

MIT OpenCourseWare
<http://ocw.mit.edu>

12.003 Atmosphere, Ocean and Climate Dynamics
Fall 2008

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.

Problem set 6: Geostrophic balance

Due date: November 5th, 2008

1. Analysis of the wind at 500 mb. Download the latest 500mb analysis of geopotential height and wind. See appendix for instructions. Choose a point in the flow, estimate the wind speed (one full quiver denotes a speed of 10 m s^{-1} ; one half quiver a speed of 5 m s^{-1} and check for consistency with the geostrophic relation:

$$v_g = \frac{g}{f} \frac{\Delta h}{d}$$

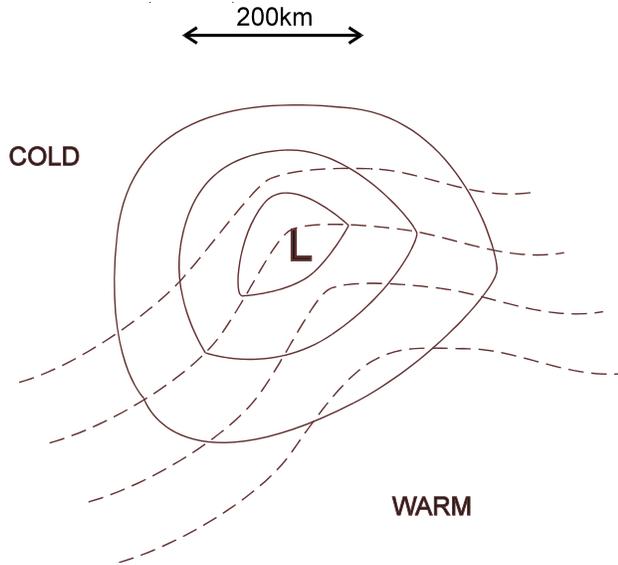
where $f = 2\Omega_{earth} \sin lat$ is the Coriolis parameter, the rotation of the Earth is $\Omega_{earth} = 7.29 \times 10^{-5} \text{ s}^{-1}$, g is the acceleration due to gravity, Δh is the difference between consecutive height contours (note the contour interval) and d is the horizontal separation of the height contours in meters. Remember that 1 degree of latitude is equivalent to 111 km.

2. A punter kicks a football a distance of 60 m on a field at latitude 30°N . Assuming the ball, until being caught, moves with a constant forward velocity (horizontal component) of 20 m s^{-1} , determine the lateral deflection of the ball from a straight line due to the Coriolis effect. [Neglect friction and any wind or other aerodynamic effects.]
3. The Gulf Stream leaves the coast near Cape Hatteras (about 36°N) and meanders into the open ocean. Current meters show that the flow speed is about 2 m s^{-1} and that the stream extends through the top 1000 m of the ocean. Assume a linear profile with speed equal to zero at 1000 m depth. The stream is typically 50 km wide. Derive the thermal wind equation for a simple unidirectional current in water, using x as the direction along the current and y as the direction across the current. Use the equation of state:

$$\rho = \rho_{ref}[1 - \alpha(T - T_{ref})],$$

where $\rho_{ref} = 1000 \text{ kg m}^{-3}$, and $\alpha = 2 \times 10^{-4} \text{ K}^{-1}$. Use the thermal wind equation to estimate the temperature difference horizontally across the current. Observed contrasts are about 3 K.

4. The following figure shows, schematically, the surface pressure contours (solid) and mean 1000hPa-500hPa temperature contours (dashed), in the vicinity of a typical northern hemisphere depression (storm). L indicates the low pressure center. Sketch the directions of the wind near the surface, and on the 500 hPa pressure surface. (Assume that the wind at 500 hPa is significantly larger than at the surface.) If the movement of the whole system is controlled by the 500 hPa wind (i.e., it simply gets blown downstream by the 500 hPa wind), how do you



expect the storm to move? [Use density of air at 1000hPa = 1.2 kg m^{-3} ; rotation rate of Earth = $7.27 \times 10^{-5} \text{ s}^{-1}$; gas constant for air = 287 J kg^{-1} .]

