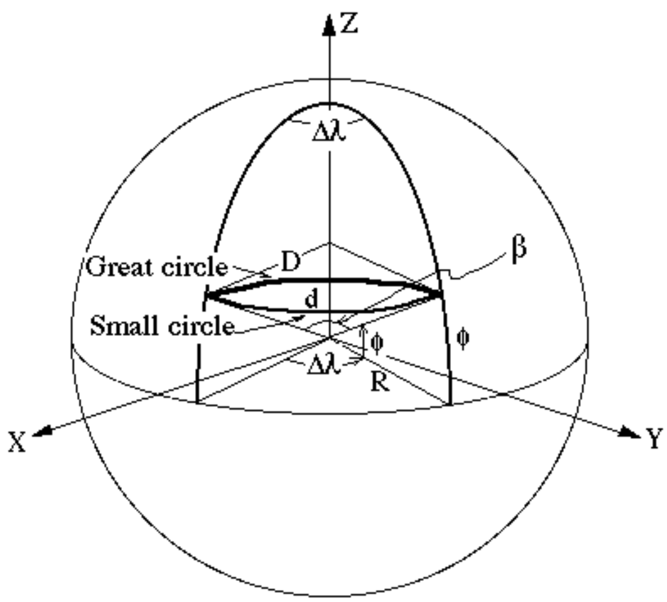
12.215 Homework #1 Solutions Oct 4, 2006

Question: (1) For latitudes of 30 degrees N and 60 degrees N, determine the difference between the great circle path and small circle path for sites at the same latitude. Plot results as a function of longitude difference between 0 and 180 degrees. (20 points).

Answer:



The small circle distance, d, is given by

$$d = R \cos\phi \, \Delta\lambda$$

where $\Delta\lambda$ is in radians.

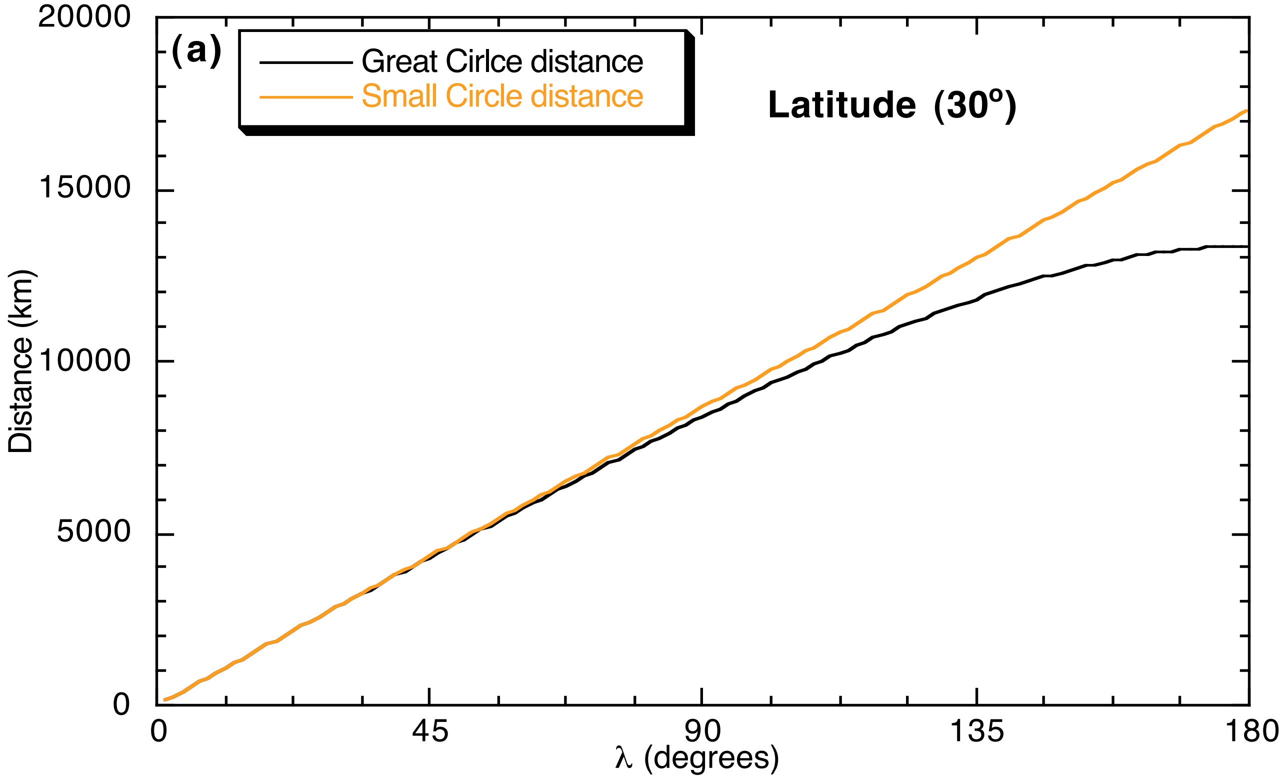
The great circle distance, D, is given by

