Radiative Equilibrium

- Equilibrium state of atmosphere and surface in the absence of non-radiative enthalpy fluxes
- Radiative heating drives actual state toward state of radiative equilibrium

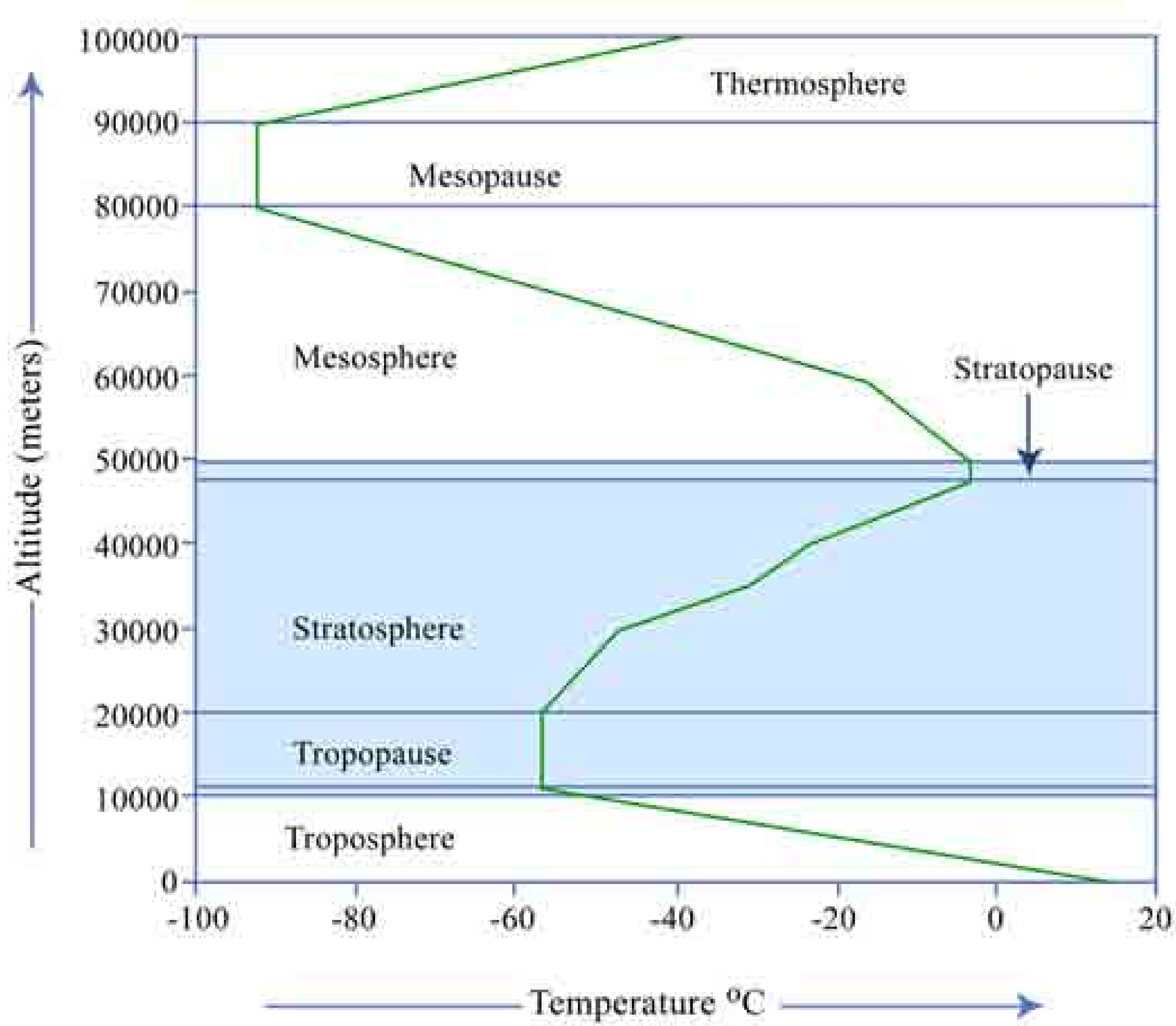
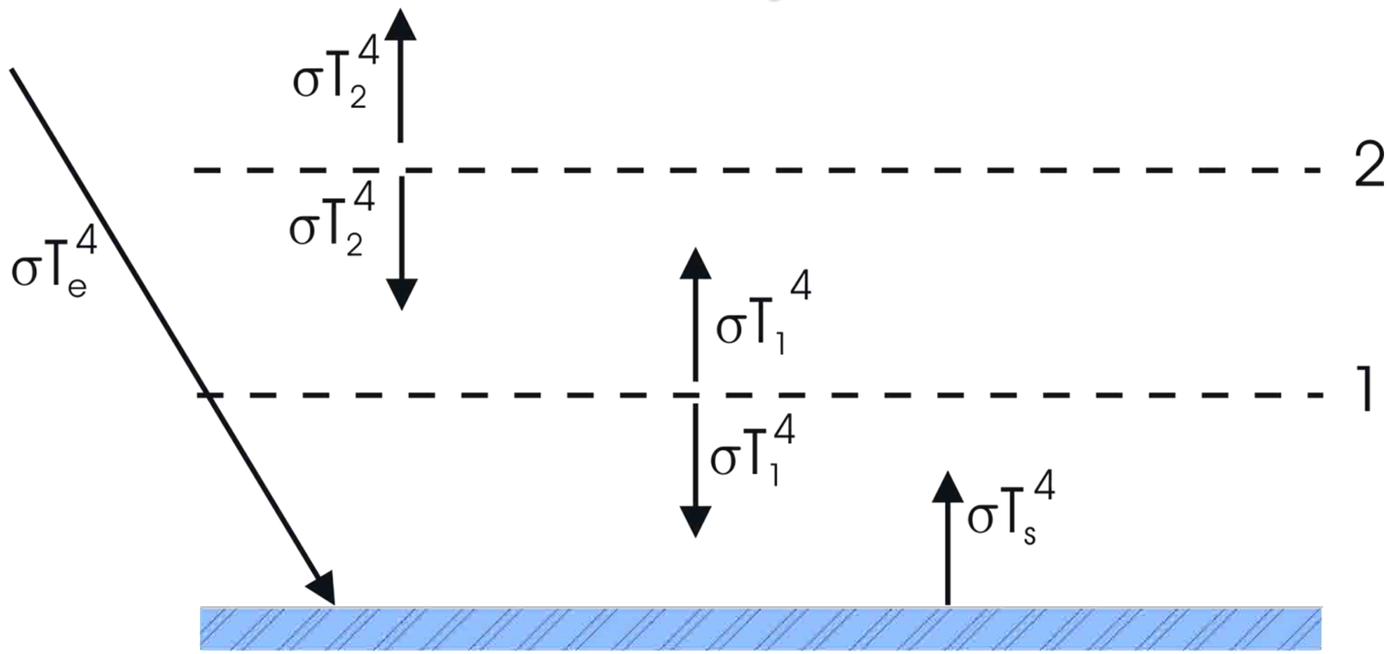


Figure by MIT OpenCourseWare.

