

**PART I**

**FLUID DYNAMICS**

# CHAPTER 1

## INTRODUCTION

### THE NATURE OF FLUIDS

**1** Fluids are substances that deform continuously and permanently when they are subjected to forces that vary spatially in magnitude or direction. The nature of the relationship between the deforming forces and the geometry of deformation varies from fluid to fluid; you will see in this chapter that the relationship is a simple linear one for air and water. Fluids can be classified as either liquids, which are relatively dense and maintain a definite volume, and gases, which are less dense and expand to fill their container. Fluids, both liquids and gases, are distinguished from solids by their inability to withstand deforming forces: in contrast to solids, they continue to deform for as long as the deforming forces are applied. This distinction is actually not as neat as I have made it out to be, but we would become sidetracked into the field of rheology for elaboration.

**2** Liquids and gases differ greatly in their structure on the atomic scale: liquids consist of closely packed molecules that exert strong forces on their neighbors as they weave around one another, sometimes forming fleeting and very small bonded aggregates, whereas gases, unless they are very compressed, consist of atoms or molecules that are almost always far apart from one another as they zip around along their free paths of motion, colliding with the walls of their container and occasionally with one another.

**3** How is it, then, that the macroscopic motions of liquids and gases need not be considered separately? The answer is that fluids can be treated as if they were **2**—as if their constituent matter, which is actually distributed discontinuously as atoms and molecules, were smeared uniformly throughout space. The idea here is that the forces among the constituent particles, which vary enormously in space on the scale of the particles themselves, average out to look as though they vary smoothly on scales much larger than the particles but very small relative to the macroscopic scales of problems in fluid dynamics—which themselves can be very small. To phrase this in a slightly different way: the structure of fluids is on such a fine scale that the actual intermolecular forces can just as well be treated as continuously and smoothly varying, from the standpoint of all problems in fluid dynamics on scales much larger than the molecules. The justification for this approach is that it works extremely well for fluid flows on scales that are much larger than the intermolecular spacing. So in these notes you never have to think again about the atoms and molecules of fluids! (Well, that's not quite true, but almost.)

## PRESSURE

4 The concept of fluid pressure is one of the most fundamental in fluid dynamics. Generally in physics the term *pressure* is used for a *force per unit area*. But we need to be more specific about the significance of pressure in fluids.

5 Suppose that you immerse a solid test sphere in a container of fluid at rest, and suppose further that you have a little meter with which you can measure the normal force per unit area exerted by the fluid at some point on the surface of the sphere (Figure 1-1). That force per unit area is the pressure exerted by the fluid on the surface of the sphere. That probably seems like a simple enough concept. (Because the fluid is not moving relative to the sphere, the fluid exerts only a normal component of force, not a tangential component; we will start looking at the nature of the tangential force exerted by moving fluid on a solid surface in the following section.) But there is more to fluid pressure than just that.

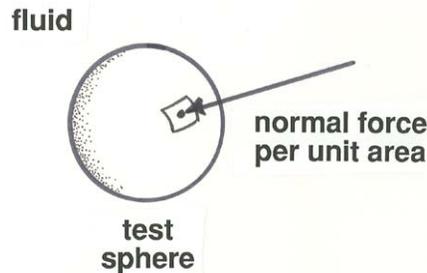


Figure 1-1. Normal force per unit area exerted by the fluid at a point on the surface of a tests sphere immersed in the fluid.

---

6 Now suppose that you make the solid sphere smaller and smaller. You can think of it as eventually becoming just a point. Then, associated with each line through that point there is a compressive force per unit area, directed inward from both directions along the line toward the point, with the same value as the force per unit area you measured on the surface of the test sphere (Figure 1-2). And the value of this compressive force per unit area is the same for every orientation of the line through the point. This is the essence of the concept of *fluid pressure*: it is a compressive force per unit area that acts equally in all directions at a point in the fluid, whether or not there is a solid surface at that point upon which the force acts. If there is no solid surface, you just have to think in terms of one part of the fluid continuum exerting a compressive force on the adjacent part of the fluid continuum.

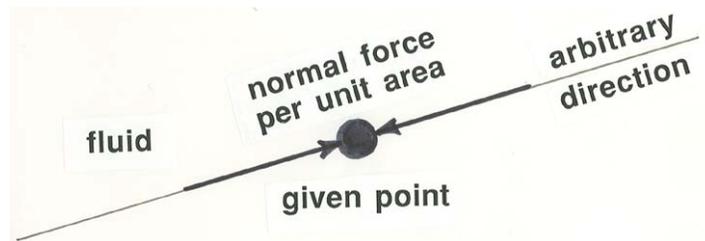


Figure 1-2. The compressive force per unit area associated with each line through a point in the fluid.