$$D = R.\ \beta$$
$$\cos\beta = \sin^2\phi + \cos^2\phi \cos\Delta\lambda$$

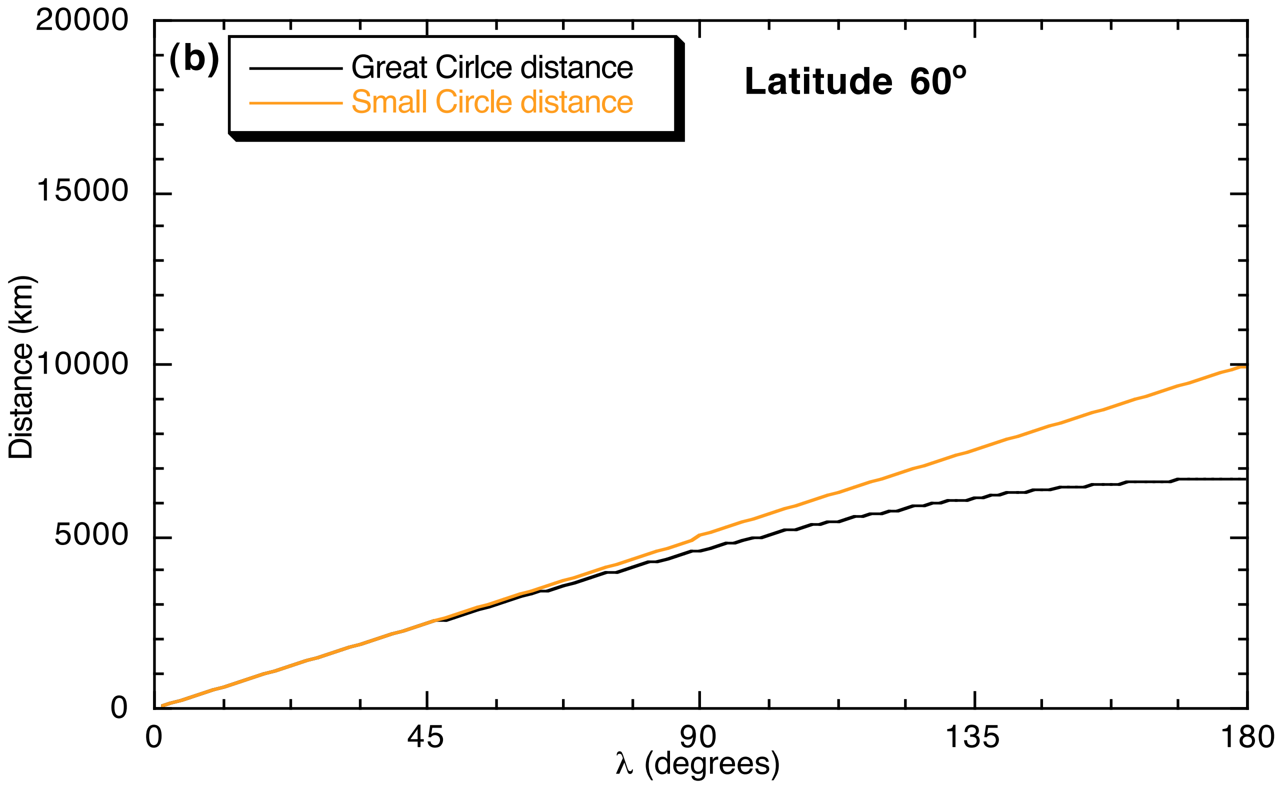
where β is in radians, and that ϕ is the same at both ends of the line.
We take R = 6371 km (radius of sphere with same volume as the Earth) and the results are shown in the figure. From the calculations that went into the figure, we have the following values