Extended Layer Models



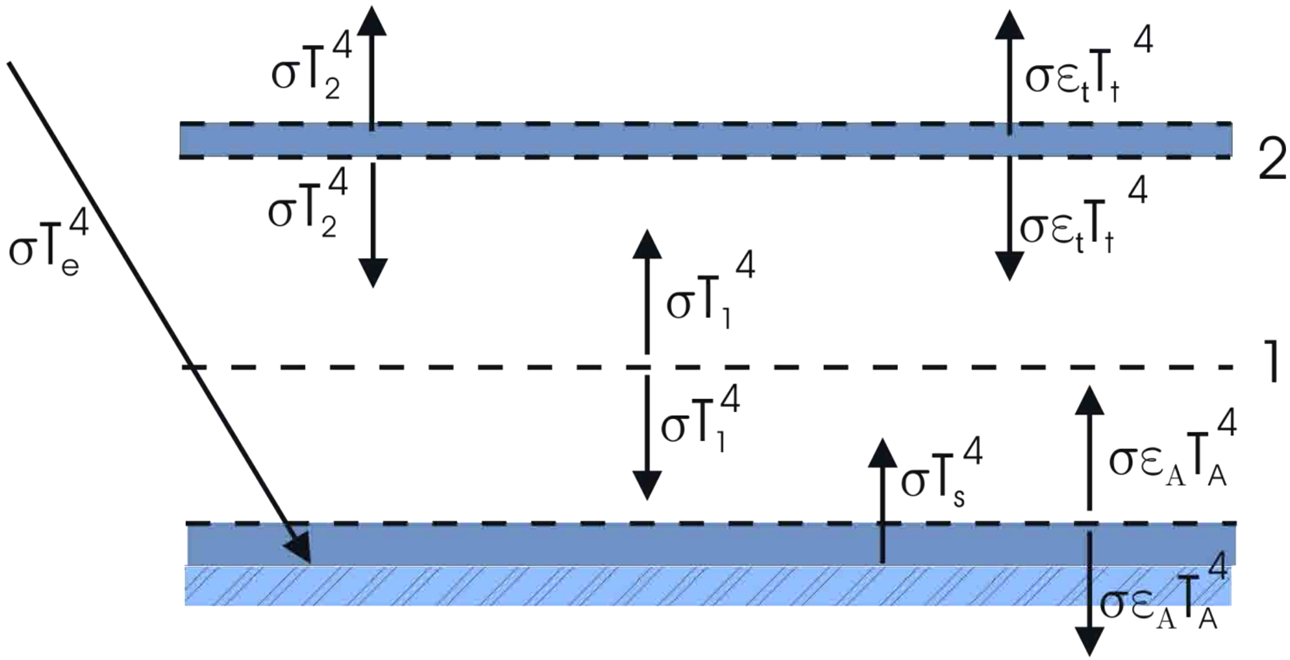
$$TOA: \quad \sigma T_2^{\,4} = \sigma T_e^{\,4} \rightarrow T_2 = T_e$$

$$Middle\ Layer: \quad 2\sigma T_1^{\,4} = \sigma T_2^{\,4} + \sigma T_s^{\,4} = \sigma T_e^{\,4} + \sigma T_s^{\,4}$$

$$Surface: \quad \sigma T_s^{\,4} = \sigma T_e^{\,4} + \sigma T_1^{\,4}$$

$$\rightarrow \quad T_s = 3^{1/4} T_e \qquad\qquad T_1 = 2^{1/4} T_e$$

Effects of emissivity<1



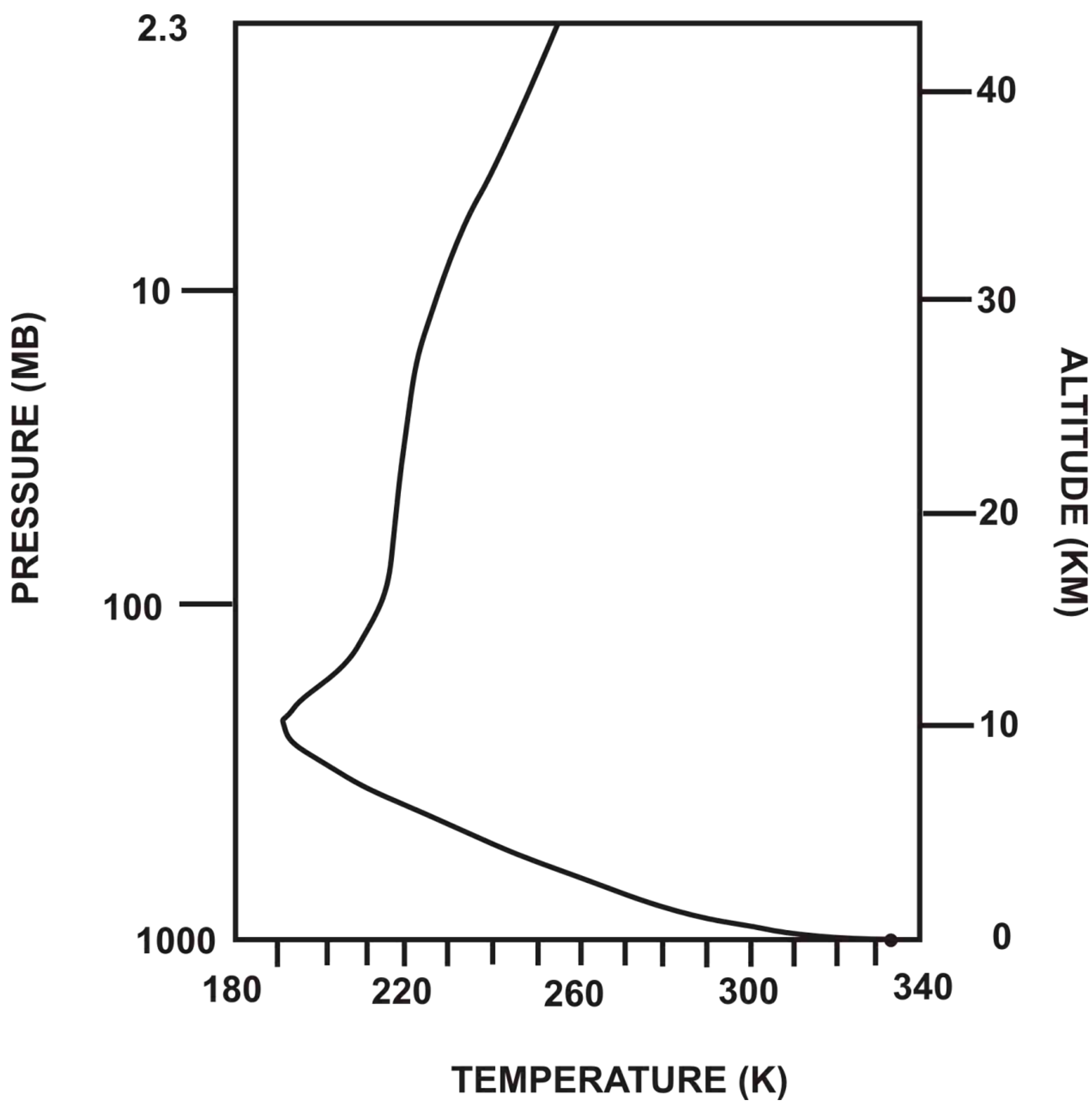
$$\textit{Surface}: \quad 2\varepsilon_A \sigma T_A{}^4 = \varepsilon_A \sigma T_1{}^4 + \varepsilon_A \sigma T_s{}^4$$