---

7 The concept of fluid pressure introduced above holds equally well for a moving fluid. Then you just have to imagine measuring the pressure at a point that is moving along with the fluid. It is convenient and natural to think of the pressure in a moving fluid as being made up of two parts, the static pressure and the dynamic pressure. The *static pressure* is the pressure that would be measured at the given point in the fluid if the fluid were not moving. The *dynamic pressure* is the difference between the total pressure—that is, the pressure you would actually measure at the given point in the moving fluid, with some appropriate instrument—and the static pressure. The dynamic pressure is the part of the pressure that is associated with the motion of the fluid. There will be much more to say about the relationship between fluid motion and dynamic pressure later in these notes; suffice it to say here that the dynamic pressure is zero in a stationary fluid, and also in a fluid that is in uniform motion, in the sense that there are no accelerations anywhere in the fluid (Figure 1-3).

8 It is not difficult to understand here, however, what determines the static pressure. In the case of fluid in a closed container, one part of the static pressure has to do just with the external compression imposed upon the walls of the container. When you blow up a balloon, the air pressure inside the balloon is greater than outside, because the distended walls of the balloon are trying to reshrink, and as a consequence they are everywhere pushing inward against the air inside (Figure 1-4). The pressure on the walls becomes adjusted to be the same at all points, because if that were not the case, then there would be pressure differences from point to point within the fluid, and by Newton's second law that would cause motions in the fluid.

---

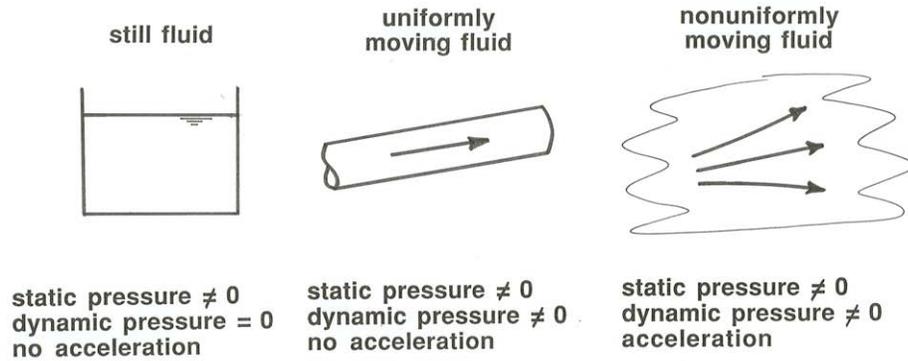


Figure 1-3. Static pressure and dynamic pressure in fluids at rest, in uniformly moving fluids, and in non-uniformly moving fluids.

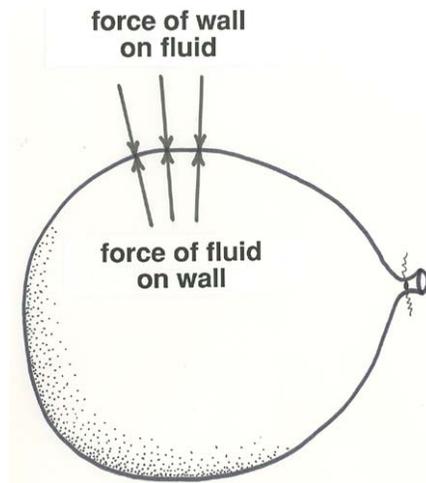


Figure 1-4. Forces at the wall of a blown-up balloon.

9 The other part of the static pressure has to do with the weight of fluid that overlies a given point in the fluid. Think about a tall upright cylindrical container filled with a liquid (Figure 1-5). You can easily compute the weight of liquid in the vertical column that overlies a little unit area on the bottom of the container: it is equal to  $\rho gh(1)(1)$ , where  $\rho$  is the density of the liquid,  $g$  is the acceleration of gravity, and  $h$  is the height of the liquid column above the bottom:

$$p = \rho gh \tag{1.1}$$

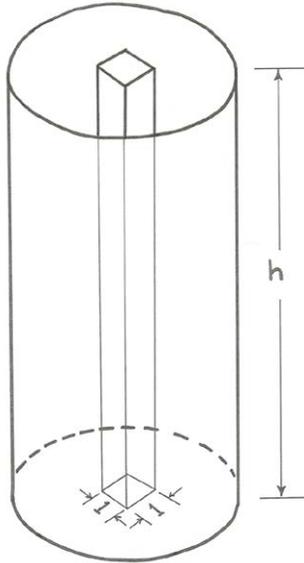


Figure 1-5. Hydrostatic pressure in a column of water.

---

It is just a matter of multiplying the weight per unit volume of the liquid,  $\rho g$ , by the volume of liquid in the vertical column,  $h(1)(1)$ . This part of the static pressure caused by the weight of overlying fluid, called the *hydrostatic pressure*, is given by the same equation not just on the bottom but also at all points in the fluid, and on the sides of the container as well; refer to the discussion, above, of the nature of pressure as a compressive force per unit area acting equally in all directions at any point in the fluid.