For $\phi = 30^o$ at $\Delta\lambda = 180^o$; d = 17,334 km; D = 13, 343 km
For $\phi = 30^o$ at $\Delta\lambda = 11.25^o$; d = 1,083.3 km; D = 1,082.9 km

For $\phi = 60^o$ at $\Delta\lambda = 180^o$; d = 10,008 km; D = 6672 km
For $\phi = 60^o$ at $\Delta\lambda = 11.25^o$; d = 625.5 km; D = 624.7 km



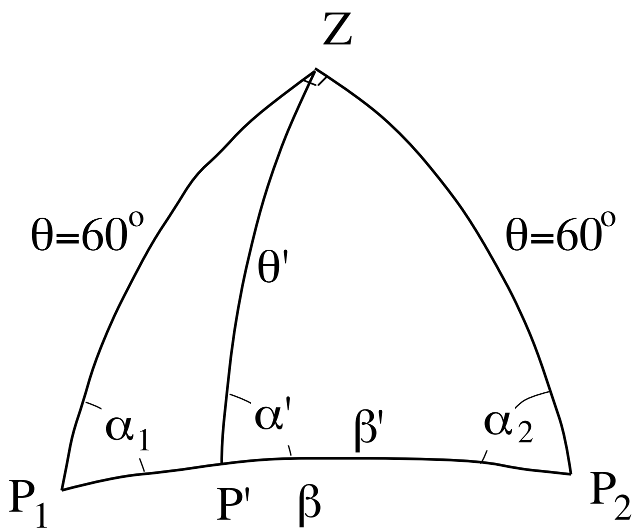
(a)
Great Cirlce distance
Small Circle distance
Latitude (30°)
Distance (km)
20000
15000
10000
5000
0
0
45
90
135
180
λ (degrees)



(b)
Great Cirlce distance
Small Circle distance
Latitude 60°
Distance (km)
20000
15000
10000
5000
0
0
45
90
135
180
λ (degrees)

Question: (2) For sites at 30 degrees latitude and separated by 90 degrees of longitude, compute the azimuths to be used along the greater circle path between the two sites. Show results graphically. What is the azimuth at the mid-point between the two locations? (20 points)

Answer: Using the figure below



$$\cos\beta = \cos^2\theta + \sin^2\theta\cos\Delta\lambda$$

Because $\Delta\lambda = 90^o$, $\cos\Delta\lambda = 0$. Therefore $\cos\beta = \cos^2 60$ and $\beta = 75.5^o$ (For R=6,371 km, D=8397.7 km).

To compute the trajectory along the great circle, we need to repeatedly solve the (spherical) triangle ZP_2P' as P' is moved along the path between P_1 and P_2.

Start:

$$\sin\alpha_1 = \sin\delta\lambda\sin\theta/(\sin\beta) \Rightarrow \alpha_1 = \alpha_2 = 63.434^o$$

Then divide b into segments, as P' is moved from P_1 to P_2, the arc along the great circle is denoted with β-β'. Given β', we have

$$\cos\theta' = \cos\theta\cos\beta' + \sin\theta\sin\beta'\cos\alpha_2$$

and

$$\sin\alpha' = \sin\alpha_2\sin\theta/\sin\theta'$$

To resolve the quandrant for a' we need an independent value of cos a'. This expression can be obtained from the cosine rule using a'