$$\rightarrow \quad T_A = \left(5/2\right)^{1/4} T_e \approx 321K \quad < T_s$$

$$\textit{Stratosphere}: \quad 2\varepsilon_t \sigma T_t{}^4 = \varepsilon_t \sigma T_2{}^4$$

$$\rightarrow \quad T_t = \left(1/2\right)^{1/4} T_e \approx 214K \quad < T_e$$

Full calculation of radiative equilibrium



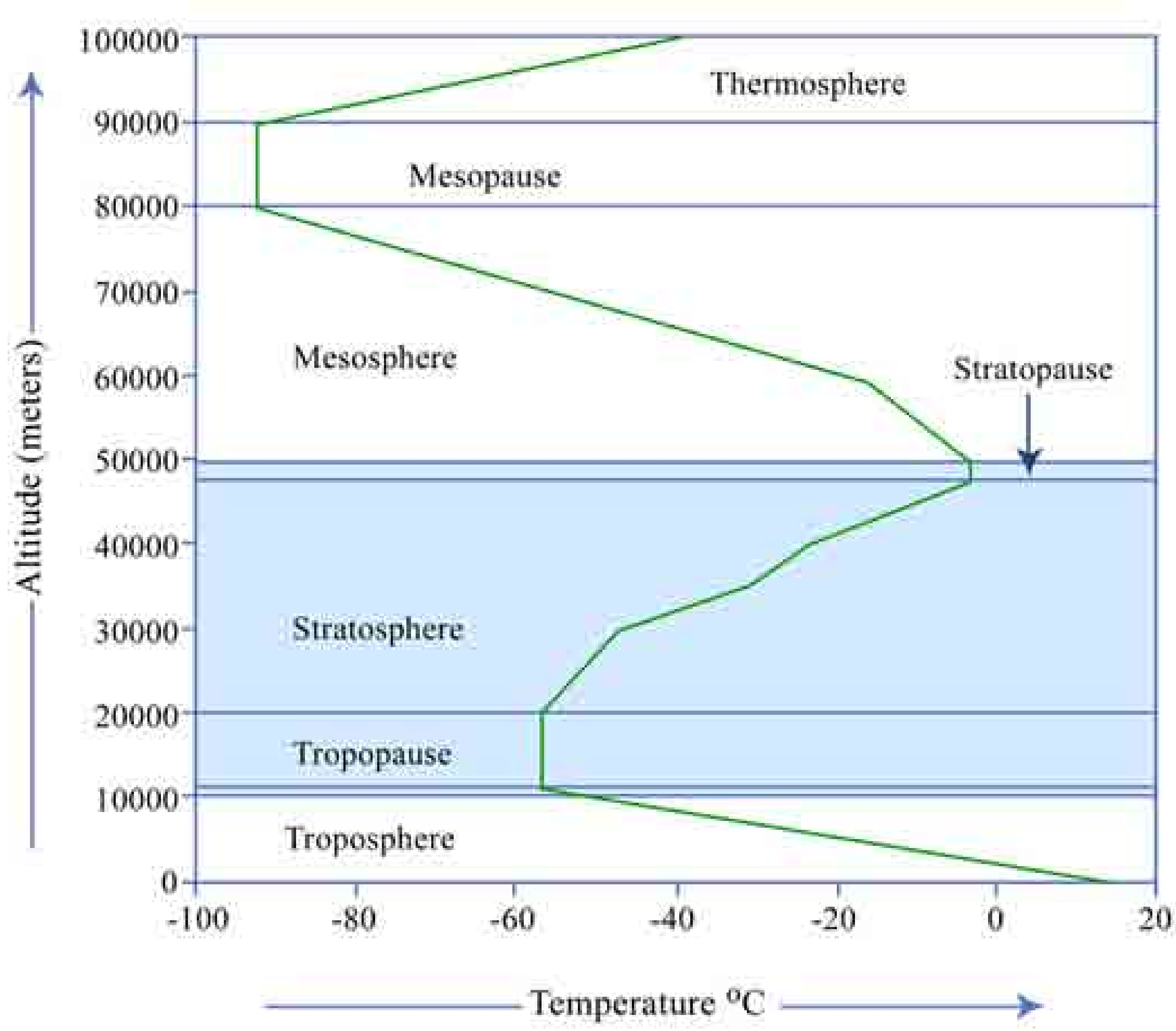
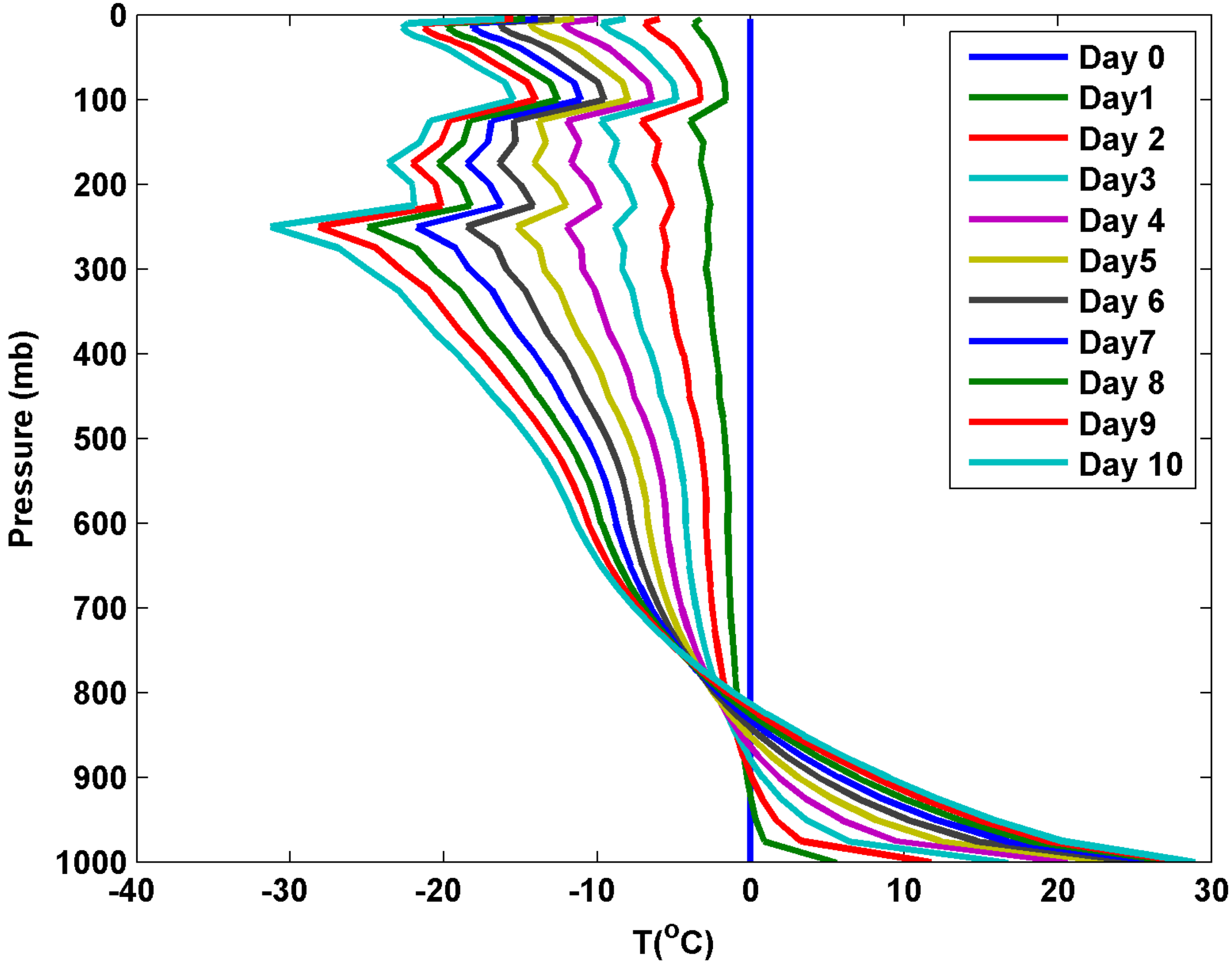
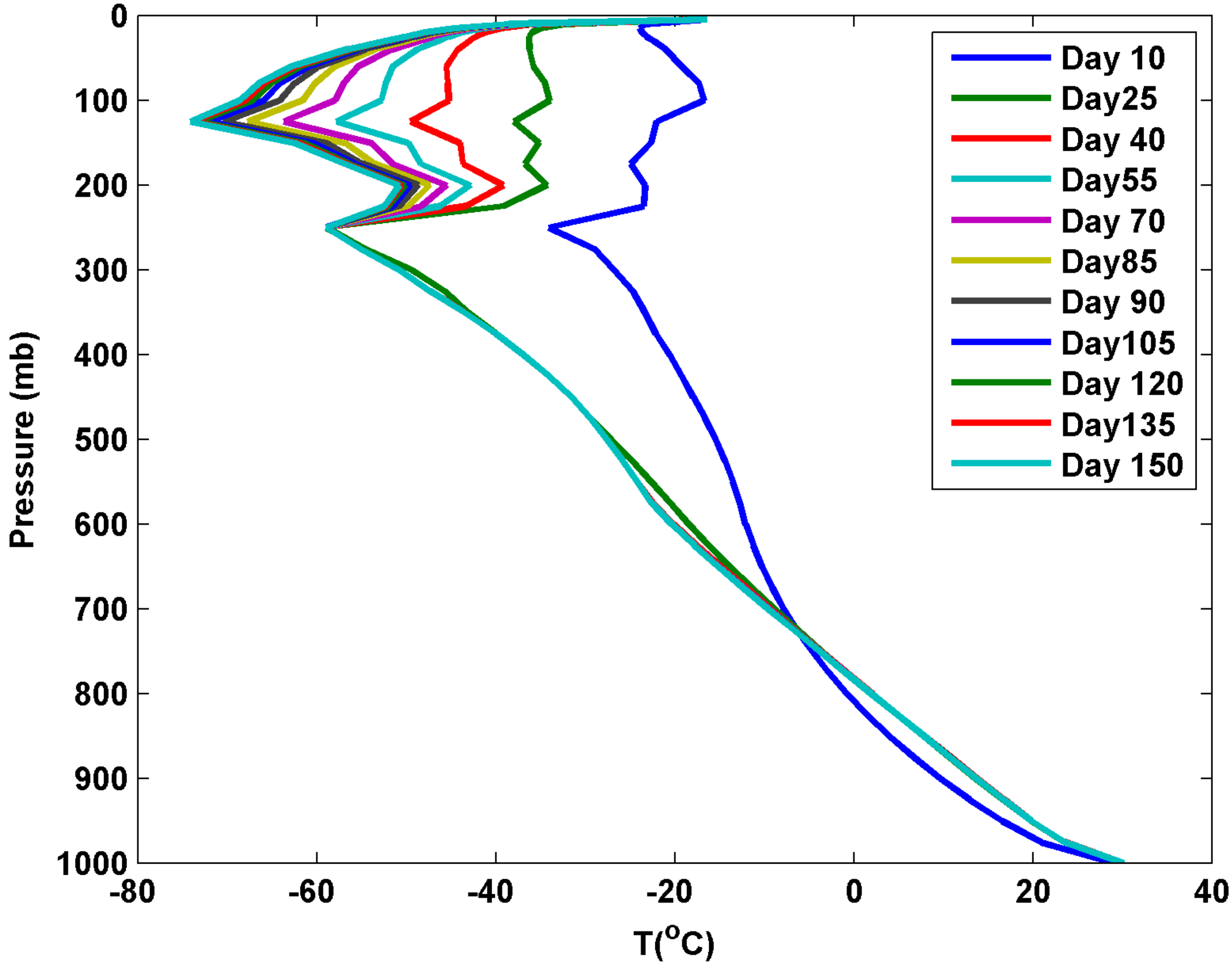