**10** So by Equation 1.1, called the *hydrostatic equation*, the hydrostatic pressure in the liquid increases linearly with depth, from zero at the surface (Figure 1-6). Compressible fluids like gases, however, are trickier; the vertical distribution of density and pressure in the atmosphere, for example, is the outcome of the balance between pressure and weight of overlying fluid, on the one hand, and the relationship between pressure and density, on the other hand.

**11** Look again at the container of motionless liquid. In your imagination, isolate a volume of liquid, bounded at the top and bottom by imaginary horizontal planes and around the sides by an imaginary vertical cylinder. Now examine the balance of forces in the vertical direction on the mass of liquid contained within that volume. One thing we know for sure is that the sum of all the vertically directed forces, upward and downward, on that mass of liquid has to be zero, because the liquid is at rest, and Newton's second law tells us that the net force acting on the volume of liquid must be zero. (This technique of isolating an

imaginary volume of material, called a *free body*, and examining the forces on it and the motions it undergoes is a common technique in the mechanics of continuous media, whether solids or fluids.)

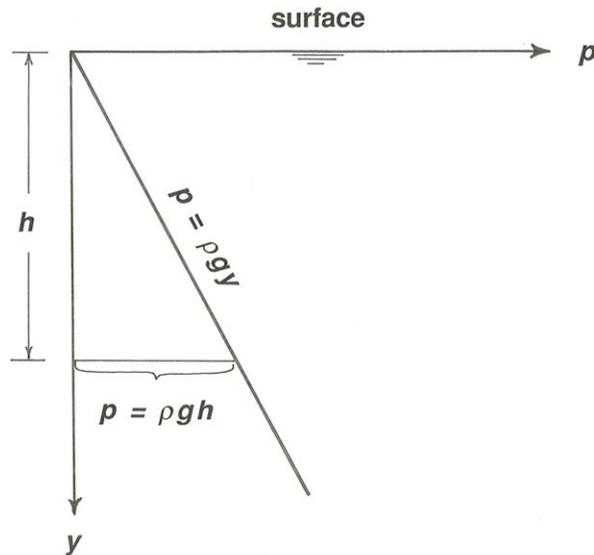


Figure 1-6. The linear increase in hydrostatic pressure with depth.

---

**12** There are vertically directed pressure forces on the top and bottom of the cylinder, but not on the sides of the cylinder, because the pressure forces on the sides of the cylinder are all horizontal. Remember that by the hydrostatic equation the pressure on the bottom of the free body is greater than the pressure on the top. Why, then, does the body not accelerate upward, in accordance with Newton's second law? The answer is that this upward-directed pressure force is exactly balanced by the weight of the liquid in the body. This is a manifestation of what is called the *hydrostatic balance*.

**13** Now suppose that you replace the imaginary free body of liquid with a real body of the same shape, with vanishingly thin but rigid walls and just empty space (if we ignore the density of air) within. The weight of the body is effectively zero, so there is no weight to balance the upward-directed net pressure force. As you all know, the body floats up to the surface. This effect, termed *buoyancy*, holds for all fluids, gases as well as liquids. It should be easy for you to imagine the great many environments, in and on the Earth, where buoyancy is an important effect.

**14** If you want a real-life demonstration of the magnitude of the buoyancy force, try taking a watertight and lightweight pail and pushing it down into a tub full of water, with its open end facing upward (Figure 1-7). You know that the

farther in you push it, the more difficult it is to push it. What is going on here is that you are pushing against the force of the hydrostatic pressure summed over the entire bottom surface of the pail. There is no water in the pail to balance that hydrostatic pressure force, so you have to establish the balance with your own hands and arms.

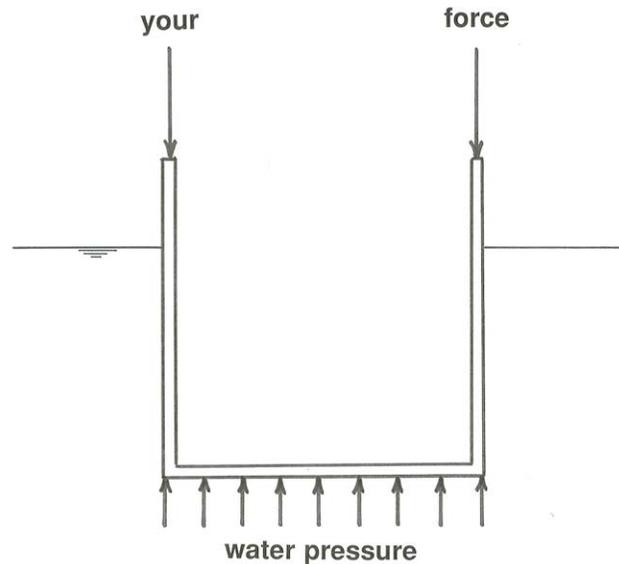


Figure 1-7. The force of buoyancy when you push an airtight empty box into the water.

