

CHAPTER 6

OSCILLATORY FLOW

INTRODUCTION

1 The first thing that comes to your mind when I mention water waves are probably the waves that appear on the water surface when the wind blows. These range in size from tiny ripples to giants up to a few tens of meters high and up to a few thousands of meters long. But many other kinds of waves make their appearance on water surfaces in nature. Here are the important kinds:

- **Flood waves in rivers.**—Very long and very low, these waves propagate downstream at a speed that is different from the speed of the flowing water itself. It is important to try to predict both the speed and the maximum height of the flood wave.

- **Seiches in lakes and estuaries.**—These are standing waves that are set up in an elongated basin by a sudden change in water-surface elevation in part of the basin, for example by a sudden drop in atmospheric pressure or by transport of surface water by a sudden strong wind. They may have just one node or more than one node.

- **Tidal bores in estuaries.**—These are waves of translation, in which the water moves along with the wave. They have steep turbulent fronts, which can be hazardous to small boats.

- **Tsunamis (seismic sea waves) in the ocean.**—These are extremely long and low-amplitude waves with high propagation speeds that are generated by sudden large-scale movements of the sea floor, usually by movement on faults but also by volcanic eruptions or submarine landslides. Their extreme destructiveness comes about because their amplitude increases spectacularly, sometimes to several tens of meters, when they shoal.

- **Internal waves in the atmosphere and the ocean.**—When a fluid is stratified by density, waves can develop within the layer through which the density varies. This is easiest to appreciate when the density change is compressed to a jump discontinuity at a well-defined surface, but internal waves can exist also in layers with only gradual change in density. Internal waves are common in many settings in the ocean, and water velocities associated with the waves are in some cases strong enough to move bottom sediment. They are also common in the atmosphere.

2 Water waves of these kinds are called *gravity waves*, because, as you will see in a minute, gravity is the important force involved. But another important kind of waves, pressure waves, are present also in fluids, both air and water.

THE NATURE OF WAVES

Introduction

3 In a very fundamental sense, the waves that are of interest to us here can be viewed as a manifestation of unsteady free-surface flow subjected to gravitational forces. That is, any unsteady flow with a deformable free surface can be considered to be a kind of wave.

4 Do not let it bother you that real water waves involve changes in the water-surface geometry even when you follow along with the waves. You know from Physics I that a function of the form $y=f(x-ct)$ represents a wave traveling with speed c in the positive x direction—and the shape of the wave does not change if you just travel along with the wave. And c could be a function of t , meaning that the speed of the wave changes everywhere with time but the shape of the wave train still stays the same. But now suppose that you took one additional step: let c a function of x rather than t . Then the shape of the wave changes as it moves: there is no speed at which you can travel, along with the wave, to keep the wave shape looking the same. The best way to think about this situation is that each point on the wave (you could call such points *wavelets*) has its own speed, so that, as all of them move, the overall shape of the wave changes with time.

5 In terms of the forces involved in wave motion, the motions of the water in the interior and the geometry of the free surface are an outcome of the interaction between pressure forces and gravity forces. Although it may or may not help you any, one way of thinking about waves is to consider that gravity tries to even out some initial nonplanarity of the water surface, and in doing so produces a usually complex unsteady flow in which the water-surface geometry changes as a function of time, but the characteristic amplitude of the water-surface disturbance has no way of actually decreasing unless viscous forces act also.

6 Real waves do decrease in amplitude, of course, because of the slight shear and therefore viscous friction in the interior of the water. But unless the waves produce water motions at the bottom, the rate of viscous dissipation of the wave motion is very slight. Mathematically, this means that the viscous term in the equation of motion can be ignored. Only when an oscillatory boundary develops at the bottom is the viscous dissipation substantial.

The Equations of Motion

7 The equation of motion that describes water waves is just the Navier–Stokes equation without the viscous term but including a term for gravity. It turns out that this equation for inviscid flow affected by gravity can be put into the form of a wave equation, so you mathematically the existence of waves should not surprise you.

8 If you had never fooled around with waves before, your natural inclination upon reading the foregoing paragraph would probably be to try to solve the equations to account for the observed behavior of water waves. And people have been doing this since the middle of the 1800s. But there are two serious impediments to simple solutions:

- The equation is nonlinear, because of the presence of the convective acceleration term, which as you know from Chapter 3 involves products of velocities and spatial derivative of velocities.
- An even more serious problem is that one of the boundary conditions—the geometry of the free surface—is itself one of the unknowns in the problem!

9 So it is unfortunately true that there is no general solution to the problem. People have therefore tried to make various simplifying assumptions that allow some mathematical progress in certain ranges of conditions for water waves. Much mathematical effort has gone into developing these partial approaches and establishing their limits of approximate validity.