5. Derive the mass conservation equation in pressure coordinates following the steps outlined in section 6.3.2 of the textbook. Then show that, if the horizontal flow is in geostrophic balance, the vertical velocity $\omega = Dp/Dt$ is independent of pressure.
6. Consider the problem illustrated by the Matlab program Pucks_on_ice. In order to get this m-file, you should download both the main program pucks_on_ice_script.m as well as the files pucks_on_ice.m, draw_fig2.m and deriv.m. Try running the program. Pucks_on_ice examines the effect of an impulsively-imposed body force upon a series of "hockey pucks" of uniform mass that are distributed about the surface of a frozen ocean. The pucks are initially at rest. The body force, which pushes the pucks to the northeast, acts for 3 days and then stops. The integration of the ensuing motion continues for period of time after the forcing has ceased.

This is an "f-plane" calculation in two dimensions, in which the Coriolis force is set to a fixed latitude (30 degrees north latitude in the first example). There are a number of "forces" in this problem, in addition to the body force. First, friction is weak and opposed to the motion of the particles. If the particle velocity is u , then the frictional force per unit mass is expressed as $-r*u$. Second, because the frozen ocean surface is not flat, gravity (g , directed downwards) accelerates the pucks down the sloping surface. In the x -direction this acceleration, or force per unit mass, is given by $-g(dh/dx)$, where dh is the change in

surface height over a distance dx . The equations used are as follows:

$$\begin{aligned}\frac{Dx}{Dt} &= u, \\ \frac{Dy}{Dt} &= v, \\ \frac{Du}{Dt} - fv &= -ru - g\frac{dh}{dx} + F_x, \\ \frac{Dv}{Dt} + fu &= -rv - g\frac{dh}{dy} + F_y,\end{aligned}\tag{1}$$

where (x,y) are the eastward and northward particle position, (u,v) are the eastward and northward particle velocity, f is the Coriolis parameter, and (F_x,F_y) are the eastward and northward components of the external body force.

The frozen sea surface has a "bump" that is Gaussian in shape, centered at (x_m,y_m) , with an amplitude, h_0 , and a length scale defined by (l_x,l_y) . The sea surface height is given by the following:

$$h(x,y) = h_0 \exp(-((x - x_m)/l_x)^2 - ((y - y_m)/l_y)^2),$$

where $x_m = y_m = 400$ km, and $l_x = l_y = 30$ km.

The model runs five different scenarios. For every scenario, the Matlab script produces two figures with particle trajectories and a figure with the bump shape. A detailed explanation of what each figures show is displayed by the script. You must comment on the trajectories of particles on the bump and away from the bump in the five different scenarios generated by the script. The five different scenarios differ in the bump height and latitude:

- (a) The bump has $h_0 = .05$ m and the latitude is 30 degrees.
- (b) The bump is changed to a dimple, making $h_0 = -.05$ m.
- (c) The latitude is changed to the north pole (lat=90).
- (d) The latitude is changed to the equator (lat=0).
- (e) The latitude is changed to -30 degrees (southern hemisphere)
- (f) Consider the equations with no body or topography forces. Derive expressions for the evolution of the oscillation for small friction and determine the frequency of the oscillation. What are the equations representing the geostrophic balance and why do the particles rotating around the bump slowly spiral out?

Appendix

To access the latest 500mb height analysis from the web go to our synoptic web site here: <http://paoc.mit.edu/synoptic/> and

- Click on upper in the Custom Plots section on the lhs sidebar.
- Click on height and wind in the yellow box on the upper rhs of the page.
- Click on Generate Map – this will send a job to our server, get the data, graph it up and send a gif back to your browser. The contours are labelled in metres and the winds plotted as vectors.

If you would like a .ps le sent back to your computer change the device setting on the page to ps. Click again and you should be able to save it to your computer. Remember to save as a .ps le.