$$\cos\alpha' = (\cos\theta - \cos\theta'\cos\beta')/(\sin\theta'\sin\beta')$$

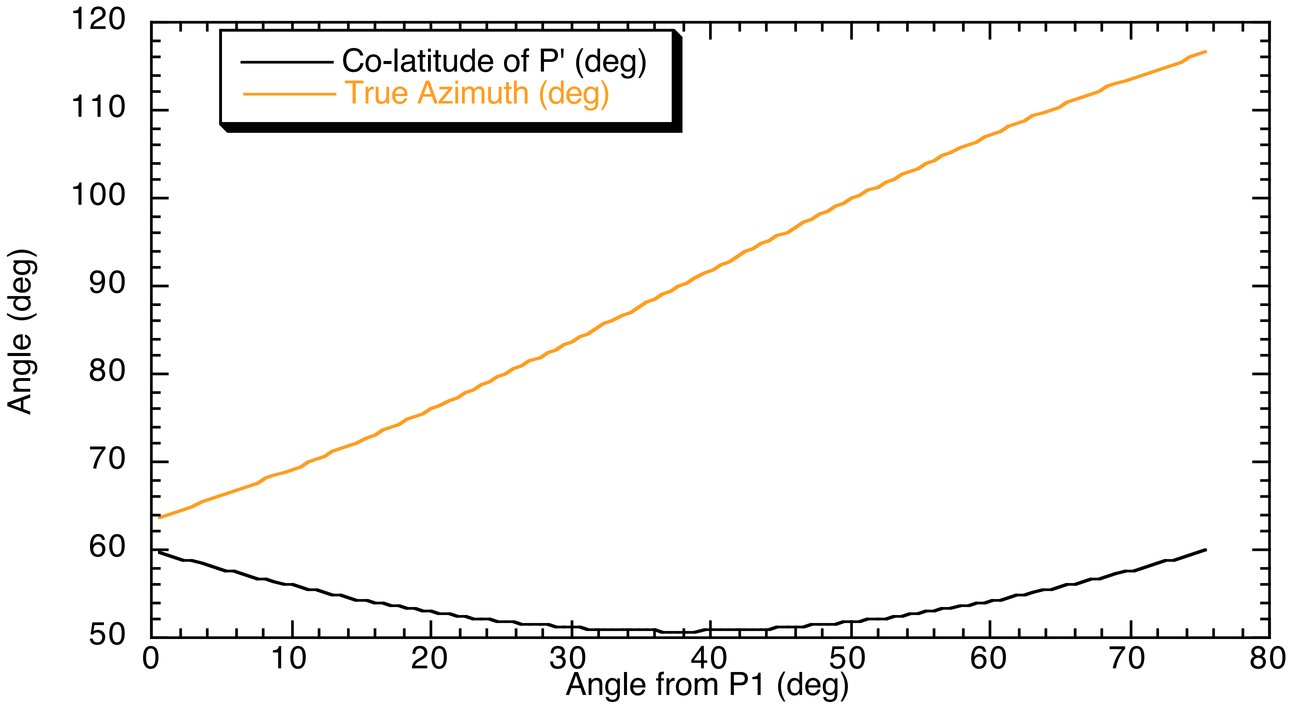
We can also at this point convert the b' angles back to changes in longitude again using the cosine rule:

$$\cos \Delta\lambda = (\cos \beta' - \cos \theta \cos \theta')/(\sin \theta \sin \theta')$$

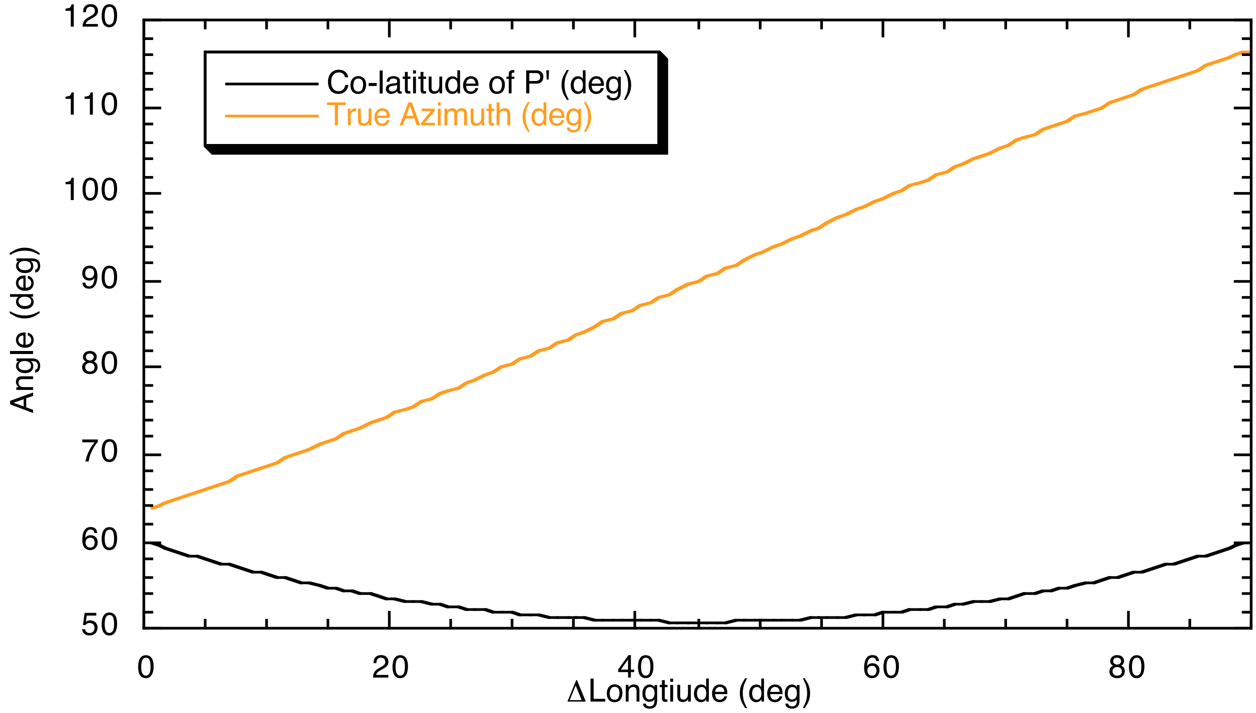
The results shown in this form at plotted below as well.