Classification of Water Waves

10 It is notoriously difficult to develop a rational classification of water waves, basically because of the mathematical complexity mentioned above. One way to classify water waves I already mentioned: does the water move with the waves (*translatory waves*), or does the water merely oscillate as the wave passes, to return to its original position after the wave has passed (*oscillatory waves*)? But I also mentioned a more fundamental approach: waves for which the convective inertia terms can be neglected are called *linear waves*, and those for which the convective inertia terms are at least in part included in the analysis are called *nonlinear waves*.

11 Yet another fundamental approach to classification is on the basis of the relative magnitudes of the three important length scales in the problem: wave height H , wavelength L , and water depth d . Out of these three you can make three characteristic ratios: H/L , H/d , and L/d . In deep water, H/d and L/d are both small, and the most important parameter is H/L , called the *wave steepness*. In shallow water, on the other hand, neither H/d nor L/d is likely to be small, and the most important parameter is likely to be H/d , called the *relative height*. In an intermediate range of water depths, the situation is more complicated.

WATER MOTIONS DUE TO WAVES

12 The aspect of surface gravity waves that is relevant to sediment movement lies largely in the back-and-forth movement of the water at the bottom: are those oscillatory currents strong enough to entrain bottom sediment? To get at the nature of bottom water motions under oscillatory waves, it is best to start with small-amplitude wave theory. The most far-reaching progress in theory of water waves came about by making the assumption that the wave steepness H/L is so small that the convective accelerations (like $u \partial u / \partial x$) in the equation of motion can be neglected compared to the local accelerations (like $\partial u / \partial t$). It turns out that this assumption simplifies the equation to such an extent that an exact solution can be obtained, by methods beyond the scope of these notes. Such waves are called *Airy waves*, after the person who first developed the solution.

13 It turns out that the small-amplitude solution holds to good approximation for values of wave steepness H/L well beyond what we have any right to expect. So we can get a good idea of the behavior of finite-amplitude waves just by relying on the small-amplitude solution.

14 What the solution gives us is the fluid velocity and fluid pressure at every point, and the geometry of the water surface, all a function of time. The water motions below the surface, and especially near the bottom, are of greatest interest in sediment transport.

15 As an oscillatory wave passes by, each little element or parcel of the water undergoes a closed-loop trajectory or orbit, making one traverse of that orbit as each wave form passes. As your intuition might tell you, the diameter of the orbits decreases with depth below the free surface. This makes sense, because the water-surface disturbance is generated and maintained at the surface by the forces of the wind on the water.

16 In deep-water waves, for which L/d is very small, the orbits are everywhere nearly circular, and the size of the orbits decreases sharply with increasing depth until they are negligible at depths equal to only one wavelength (Figure 6-1). The orbits are perfectly circular only in the limit of vanishingly small H/L , but even for non-negligible H/L the orbits are very close to being circular. In shallower water the passage of the waves causes water motions all the way to the bottom, although except at the largest values of L/d the diameter of the orbits still decreases with depth. The orbits even at the surface are somewhat elliptical, flattened in the vertical, and they become more elliptical with depth, until at the bottom they are back-and-forth straight lines (Figure 6-2).

17 The effect of finite wave steepness is to distort the wave form such that crests are sharper and troughs are broader, and to distort the orbits such that the speeds are greater when the water is moving in the direction of wave propagation than in the direction opposite wave propagation. (But the net transport remains nearly zero, because the time during which the higher forward speeds prevail are shorter than the times during which the lower reverse speeds prevail.)

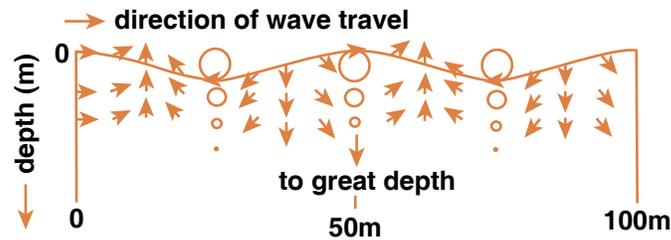


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Figure 6-1. Instantaneous velocity vectors at a given time, and orbital paths of fluid particles over one complete cycle, in a wave motion in deep water. (From Neumann and Pierson, 1966.)

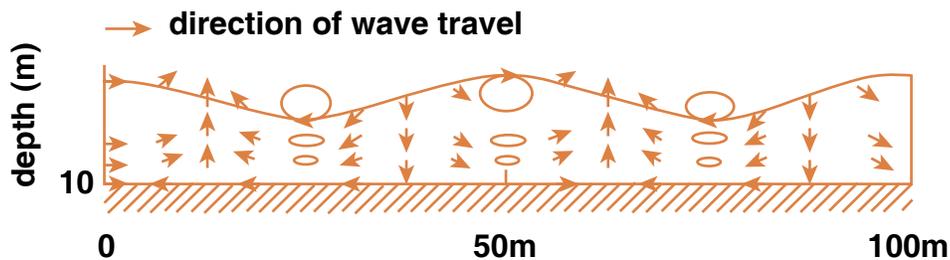


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Figure 6-2. Instantaneous velocity vectors at a given time, and orbital paths of fluid particles over one complete cycle, in a wave motion in water of constant shallow depth. Approximate conditions are shown for a wave with amplitude of 2.5 m and a wavelength of 50 m in water 10 m deep. (From Neumann and Pierson, 1966.)