---

**15** The various bodies within the fluid we have been dealing with need not be of the special shape, with only vertical and horizontal surfaces, we have been assuming. The same considerations hold true for bodies of arbitrary shape. At a point on a sloping part of the surface of such a body, you just have to take the vertical component of the pressure that is acting normal to the surface at the given point, and therefore in a sloping direction (Figure 1-8).

**16** One final matter has to do with the weight per unit volume, or *specific weight*, of an object or of some part of a continuous material, usually denoted by  $\gamma$ . Because the weight of a body is  $mg$ , the specific weight is  $mg/\text{volume}$ , which can be rearranged as  $(m/\text{volume})g$ , or  $\rho g$ , because density  $\rho$  is just mass per unit volume. So the relationship between density (mass per unit volume) and specific weight (weight per unit volume) is  $\gamma = \rho g$ . (Minor note: density could be called “specific mass”, but it never is—although its inverse, the volume per unit mass, is indeed called the specific volume.)

---

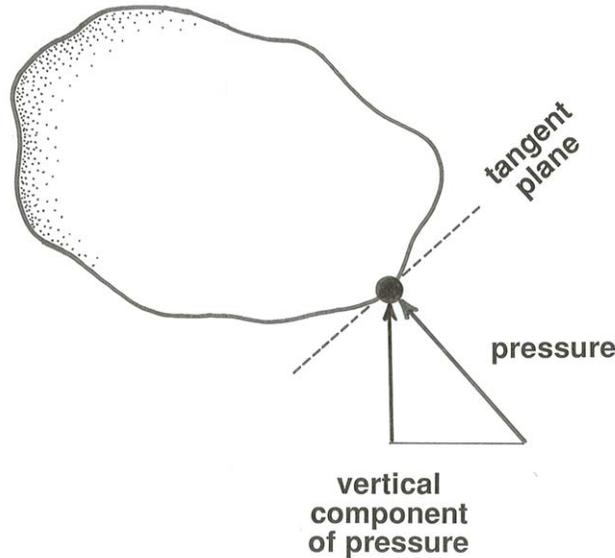


Figure 1-8. The vertical component of the fluid pressure on a submerged object of arbitrary shape.

**17** You also have to think about the submerged specific weight of an object that is entirely immersed in a fluid. It should make sense to you, and it follows from the earlier considerations on the pressure forces on submerged bodies, that the effective weight of a submerged body is less than its actual weight by the weight of the fluid it displaces. (This is the effect that I think is supposed to have caused Archimedes to shout “Eureka!” in his bathtub.) In these notes the submerged specific weight is denoted by  $\gamma'$ . Here is the mathematics:

$$\gamma' = \rho_{body}g - \rho_{fluid}g = (\rho_{body} - \rho_{fluid})g \quad (1.2)$$

**18** If this does not make immediate sense to you, just imagine that the density of a certain submerged body, initially denser than the fluid, is gradually decreased somehow until its density is the same as that of the fluid, whereupon it has the same specific weight as the fluid and is in hydrostatic balance, and therefore neither rises nor sinks. A body of this kind is said to be *neutrally buoyant*. Tiny neutrally buoyant particles make excellent markers for tracing and visualizing the motions of fluids, both in reality and in the imagination.

## VISCOSITY

**19** Another concept in fluid dynamics, viscosity, is one that is less likely to be within your range of intuition and experience than pressure. *Viscosity* is a property of fluids that characterizes their resistance to deformation. This section

is devoted to making that idea clear to you, and to making a start on exploring its consequences for fluid motions.

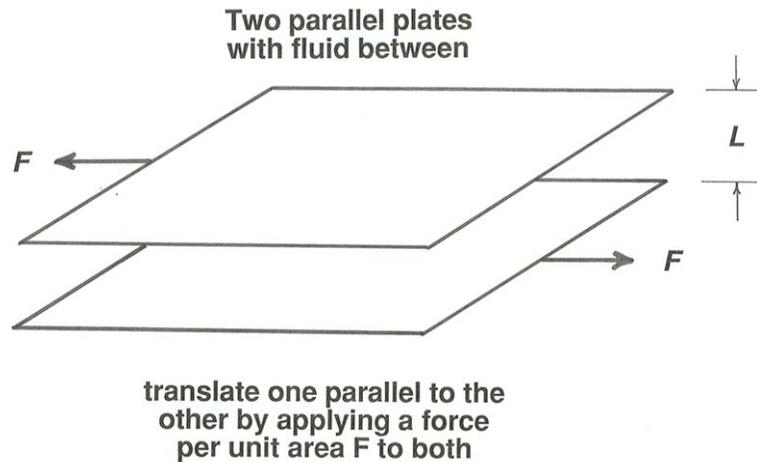


Figure 1-9. Shearing a fluid between two parallel plates.