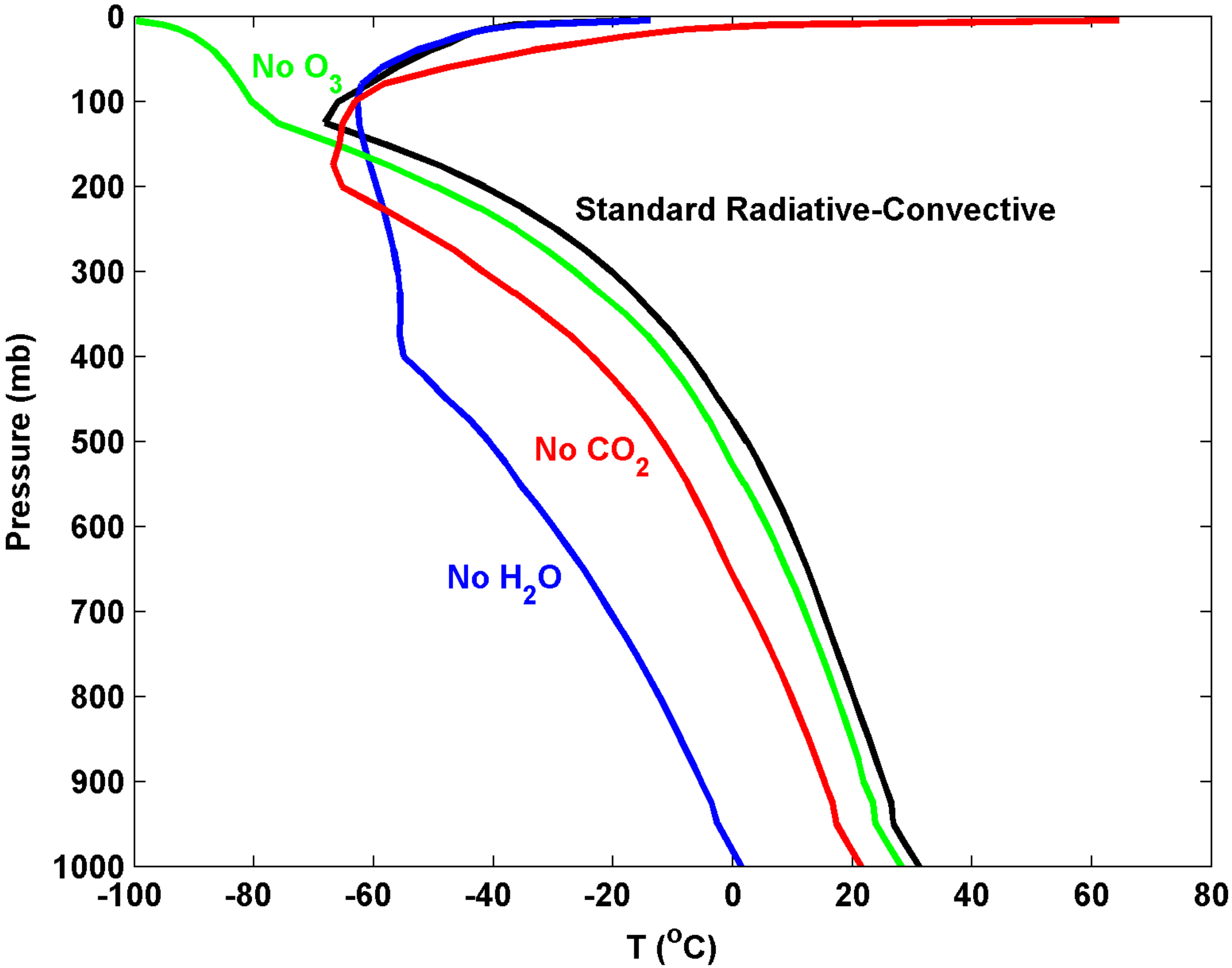
Figure by MIT OpenCourseWare.