18 The size of wind-generated surface gravity waves depends on three things: wind speed, duration, and fetch. It is obvious to even the most casual observer that wave size increases with wind speed. The reason is that the wind transfers some of its kinetic energy to the waves, both by shear stresses on the water surface and by the front-to-back pressure differences you learned about in Chapter 3, and this adds mechanical energy to the waves, in the form of kinetic energy of the water motions in the wave and potential energy associated with

elevation differences of the undulatory water surface. But the waves are not created instantly by the wind: it takes time for the waves to build to a state of equilibrium with the wind. One or both of two processes eventually begin to dissipate the energy of the waves: (1) breaking of the wave crests (called whitecapping), as the waves become steeper, which converts the organized mechanical energy of the waves to turbulence and thence to heat; and (2) if the wave are large enough relative to the water depth, bottom friction, which bleeds off energy at the bottom of the water column, in the oscillatory-flow boundary layer (more on that later) and converts it to heat. The term *fetch* is used for the distance over which the wind can work on the waves. You know, from standing at the shore of the ocean or a lake with the wind blowing offshore, that the waves grow from little ripples to much larger waves in the offshore direction, ultimately to reach equilibrium with the wind if the fetch is long enough.

19 If you, as an otherwise knowledgeable sedimentationist, were approaching the problem of bottom water movement by waves for the first time, one of the first things that would occur to you would probably be to ask how to predict the nature of the bottom water motions as a function of the wind speed. The bottom water motions might be characterized by the oscillation period T , the size of the excursions of water elements near the bottom, called the orbital diameter d_o , or the maximum near-bottom water velocity during one oscillation period, which I will denote by U_m . These three quantities are not independent of one another: they are related by the equation $U_m T = \pi d_o$. You might try your hand at deriving this simple relationship sometime, when you have some time to spare.

20 Unfortunately there is no simple way to make such a prediction, because it depends on the balance, mentioned above, between energy input from the wind and energy dissipation both at the surface, by whitecapping, and at the bottom, by bottom friction. One thing, albeit only qualitative, that can be said is that the shallower the water depth, the more important the bottom friction is in determining the balance. In very deep water its role is non-existent, whereas in very shallow water it becomes the dominant effect. Where the water is sufficiently shallow that a wind with effectively unlimited fetch has produced a fully developed wave state, such that energy loss by bottom friction does not allow the waves to grow any larger, the sea state has been called a *depth-limited sea*. Such wave states must be common in the shallow ocean during powerful storms, and much of the work of the waves on the sediment bottom takes place at those times.

21 It is easy to make a regular train of waves in a wave tank: all you need to do is put a wave generator of some kind in one end (a flap that is hinged at the bottom and moved back and forth in simple harmonic motion at the top does nicely) and energy-absorbing material at the other end, to prevent reflection of the incoming waves. You know, however, that waves that have attained equilibrium with a wind, or are building toward equilibrium, are far less regular: the crests are not regular and straight-crested, and if you try to watch a single wave crest as it progresses, it soon disappears, to be replaced by other wave crests. The explanation is that the wave state actually is a combination, or superposition, of a

set of “component” waves, each with a slightly different period, speed, and direction of propagation. The reality of these component waves is manifested not in the region of wave generation but along distant shores. You are at the beach, during a nice settled spell of summer weather, watching the waves come into shore and break on the beach. Close observation would reveal that these waves, called swell, come from a single direction and are very regular in their size and period. These are waves that were generated by some distant storm and have crossed a great expanse of open ocean to arrive at your beach. Deep-water waves are said to be *dispersive*: the Airy wave solution, mentioned above, for the speed of deep-water waves is $c^2 = gL/2\pi$, where c is the wave speed and L is the wavelength of the waves. That means that longer waves outrun shorter waves, to arrive at distant locations earlier. And the waves sort themselves out by direction as well. It is those two effects—dispersion and directionality—that make the incoming swell so regular.

22 In the midst of a storm at sea, however, matters are different: the sea state comprises a range of wave sizes and directions, making for a complicated surface topography. Such a wave state is usually represented by what is called a *two-dimensional wave spectrum*, and the waves are referred to as *spectral waves*. Think in terms of a joint frequency distribution of wave energy in terms of wave period (or wave frequency) and direction. The sea state generated by a wind that blows in one direction for a long time shows a very elongated distribution, with a wide range of periods but a much narrower range of directions, but if the wind direction is changing, as commonly happens during the passage of a storm, the wave state has to readjust, and while it does so the two-dimensional wave spectrum is likely to broaden in direction as well as in period.