---

**20** For a first look at how fluids behave when they are deformed, here is an experiment you could attempt on your own kitchen table. Arrange two horizontal parallel plates, spaced a distance  $L$  apart, with a fluid at rest between them (Figure 1-9). You could justifiably argue that it would be hard to make such an experiment, because how could you keep the fluid from leaking out at the margins of the plates? Do not worry about such practicalities; just suppose that the plates are very broad relative to their spacing, or that the fluid you have chosen is “thick”, i.e., has a high viscosity (you are likely to have a number of high-viscosity household fluids, like honey, or molasses, or corn syrup, or motor oil, available), which would ooze out from the between the plates only slowly, giving you time to do the experiment described below.

**21** Accelerate the upper plate rapidly to a constant velocity  $V$  parallel to itself by applying a force per unit area, call it  $F$ , over its entire surface, while you hold the lower plate fixed by applying to it an equal and opposite force per unit area. You could do that by taping the lower plate to the table and attaching handles with suction cups to the top of the upper plate. The fluid is set in motion by friction from the moving plate.

**22** How does the fluid move? You might picture the motion as a series of tabular layers of fluid, parallel to the bounding plates, sliding past one another in a shearing motion, but of course in reality the shearing is continuous rather than as discrete layers. Shear of this kind always acts whenever fluids are in motion relative to solid boundaries—which is just about all the flows we will consider in these notes. If you were somehow able to measure the velocity of the fluid at a

large number of points along some imaginary line normal to the plates (Figure 1-10), what would be the distribution of velocity? You would find that after an initial transient period of adjustment during which progressively lower layers of the fluid are brought into motion, the velocity would vary linearly from zero at the stationary plate to  $V$  at the moving plate.

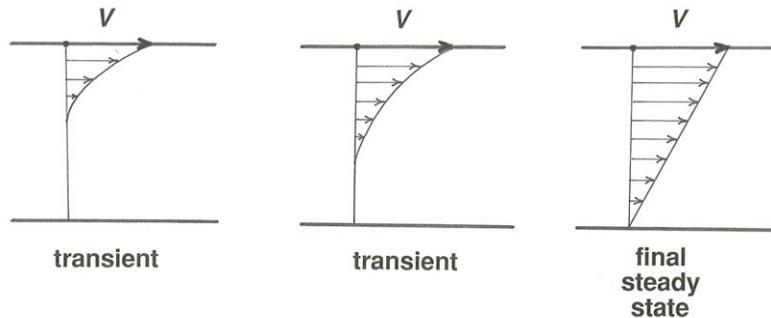


Figure 1-10. Development of the velocity profile in a fluid sheared between two parallel plates.

---

**23** From Figure 1-10 you can see that the fluid in contact with each of the plates has exactly the same velocity as the plates themselves. This is a manifestation of what is known as the *no-slip condition*: fluid in contact with a solid boundary has exactly the same velocity as that boundary. Although this no-slip condition might seem counterintuitive to you, it is a fact of observation, and it can be justified by considerations on intermolecular forces. The flow of the continuously deforming fluid past the solid boundary is not the same as sliding a rigid slab, like a brick, across a table top. Intermolecular forces act between the fluid and the solid at the boundary just as they do across planes of shearing in the interior of the fluid, so there's no more reason to expect a discontinuity in velocity at the boundary than within the fluid.

**24** To see why the velocity distribution between the plates is linear, pass an imaginary plane, parallel to the plates, anywhere through the fluid (Figure 1-11). Because the fluid contained between this plane and either the lower plate or the upper plate is not being accelerated after the steady state is attained, the fluid on either side of this plane must be exerting on the fluid on the other side of the plane

---

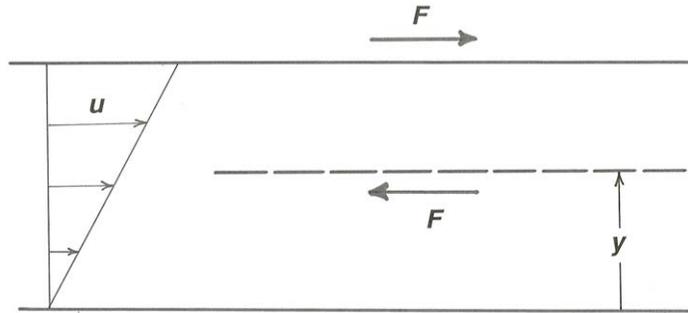


Figure 1-11. An imaginary plane, parallel to the plates, in the sheared fluid.  $F$  is the shearing force per unit area.

the same force per unit area  $F$  as that on the plates themselves. Because the imaginary plane can be located anywhere between the two plates, the shearing force per unit area across *all* such planes in the fluid, called the **shear stress**, must be the same. (From here on, keep firmly in mind the distinction between *shear*, an aspect of the geometry of fluid deformation, and *shear stress*, the shearing force per unit area associated with the shearing.) And because the fluid must be expected to shear or deform at the same rate for the same applied shearing force, the rate of change of velocity normal to the plates must be constant: assuming the  $y$  axis to be normal to the plates, and letting  $u$  be the velocity of the fluid at a point,

$$du/dy = k \tag{1.3}$$

where  $k$  is some constant. So the velocity itself must vary linearly: taking  $y = 0$  at the lower plate,

$$u = \int k dy = ky + c \tag{1.4}$$

Evaluating the constant of integration  $c$  by using the no-slip condition that  $u = 0$  at  $y = 0$ , we find  $c = 0$ , so

$$u = ky \tag{1.5}$$

For more on what determines the magnitude of that constant  $k$ , see a later paragraph.