Time scale of approach to equilibrium

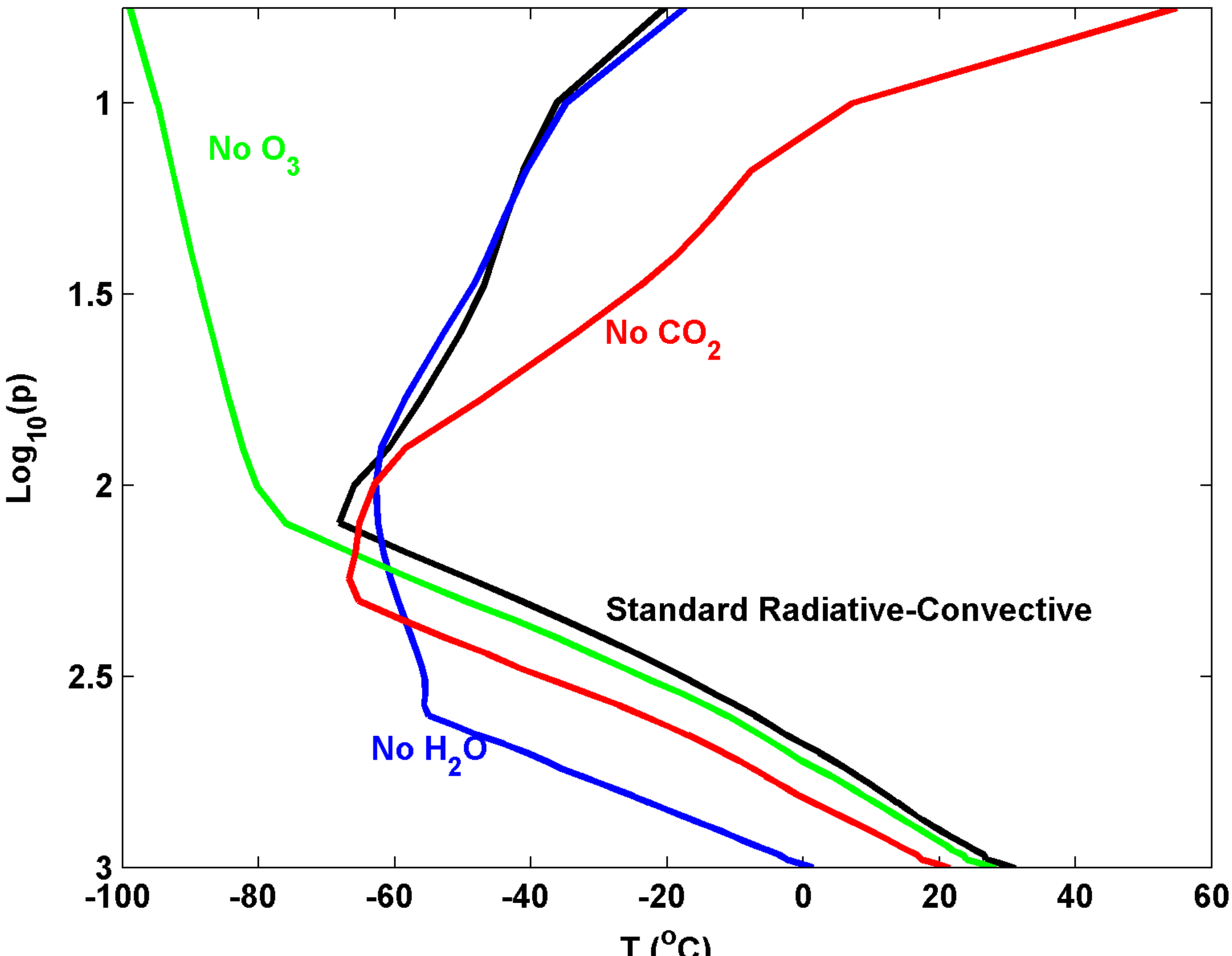



0
100
200
300
400
500
600
700
800
900
1000
Pressure (mb)
-80
-60
-40
-20
0
20
40
T(°C)
Day 10
Day25
Day 40
Day55
Day 70
Day85
Day 90
Day105
Day 120
Day135
Day 150