23 The relevance of the wave spectrum is that it translates to what the water motions are at depth, although the picture of motions at depth is mediated, or filtered, by the rapid decrease in amplitude of the component wave motions with depth. Because of this rapid decrease, the water column acts as a kind of low-pass filter, emphasizing the contribution of the larger wave components at the expense of the smaller, thus making the water motions at the bottom less variable, in both period and orientation, than at the surface. Even so, during strong but changing winds you might expect the water motions at the bottom to be not as simple as a regular back-and-forth motion with a single orientation.

WAVE BOUNDARY LAYERS

24 The linearized small-amplitude solution for velocity mentioned in the previous section predicts spatially slowly varying velocities both at and below the water surface at any given time. In the case of waves in shallow to intermediate water depths (that is, for which the wavelength is not very small relative to the water depth), these velocities are predicted to be still appreciable even at the bottom—and when the wavelength is large relative to the water depth, the magnitudes of near-bottom velocities are about the same as the velocities at the surface. Remember that the assumption of inviscid flow means that these nonzero near-bottom velocities extend all the way to the bottom.

25 Viscosities of real fluids like water are small enough that the free-surface profile and the spatial and temporal distribution of velocities is well accounted for by the inviscid solutions, and viscous damping is small—so small that large waves can travel across wide ocean basins without great loss of energy. But the predicted nonzero velocities at the solid bottom boundary under the waves is clearly contrary to fact: just as in unidirectional flows of real fluids, the velocity must go to zero at the bottom boundary. This leads to the concept of *the bottom boundary layer in oscillatory flows*: this is commonly called the **wave boundary layer**

26 Many of the physical effects associated with wave boundary layers are analogous to or parallel to those of unidirectional-flow boundary layers. Here is a mostly qualitative account of some of the important things about wave boundary layers. The first thing to note is that, just as with unidirectional flows, at relatively low values of a suitably defined Reynolds number the boundary layer is laminar, and at higher values of the Reynolds number the boundary layer is turbulent, even though the flow in the domain above the boundary layer, where the inviscid assumption holds, is effectively nonturbulent (provided that there is no coexisting unidirectional current; see a later section).

27 If the wave Reynolds number is defined as $\rho U_m d_o / \mu$, where U_m is the maximum bottom velocity predicted by the inviscid theory and d_o is orbital diameter given by the inviscid theory, then the critical value for the transition from laminar to turbulent flow in the wave boundary layer is known from observations in wave tanks and oscillatory flow ducts in the laboratory to be about 10^4 .

28 An exact solution can be found for the velocity profile in the laminar boundary layer. The mathematics is straightforward but beyond the scope of these notes. The result, when expressed as the velocity deficit u_d , the difference between the inviscid bottom velocity (which in the context of the laminar boundary can be viewed as the velocity at the top of the boundary layer) and the actual velocity at some height z above the bottom, is

$$u_d = \exp\left(-\sqrt{\frac{\omega}{2\nu}} z\right) \cos\left(kx - \omega t + \sqrt{\frac{\omega}{2\nu}} z\right) \quad (6.1)$$

where ω is the angular frequency of the oscillation (related to the period T by $\omega = 2\pi/T$), k is the wave number (related to the wavelength L by $k = 2\pi/L$), ν is the kinematic viscosity μ/ρ , and z is measured upward from the bottom.

29 The solution in Equation 6.1 has two factors, one expressing a negative exponential dependence and the other expressing a cosine dependence. The former causes u_d to drop off sharply with height above the bottom, and the latter just takes account of the time variation in the velocity—but it is important to note that there is a phase difference with the overlying inviscid flow, and the phase difference itself depends on z , ranging upward from zero at the bottom at the same time u_d is getting smaller.

30 The negative exponential dependence of u_d on z in Equation 6.1 means that the effective boundary-layer thickness is fairly well defined, although technically one has to take some arbitrary value like 0.01 for u_d in order to obtain a definite thickness for the boundary layer. It turns out that the value of z that corresponds to $u_d = 0.01$, which is usually denoted by δ_L , is

$$z = \delta_L = 5 \sqrt{\frac{2\nu}{\omega}} \quad (6.2)$$

31 But most boundary layers under waves in the real ocean under conditions of interest in sediment transport are *turbulent* rather than laminar. Theoretical analyses of the turbulent wave boundary layer have been carried out by replacing the molecular viscosity by a turbulent eddy viscosity, making some assumption about how the eddy viscosity varies vertically, and obtaining an expression for the vertical velocity distribution. The velocity profile is found in this way to be logarithmic. Again there is the problem of how to define arbitrarily the boundary-layer thickness, but the height δ_T of the turbulent boundary layer is usually taken to be

$$\delta_T = \frac{2\kappa u_*}{\omega} \quad (6.3)$$

where κ is von Kármán's constant, the inverse of the constant A introduced in Chapter 4, and u_* is again the shear velocity (which could be taken as the maximum or the time average).