A plot as a function of β-β' = angle from P_1 is show below.

(b) At the midpoint, α' =90 (θ' = 50.768°) i.e., mid-way along the path, you travel due East.



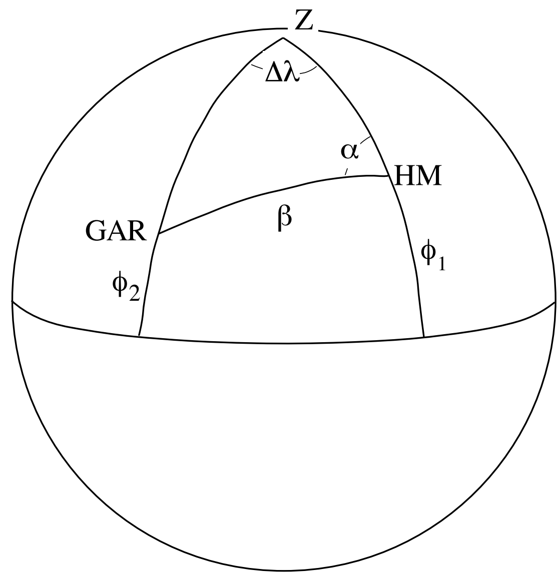
Results as a function of longitude difference:



120
110
100
90
80
70
60
50
Angle (deg)
Co-latitude of P' (deg)
True Azimuth (deg)
0
20
40
60
80
ΔLongtiude (deg)

Question: (3) The Garmin factory is located at 38.95005 N, 94.74612 W, and it is supposed to be 2029 km at a bearing from True North of 267 degrees, from N 42.26615, 71.08850 W. Compute what you think the distance and bearing should be. How well do your results agree with Garmin. (20 points)

Answer: (a) "Quick and dirty solution"
Treat the geodetic latitudes as geocentric latitudes and solve the spherical triangle below.



Solving spherical triangle HM-Z-GAR we have

$$\cos\beta = \sin\phi_1 \sin\phi_2 + \cos\phi_1 \cos\phi_2 \cos\Delta\lambda$$
$$\sin\alpha = \sin\Delta\lambda \cos\phi_2/\sin\beta$$

To resolve the quadrant for a we also need an unique definition for cos a. (Using $\cos\alpha = \sqrt{1-\sin^2\alpha}$ is not enough because the sign of the square root is unknown).

This yields $\beta = 18.202^o$ (= 0.317682 radians).
Using the mean radius of the Earth as 6,371 km, we obtain

D = 2024 km (*cp*. 2029 km)
$\alpha = 267.45^o$ (*cp*. 267°)

(b) "Better solution"
Convert the geodetic latitudes to geocentric latitudes and then solve the above spherical triangle using the geocentric values. The results are

$\phi_{MH} = 42.074664$ and $\phi_{GAR} = 38.762034$
$R_{HM} = 6368.5$ km and $R_{GAR} = 6369.7$ km

yielding $\beta = 18.252$ (= 0.318560 radians). Using mean R = 6,369 km we obtain:

D = 2028.9 km (*cp*. 2029 km)
$\alpha = 267.4^o$ (*cp*. 267°)