Contributions of various absorbers



Note: All simulations have variable clouds interacting with radiation



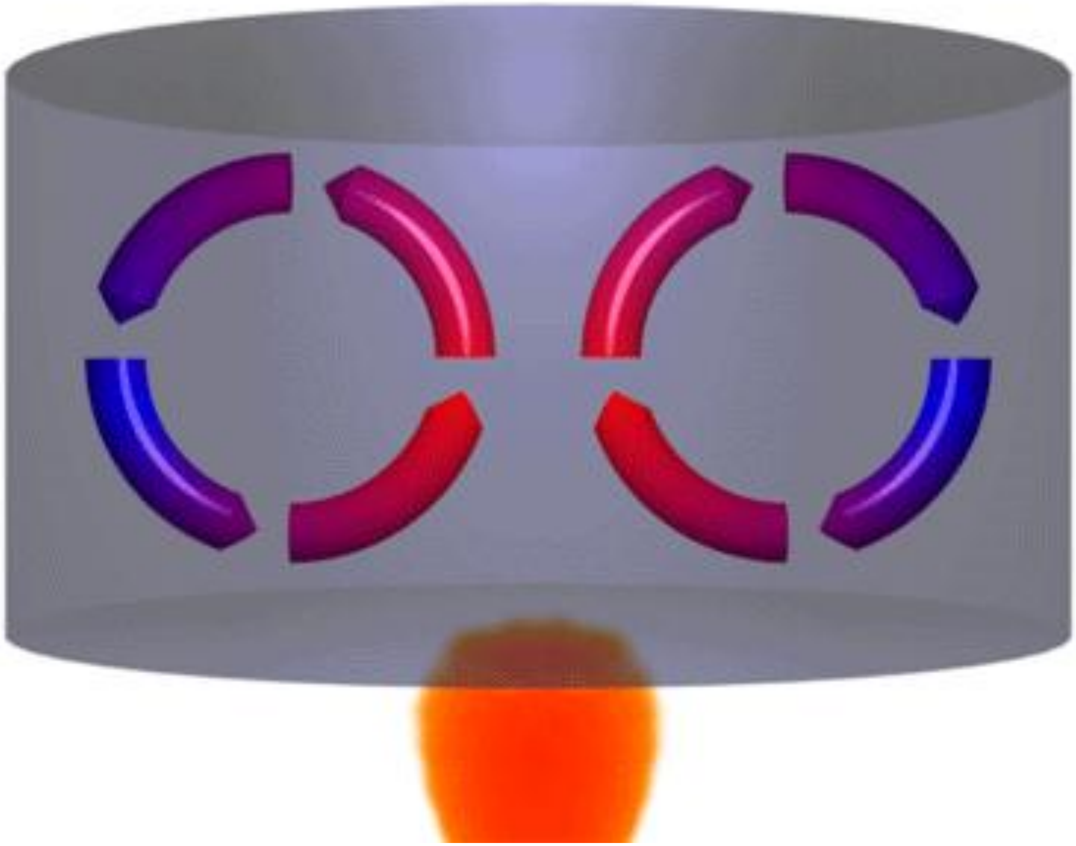
No O_3
No CO_2
Standard Radiative-Convective
No H_2O
1
1.5
2
2.5
3
$Log_{10}(p)$
-100
-80
-60
-40
-20
0
20
40
60
T (^oC)

Problems with radiative equilibrium solution

- Too hot at and near surface
- Too cold at a near tropopause
- Lapse rate of temperature too large in the troposphere
- (But stratosphere temperature close to observed)

Missing ingredient: Convection

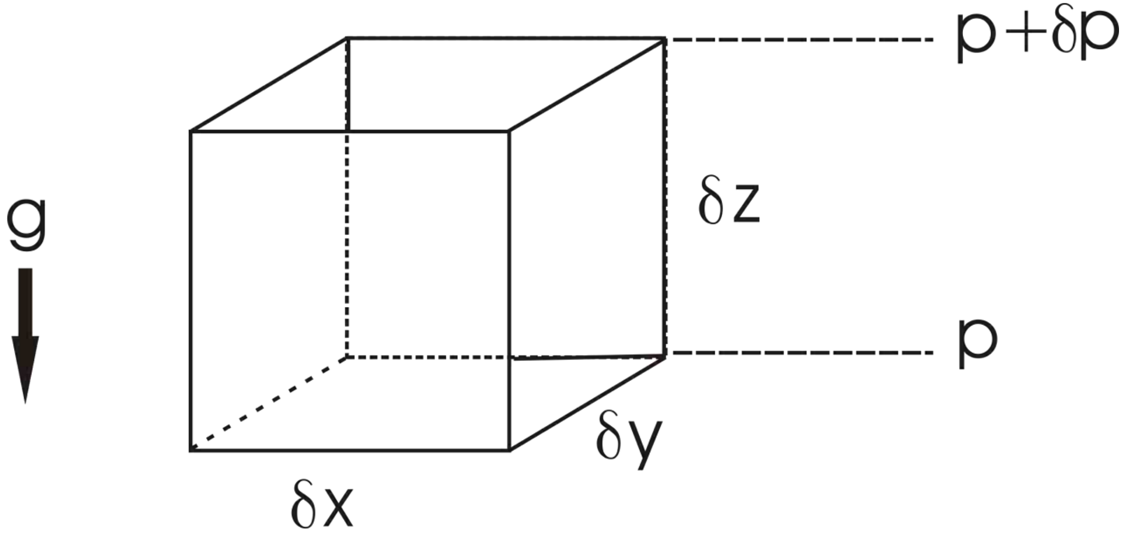
- As important as radiation in transporting enthalpy in the vertical
- Also controls distribution of water vapor and clouds, the two most important constituents in radiative transfer



When is a fluid unstable to convection?

- Pressure and hydrostatic equilibrium
- Buoyancy
- Stability

Hydrostatic equilibrium



$$Weight: \quad -g\rho\delta x\delta y\delta z$$

$$Pressure: \quad p\delta x\delta y-\left(p+\delta p\right)\delta x\delta y$$

$$F=MA: \quad \rho\delta x\delta y\delta z\frac{dw}{dt}=-g\rho\delta x\delta y\delta z-\delta p\delta x\delta y$$

$$\frac{dw}{dt}=-g-\alpha\frac{\partial p}{\partial z}, \qquad \alpha = {}^{1}\!/_{\rho} = specific\ volume$$

Pressure distribution in atmosphere at rest

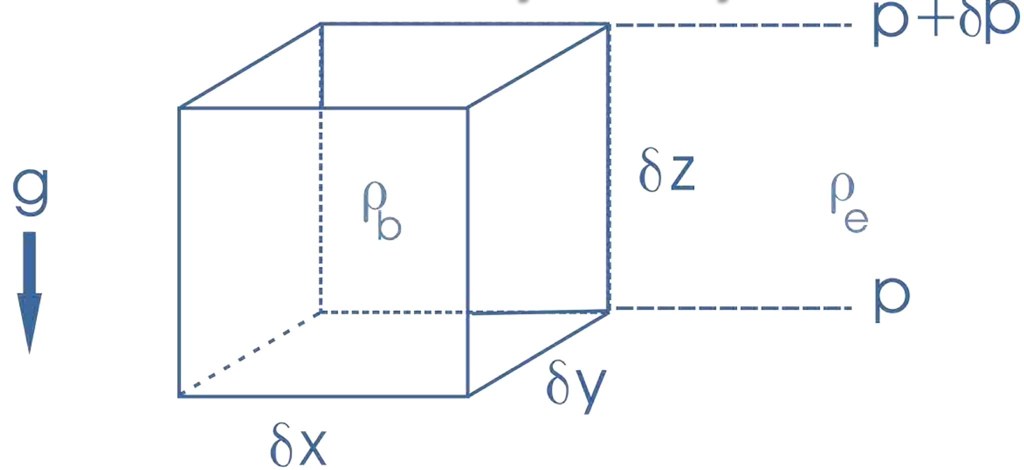
$$Ideal\ gas: \quad \alpha = \frac{RT}{p}, \qquad R \equiv \frac{R^*}{\bar{m}}$$

$$Hydrostatic: \quad \frac{1}{p}\frac{\partial p}{\partial z} = -\frac{g}{RT}$$

$$Isothermal\ case: \quad p = p_0 e^{-z/H}, \qquad H \equiv \frac{RT}{g} = \text{"}scale\ height\text{"}$$