32 A significant aspect of wave boundary layers is that they do not keep growing upward into the interior of the flow indefinitely, the way unidirectional-flow boundary layers do, provided that density stratification does not inhibit their upward growth. The reason is that the thickness of the wave boundary layer is limited by the cessation and turnaround of the flow in each cycle. For turbulent wave boundary layers over rough beds, the thickness of the wave boundary layer is likely to be something less than one meter—far smaller than the typical boundary layer beneath currents in deep-water natural environments.

33 Just to give you a feel for the velocity structure in the oscillatory-flow boundary layer is like, Figure 6-3 is a graph (admittedly somewhat complicated) that shows flow velocity in the laminar oscillatory boundary layer vs. height above the bottom for four equally spaced times during one complete oscillation cycle ($0, \pi/2, \pi, 3\pi/2, \text{ and } 2\pi$). The vertical coordinate is labeled with values of the length variable under the radical sign in Equation 6.2 but with slightly different notation. The curves in Figure 6-3 are labeled in two ways: 1–4 unprimed, and 1–4 primed. The unprimed numbers are for what the velocity of the bottom would look like to a neutrally buoyant observer riding with the oscillatory flow just outside the boundary layer. This corresponds to oscillating a flat plate in a tank of still water. The primed numbers are for the equivalent curves, more relevant to our interests here, that show the velocity profile of the

fluid relative to an observer planted firmly on the bottom. You can see how, during each half cycle (for example, from curve 0' to curve 1' to curve 2') there is a considerable mismatch in phase between the upper part of the boundary layer and the lower part.

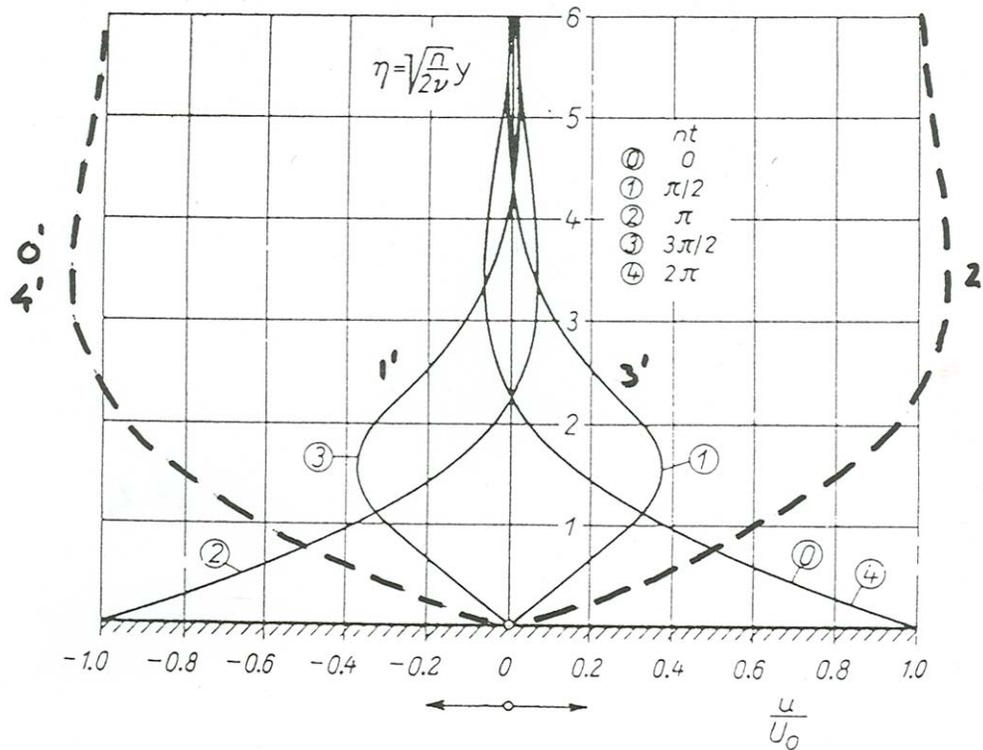


Figure 6-3. Plot of fluid velocity u , normalized by dividing by the velocity U_0 outside the laminar oscillatory boundary layer, vs. distance above the bottom, for one complete oscillation cycle. See text for explanation. See Equation 6.2 for the nature of the vertical coordinate. The top of the graph corresponds approximately to the top of the boundary layer. (Modified from Schlichting, 1960, p. 76.)

34 Finally, the magnitude of the bottom shear stress is important both for its role in sediment transport and also for its effect on attenuation of wave energy by bottom friction, so a lot of effort has gone into developing ways to predict the bottom shear stress. Basically it boils down to dealing with a **wave friction factor** f_w , analogous to the unidirectional-flow friction factor, and working with an experimentally determined **wave friction-factor diagram** expressing the dependence of the wave friction factor on the Reynolds number and, for rough beds, a **relative roughness** d_o/D , where D is the size of the roughness elements.

35 One interesting aspect of the bed shear stress in oscillatory flow is that in laminar boundary layers the maximum shear stress leads the maximum velocity by a phase angle of $\pi/4$, meaning that the maximum shear stress acts on the

bottom at a time equal to $T/8$ (where T is the period of the oscillation) before the velocity reaches its maximum at the top of the boundary layer. In turbulent boundary layers the same effect is present but the phase difference is somewhat smaller.

