. oceans, or midoceanic ridges vs. abyssal plains). To do this, do T&S, problem 2-6.

But before starting, note that the choice of wording in this problem is very poor (i.e., wrong). When T&S say "deviatoric stress," substitute "nonlithostatic stress." When they say "pressure," substitute "lithostatic stress."

Before answering T&S 2-6, describe the difference between "deviatoric stress" and "nonlithostatic stress." Also, describe the difference between "pressure" and "lithostatic stress."