**25** What determines the value of  $F$  needed to produce a given difference in velocity between the two plates (Figure 1-12)? For many fluids the ratio of  $F$  to the quantity  $V/L$ , which represents the rate of shear in the fluid, would be found to be the same for all values of  $F$ :

$$F/(V/L) = \text{const, or } F = \text{const} \cdot (V/L) \quad 1.6)$$

This constant ratio, the ratio of applied shear stress to the resulting rate of shear, usually denoted by  $\mu$ , is called the **viscosity** of the fluid. The viscosity is this the fluid property that characterizes the resistance of the fluid to deformation. For fluids like air and water, the viscosity is indeed an intrinsic property of the fluid, in that it does not depend on the state of motion but only on the nature of the fluid itself. Different fluids have different viscosities. Fluids with higher viscosities require a greater shearing force per unit area to produce a given rate of shearing, and fluids with higher viscosities have a greater rate of shearing for a given shearing force per unit area. For a given fluid the viscosity is a function of temperature; for water, viscosity decreases with temperature, but for air, viscosity increases with temperature.

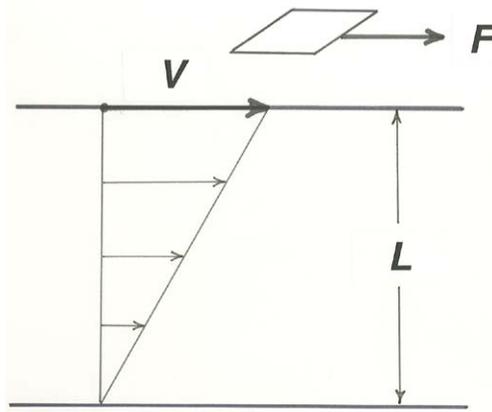


Figure 1-12. The force per unit area  $F$  needed to produce a given difference in velocity between two parallel plates with sheared fluid between.

**26** Now we need to generalize beyond the kitchen-table experiment. Here comes a substantial conceptual jump. The parallel-plate experiment is a rather specialized case of shearing in a fluid. In a more general flow, the geometry of the flow is more complicated and the rate of shear and the corresponding shearing force per unit area generally varies from place to place. Even so, the deformation of the fluid in any tiny volume can be visualized in the same way as in the parallel-plate experiment. A relationship like Equation 1.6 holds at every point in a sheared fluid, no matter how much the rate and orientation of the shearing vary from place to place:

$$\frac{\tau}{du/dy} = \mu \quad (1.7)$$

or

$$\tau = \mu \frac{du}{dy} \quad (1.8)$$

where  $\tau$  is the *shear stress* (i.e., the local shearing force per unit area) exerted across the shearing surfaces at some point in the fluid, and  $du/dy$  is the rate of change of the local fluid velocity  $u$  in the direction  $y$  normal to the shearing surfaces at the point (Figure 1-13). We will often have occasion to make use of Equation 1.8 later in the course. I have sidestepped its derivation from first principles; that would necessitate starting from Newton's second law, written in differential form, for the general fluid motion. I hope that the shortcut I have presented here gives you a good understanding of the significance of Equation 1.8.

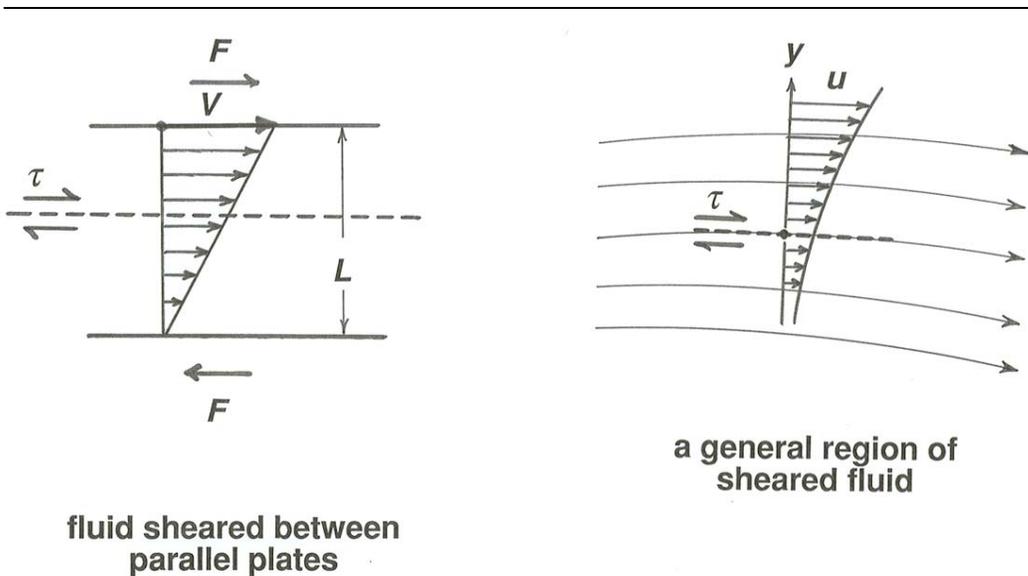


Figure 1-13. Viscosity as the ratio of applied shear stress to rate of shear at a point in a sheared fluid.