COMBINED FLOW (WAVES PLUS CURRENT)

Introduction

36 It is common, in lakes and in the ocean, for currents and waves to coexist. Such a flow, involving both waves and a current, is called *combined flow*. In the interior of the current, far away from the bottom boundary, the waves superimpose upon the current a wholesale oscillatory motion of the water that does not affect in any substantial way the structure of turbulence in the current, because very little shear is introduced into the fluid. Near the bottom, however, in the bottom boundary layer, the oscillatory wave motion and the unidirectional current interact in complex ways, with important consequences for sediment entrainment and movement. Such a boundary layer is called a *combined-flow boundary layer* or a *wave-plus-current boundary layer*.

Varieties of Combined Flow

37 The simplest of combined flows are those involving a train of waves of the “wave tank” variety (with only one wave component, with a single period and a single direction) and the direction of wave propagation is the same as the direction of the current. Then you can imagine a range of combined flows, with one end member being a symmetrical purely oscillatory flow and the other end member being a current without superimposed waves (Figure 6-4). In the range for which the oscillatory flow speed is greater than the unidirectional flow speed, the water elements oscillate but undergo a net shift in position. In the range for which the unidirectional flow speed is greater than the oscillatory flow speed, the water elements move in one direction only but at time-varying speed. The “middle” case is that of a stop–start movement.

38 Just to complicate matters further, the asymmetry in the maximum forward and backward velocities in oscillation-dominated combined flow is not the only way that such a velocity asymmetry is produced: it can exist even in purely oscillatory flow for which the forward stroke is at a higher velocity but for a shorter time, and the backward stroke is at a lower velocity but for a longer time (Figure 6-5). the net water movement is still zero, but there is asymmetry in the velocities, and, if the waves are moving sediment, an even greater asymmetry in sediment transport. Such asymmetrical purely oscillatory flow develops when shallow-water waves are propagating into shallower water (that is called shoaling).

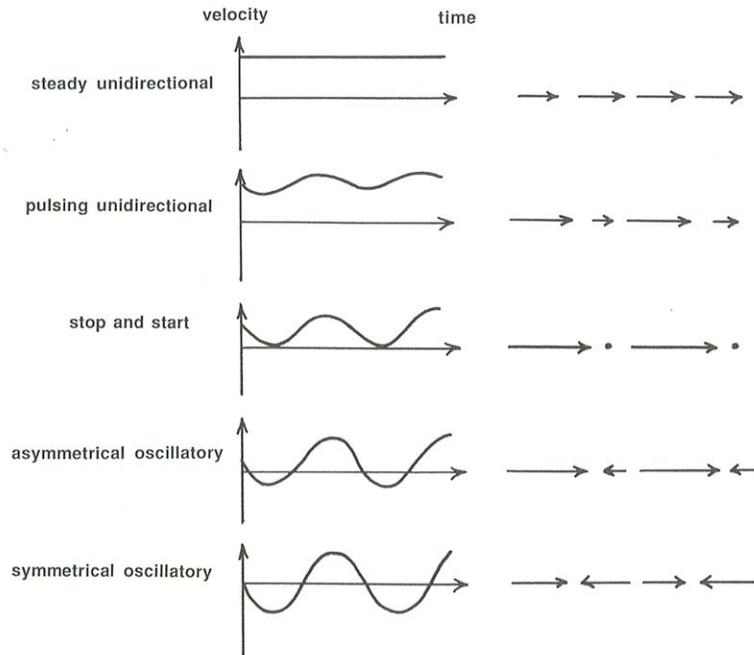


Figure 6-4. The range of combined flows, when the orientations of the unidirectional component and the oscillatory component are the same.

39 Of course, the range of combined flow described above are a special case, because there can be any angle between the direction of wave propagation and the direction of the current, from zero degrees to ninety degrees. Most of the experimental work on combined flow has been done in the zero-degree case, because that is by far the easiest to set up in the laboratory. In the natural environment, however, the zero-degree case is the exception rather than the rule. Some thought on your part should convince you that the non-zero-degree case involves water movements that are either looping or zigzag, depending upon whether the flow is oscillation-dominated or current-dominated, respectively.

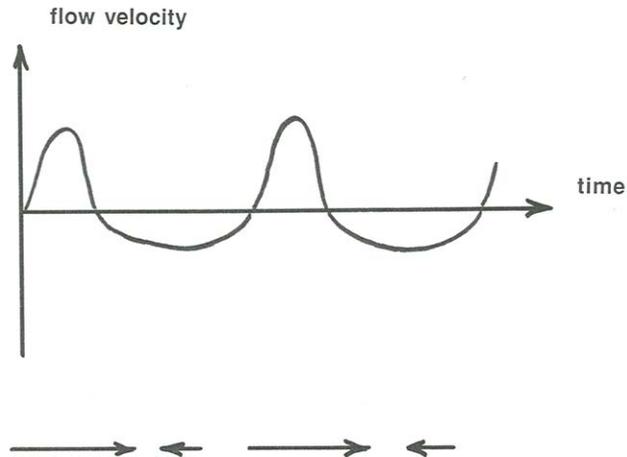


Figure 6-5. Time history of flow velocity in asymmetrical purely oscillatory flow.