Earth: H~ 8 Km

Buoyancy



$$Weight: \quad -g\rho_b \delta x \delta y \delta z$$

$$Pressure: \quad p\delta x \delta y - (p + \delta p)\delta x \delta y$$

$$F = MA: \quad \rho_b \delta x \delta y \delta z \frac{dw}{dt} = -g\rho_b \delta x \delta y \delta z - \delta p \delta x \delta y$$

$$\frac{dw}{dt} = -g - \alpha_b \frac{\partial p}{\partial z} \qquad but \quad \frac{\partial p}{\partial z} = -g/\alpha_e$$

$$\rightarrow \quad \frac{dw}{dt} = g\frac{\alpha_b - \alpha_e}{\alpha_e} \equiv B$$

Buoyancy and Entropy

Specific Volume: $\alpha = 1/\rho$

Specific Entropy: s

$$\alpha = \alpha(p, s)$$

$$\left(\delta\alpha\right)_p = \left(\frac{\partial\alpha}{\partial s}\right)_p \delta s = \left(\frac{\partial T}{\partial p}\right)_s \delta s$$

$$B = g\frac{\left(\delta\alpha\right)_p}{\alpha} = \frac{g}{\alpha}\left(\frac{\partial T}{\partial p}\right)_s \delta s = -\left(\frac{\partial T}{\partial z}\right)_s \delta s \equiv \Gamma\delta s$$

The adiabatic lapse rate

$$\textit{First Law of Thermodynamics:}$$

$$\dot{Q} = T\frac{ds_{rev}}{dt} = c_v\frac{dT}{dt} + p\frac{d\alpha}{dt}$$

$$= c_v\frac{dT}{dt} + \frac{d(\alpha p)}{dt} - \alpha\frac{dp}{dt}$$

$$= (c_v + R)\frac{dT}{dt} - \alpha\frac{dp}{dt}$$

$$= c_p\frac{dT}{dt} - \alpha\frac{dp}{dt}$$

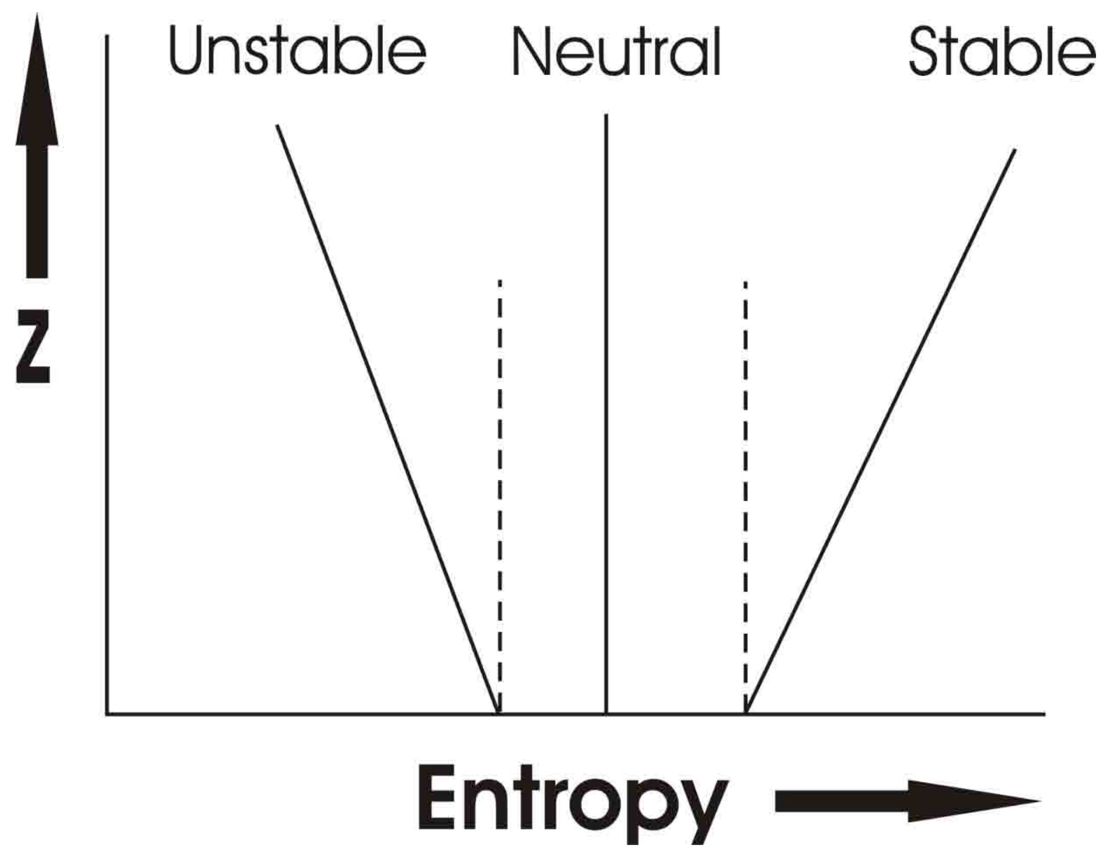
$$\textit{Adiabatic:} \quad c_p dT - \alpha dp = 0$$