---

**27** You can see now the significance of  $k$  in the linear velocity distribution in Equation 1.5 for shearing of a fluid between parallel plates. Combine Equation 1.8, the general relationship between shear stress and shear rate, with the result, found above for the parallel-plate experiment, that  $du/dy = k$  (Equation 1.3), which expresses that the spatial rate of change, in the direction normal to the planes of shearing, of local fluid velocity is the same at all levels between the plates:

$$\frac{du}{dy} = k = \frac{\tau}{\mu} \quad (1.9)$$

**28** So the constant  $k$  reflects the relative magnitude of the applied shearing force per unit area and the viscosity: for given viscosity, a greater applied shearing force per unit area on the upper plate produces a steeper velocity gradient in the fluid, and for given applied force per unit area, a given viscosity produces a less steep velocity gradient.

**29** Why do fluids resist deformation? The shear stress that is mutually exerted across the surfaces of shear in the fluid, like the shear planes in the fluid between the parallel plates on your kitchen table, can be thought of as *internal friction*. For liquids, to account for the origin of this internal friction you can appeal to the necessity of stretching and breaking the fleeting bonds between the adjacent close-lying molecules of the fluid whose centers lie on one side or other of the imaginary shear plane. For gases, however, the picture is not as straightforward (although ultimately simpler mechanically!) because gases consist of isolated atoms or molecules pursuing their free paths and interacting among themselves only relatively infrequently. In gases, the shear planes pass through mostly empty space, and the constituent particles are for the most part moving freely as they pass across the shear planes in one direction or the other. To present a satisfactory account of the origin of internal friction in sheared gases, we have to deal with the phenomenon of *diffusion*. Diffusion is an important physical process that will figure in a number of later topics in these notes as well, so this is a good place, in the following section, to address it.

## DIFFUSION

**30** *Diffusion* is the process by which matter, or properties carried by matter, like momentum, heat, solute, or suspended sediment, is transported from one part of a medium to another by random motions, molecular or macroscopic, in the presence of a spatial variation, or gradient, in average concentration of matter or the property. The essential factors in diffusion are thus the presence of random movements within the medium and a spatial gradient of some quantity or property. There cannot be diffusive transport without the concurrent existence of both conditions.

**31** A good way to understand the nature of diffusion is to think about a simple hypothetical example. Suppose that you erect a vertical wall or barrier across the middle of a large room and manage somehow to fill one side of the room with white air molecules and the other side of the room with black air molecules (Figure 1-14). At some particular time you instantaneously remove the barrier. Watch the exchange of speeding molecules across the plane once occupied by the barrier. In any small time interval the numbers of molecules passing in one direction across that plane is almost exactly equal to the numbers of molecules passing in the other direction, because the concentration of molecules stays the same everywhere, on the average, or else pressure differences

from place to place would cause a net movement of air from higher pressure to lower pressure.

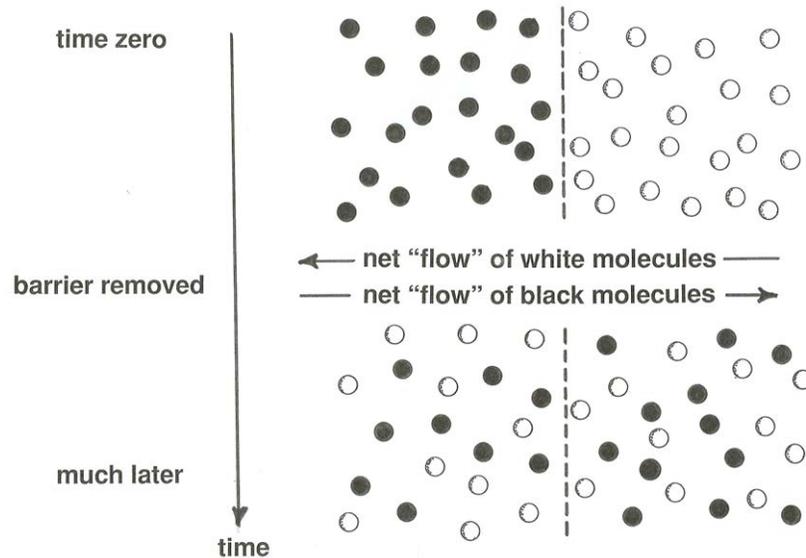


Figure 1-14. Diffusion of air molecules.