40 Matters are considerably more complicated when the waves are spectral waves, with more than one component. A bit of elementary physics might help here to guide your thinking. When two sinusoidal oscillations in a horizontal plane, with the same frequency, are combined, the resulting trajectories of material particles describe an ellipse. If, however, the frequencies are different, the trajectories acquire a more complicated, but infinitely repeating, pattern, called Lissajous figures. These figures are complexly looping, but they are infinitely repeating, tracing out the same trajectory over and over again, always within the confines of a rectangular box defined by the amplitudes of the two oscillations. (Try googling “Lissajous figures” on the Internet to see some engaging animations.) Such figure simulate what the bottom water movements would be under the action of two wave components with different periods and amplitudes.

41 Matters become more complicated when there are three or more oscillatory components, with different orientations and frequencies. Then the particle trajectories appear totally irregular and non-repeating (see Figure 6-6, which is fake but it gives you the idea). Such would be the nature of bottom water movements in purely oscillatory flow produced by spectral waves for which more than one component contributes to the bottom water movement. Keep in mind that all of this is before a current is superimposed on the oscillatory motion.

42 The effects of this most general case of combined flow on sediment movement has, to my knowledge a least, been little studied in the laboratory, owing to the great difficulty in arranging for such flows, but sediment movement under such flows in common natural environments like lakes and the shallow ocean must be more the rule than the exception.

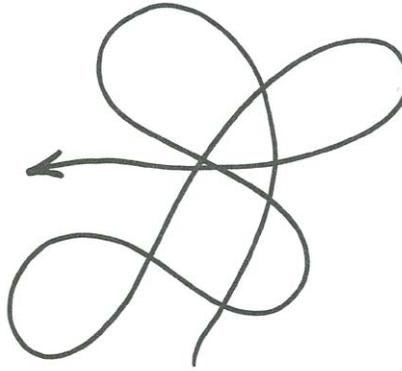


Figure 6-6. Cartoon to give you some idea of what the trajectories of near-bottom water elements would be in the case of three or more oscillatory components, with different orientations and frequencies.

43 What helps to save sedimentationists from this morass of complexity is the low-pass-filter effect of the water column, mentioned earlier: even in the case of a broadly two-dimensional wave spectrum, the bottom water movements are likely to be dominated by only the strongest component(s), thereby making the water movements more like the simpler cases discussed above. At times when the wave spectrum is undergoing major adjustments to the changing direction of strong winds, however, the more general case is likely to be important.

The Combined-Flow Boundary Layer

44 In combined flows, the boundary layer consists of a wave boundary layer, immediately adjacent to the bottom, commonly some tens of centimeters thick if the bottom is sufficiently rough as to cause the wave boundary layer to be turbulent, and a much thicker current boundary layer, extending upward for tens of meters (see Chapter 7 for more on current boundary layers in large-scale natural environments influenced by the Earth's rotation.) In other words, the wave boundary layer is embedded in the lowermost part of the current boundary layer. Such boundary layers are called **combined-flow boundary layers** or **wave-current boundary layers**. Such boundary layers are the rule during storms in the shallow ocean, over continental shelves, and as such they are a key aspect of continental-shelf sediment transport.

45 A simple-minded approach to the structure of wave-current boundary layers would be to simply add the time-varying velocity of a wave boundary layer to the unidirectional-flow velocity profile. The problem is that such an approach does not take into account a substantial element of nonlinearity, as follows. Within the wave boundary layer, the superimposition of the current causes the forward stroke of the oscillatory bottom water motion to be of greater velocity than the backward stroke. You learned back in Chapter 4 that, for fully developed

turbulent flow (sufficiently large Reynolds number) the flow resistance goes approximately as the square of the velocity—so you should expect the time-average boundary shear stress beneath the wave–current boundary layer to be greater than what it would be for the given current without the accompanying waves. You should expect that the characteristics of the turbulence within the wave boundary layer would be much different from the purely oscillatory case. Above the wave boundary layer, however, the turbulence is unaffected by the presence of the wave boundary layer below: there, you can indeed think in terms of a bodily back-and-forth oscillatory motion being superimposed on the unidirectional current. One way of expressing this is that the wave boundary layer “sees” this region or layer of the flow—within the current boundary layer but above the wave boundary layer—as the interior of the flow. If you could observe the details of the flow in this layer while moving your head in orbits that match the oscillatory component of the flow, they would look the same as if there were no oscillatory component to the flow.