$$\textit{Hydrostatic:} \quad c_p dT + g dz = 0$$

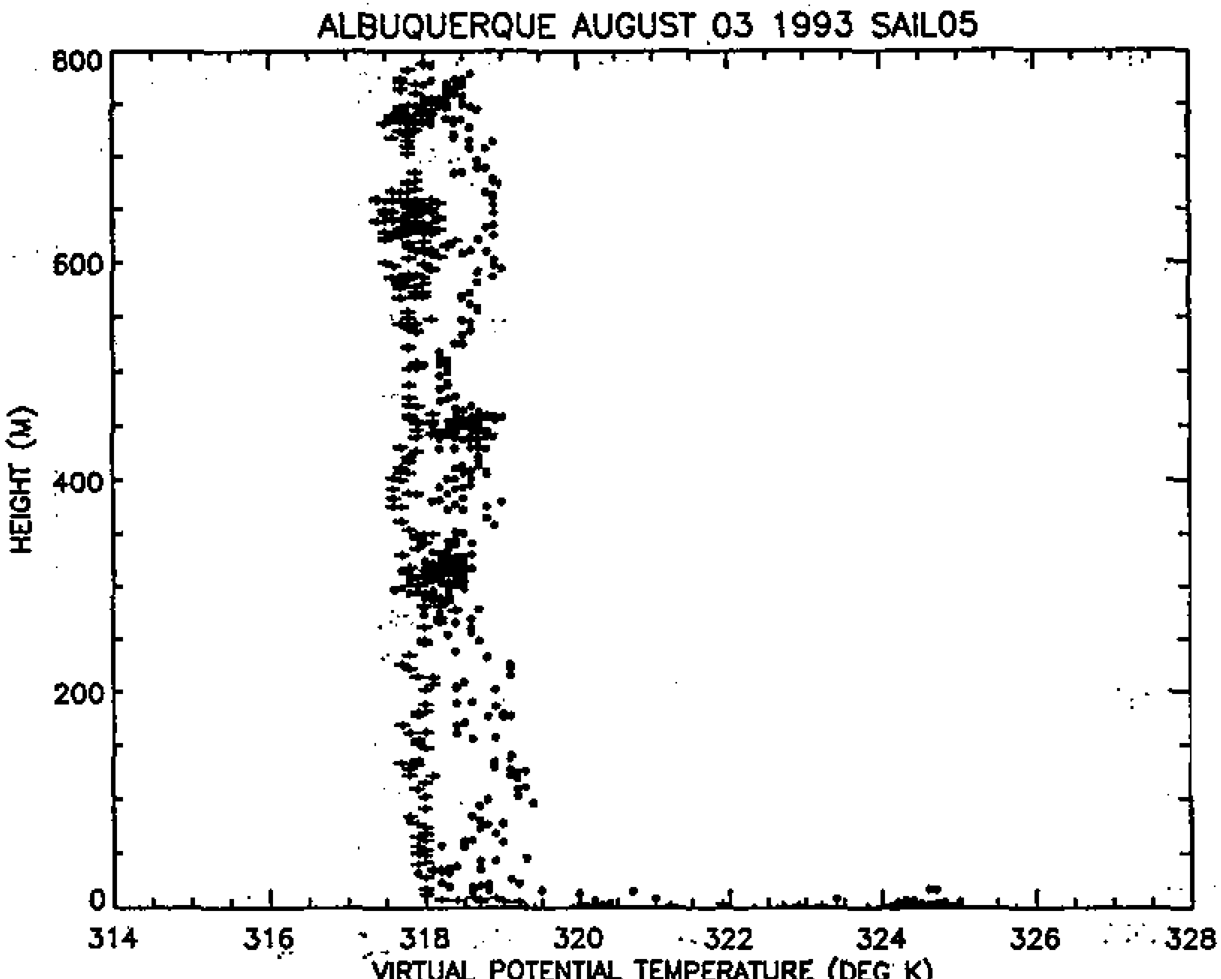
$$\rightarrow \quad \left({dT}/{dz} \right)_s = -{g}/{c_p} \equiv -\Gamma_d$$

$$\Gamma = g / c_p$$

Earth's atmosphere: $$\Gamma = 1K / 100\,m$$



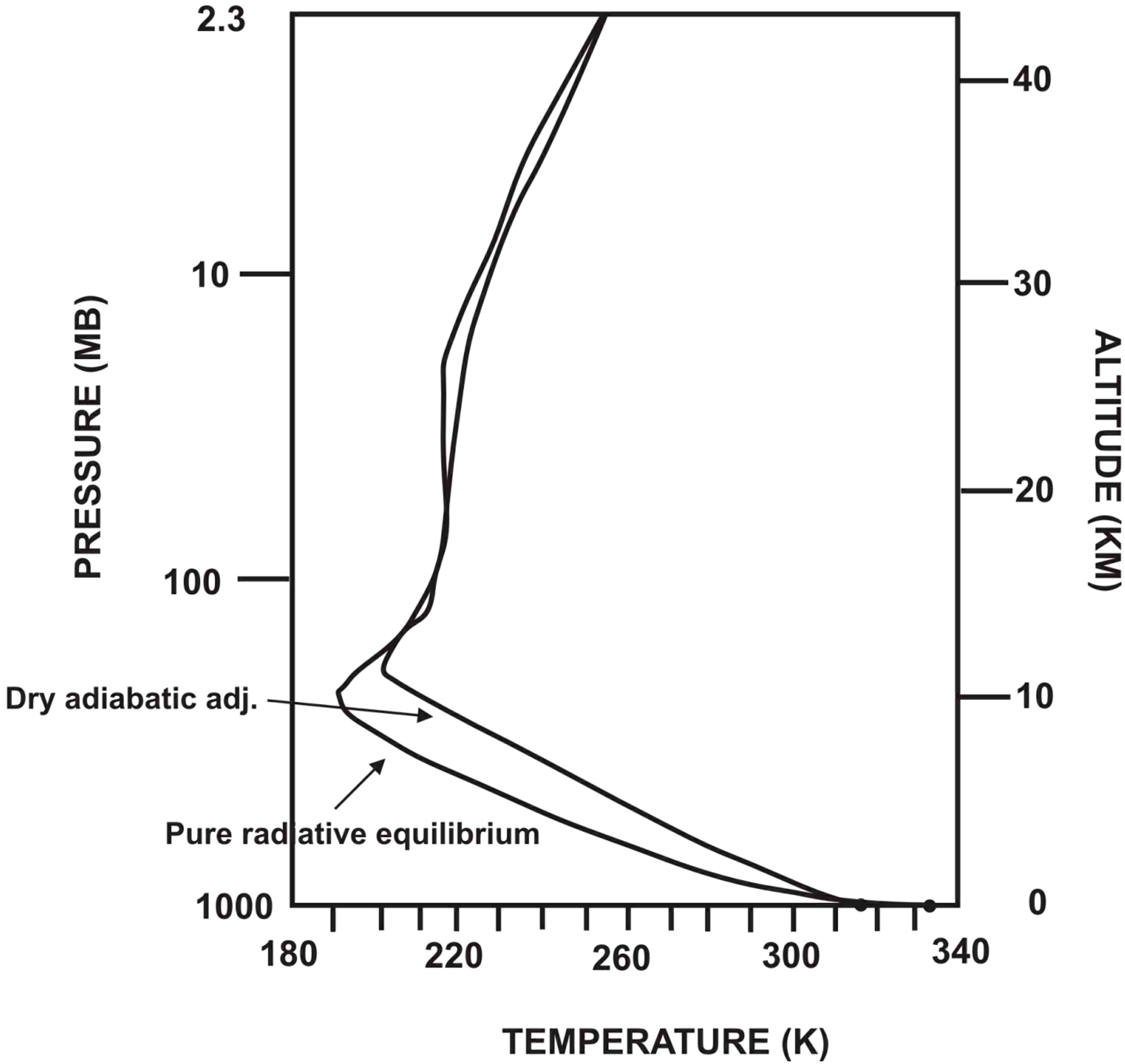
Model Aircraft Measurements (Renno and Williams, 1995)



Radiative equilibrium is unstable in the troposphere

Re-calculate equilibrium assuming that tropospheric stability is rendered neutral by convection:

Radiative-Convective Equilibrium



Better, but still too hot at surface, too cold at tropopause