**32** Immediately after the barrier is removed, all of the molecules moving from the “white” side to the “black” side are white and all of the molecules moving from the “black” side to the “white” side are black. At later times, after some of the white molecules have made their way over to the originally black side and some of the black molecules have made their way over to the white side, both white and black molecules move across the plane in each direction, but for a long time more white molecules than black move from the originally white side to the originally black side, and likewise more black molecules than white move from the originally black side to the originally white side.

**33** This reveals the essence of diffusive transport: as the gradient of concentration of black and white molecules normal to the plane is made smaller by the diffusive transport, the rate of diffusive transport itself decreases; gradually the concentration gradients are evened out. Eventually, equal numbers of blacks and whites pass across the plane in either direction, and there is no more diffusive transport.

**34** In simple diffusion of the kind exemplified above, the rate of diffusive transport, expressed as mass per unit time per unit area normal to the direction of diffusion (called the *diffusive flux*) is directly proportional to the concentration gradient in the direction of the diffusive flux. In the example above, you could

convince yourself of this with just a little thought. This is expressed by the equation

$$F = -D \frac{\partial c}{\partial x} \quad (1.10)$$

where  $F$  is the rate of transport of mass per unit area,  $c$  is the mass concentration of the diffusing substance, and  $D$  is a proportionality coefficient called the **diffusion coefficient**. The minus sign is there because the diffusive transport is in the direction of decrease in  $c$  (“down the gradient”, in the parlance of physics). Diffusion can be more complicated than this: the diffusion coefficient might be a function of the concentration, and it might vary with direction at a point. We will not need to deal with such complications in these notes.

### VISCOSITY AS A DIFFUSION COEFFICIENT

**35** Viscosity can be interpreted as a *diffusion coefficient for molecular momentum*. Look at a small region of sheared fluid, and focus in on the collection of molecules in the vicinity of one of the shear planes (Figure 1-15). Molecules are continually passing back and forth across the plane. The total mass of molecules passing in one direction is the same as the total mass passing in the other direction, but (and this is the essential point) the same kind of statement is *not* true for transport, in the direction normal to the shear plane, of the component of molecule momentum taken normal to the shear plane and in the direction of average movement—just because, by the nature of shearing, the average velocity is greater on one side of the plane than the other. So random movements of molecules across the shear plane cause a normal-to-the-shear-plane transport of downflow component of momentum in the direction from higher flow speed to lower flow speed.

**36** Applying the diffusion equation (Equation 1.10) here,

$$\begin{aligned} \text{momentum transport rate} &= D \frac{d(\rho u)}{dy} \\ &= \rho D \frac{du}{dy} \end{aligned} \quad (1.11)$$

By Newton’s second law this spatial rate of change of momentum is equivalent to a force per unit area  $\tau$  on the shear planes:

$$\tau = \rho D \frac{du}{dy} \quad (1.12)$$

**36** By comparing Equation 1.11 with Equation 1.12 you see that the viscosity  $\mu$  can be viewed as the diffusion coefficient for downflow momentum,

multiplied by the fluid density  $\rho$ . The physical interpretation here is that the on-the-average faster molecules that pass across the shear plane from the higher-velocity side to the lower-velocity side tend to speed up the molecules on the lower-velocity side, by eventually colliding with them and exerting actual forces on them, and conversely the on-the-average slower molecules that pass across the shear plane from the lower-velocity side to the higher-velocity side tend to slow down the molecules on the higher-velocity side. This mutual speeding-up and slowing-down has the effect of an actual surface contact force across the plane, of the sort you would associate with, say, the sliding of a brick on a tabletop—but remember that there is no slip along the shear planes in the fluid.

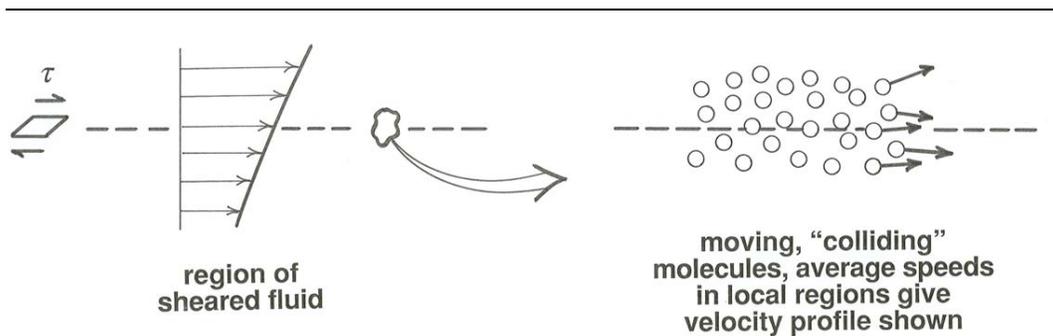


Figure 1-15. How the random motions of fluid molecules in the presence of macroscopic shear in a fluid continuum gives rise to shear stress.