46 The velocity profile in the combined flow above the wave boundary layer is different from that of a purely unidirectional current, though, because the “anchoring” effect on the profile in the region of the flow nearest the bed is different. Do you recall from Chapter 4 (see Figure 4-39 in particular) how, at any given level above the bed in a unidirectional current the slope of the law-of-the-wall velocity profile is the same for dynamically rough flow as for dynamically smooth flow but the actual velocity is smaller, because of the greater boundary resistance in rough flow? In a wave–current boundary layer the presence of the wave boundary layer has an effect of the same kind: the slope of the velocity profile, a reflection of the turbulence structure of the current, is the same as for a purely unidirectional current, but the velocity is smaller because of the greater flow resistance at the bottom of the wave boundary layer. This is shown clearly in Figure 6-7: look at the two curves in the left-hand part of the graph, and you will see that at a given elevation the velocity is smaller for the combined flow than in the purely unidirectional flow.

47 It is also clear from Figure 6-7 that the oscillatory velocity in the wave boundary layer in a combined flow is not much different from the oscillatory velocity in the wave boundary layer of a purely oscillatory flow, at least for flows in which the oscillatory component is larger than the unidirectional component. The bottom line, therefore, is that the velocity profile in the wave boundary layer

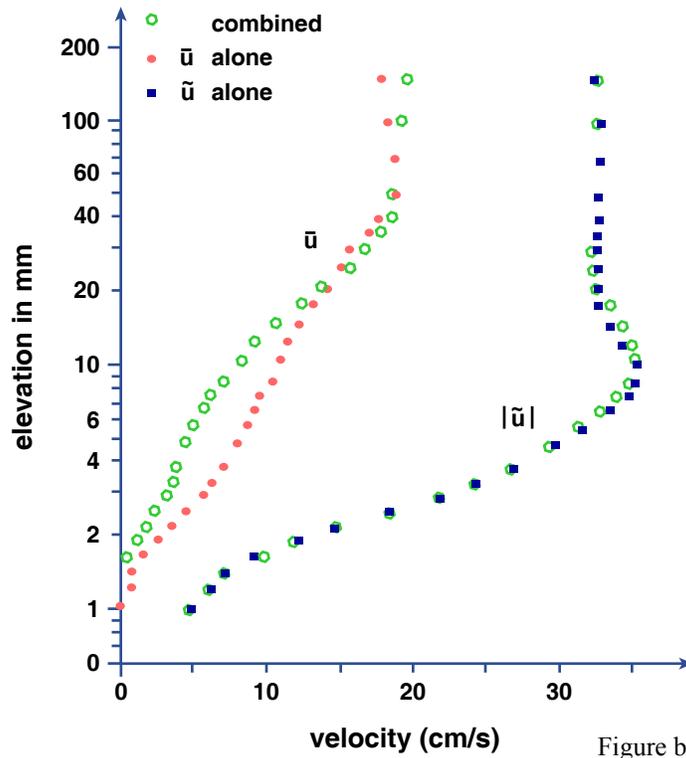


Figure by MIT OpenCourseWare.

Figure 6-7. Velocity profiles in purely unidirectional flow, purely oscillatory flow, and combined flow. The plus signs are for purely unidirectional flow; the x marks are for purely oscillatory flow; and the black dots are for combined flow. The two curves on the left show the unidirectional velocity component, and the curves on the right show the oscillatory velocity component. (From Nielsen, 1992, p. 62.)

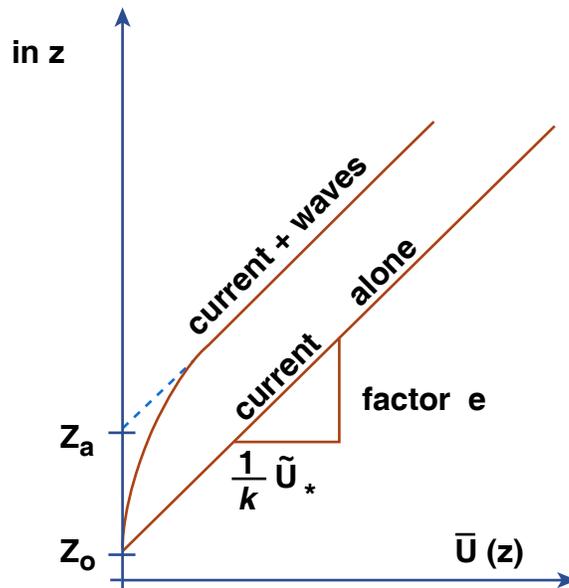


Figure by MIT OpenCourseWare.

Figure 6-8. How the addition of an oscillatory component of the flow changes the velocity profile of the unidirectional component of the flow. (From Nielsen, 1992, p. 79.)

is not greatly changed by the addition of a current, but the velocity profile in the flow above the wave boundary layer is substantially different than in a purely unidirectional flow (Figure 6-8).

References cited:

Nielsen, P., 1992, Sea Bed Mechanics.

Schlichting, H., 1960, Boundary Layer Theory: New York, McGraw-Hill, 647 p.