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12.003 Atmosphere, Ocean and Climate Dynamics
Fall 2008

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Problem set 5: Convection and moist processes

Due date: October 15th, 2008

1. Consider an atmospheric temperature profile at dawn with a temperature discontinuity (inversion) at 1 km, and a tropopause at 11 km, such that

$$T(z) = \begin{cases} 5^\circ C & z < 1 \text{ km} \\ [10 - 8(z - 1)]^\circ C & 1 < z < 11 \text{ km} \\ -70^\circ C & z > 11 \text{ km} \end{cases} \quad (1)$$

(where here z is expressed in km). Following sunrise at 6 a.m. until 1 p.m., the surface temperature steadily increases from its initial value of 5°C at a rate of 3°C per hour. Assuming that convection penetrates to the level at which air parcels originating at the surface attain neutral buoyancy, describe the evolution during this time of convection:

- (a) if the surface air is completely dry;
- (b) if the surface air is saturated.

You may assume that unsaturated (saturated) air parcels follow the dry (moist) adiabatic lapse rate of 10 K km^{-1} (7 K km^{-1}) under all conditions (even at finite displacements).

2. Investigate under what conditions we may approximate

$$\frac{L}{c_p T} dq \quad (2)$$

by

$$d\left(\frac{L}{c_p T} q\right) \quad (3)$$

in the derivation of equivalent temperature. Is this a good approximation in typical atmospheric conditions?

3. Raindrops are water drops sufficiently large that they fall with appreciable speeds. A typical raindrop has a diameter of 1 mm. That of a typical cloud droplet is perhaps 100 times less. Although it may be thought that cloud droplets grow into raindrops by condensation, this can be shown to be false by simple thermodynamic arguments. Please do so. That is, calculate the maximum size to which a cloud droplet can grow solely by condensation. You will need the following:

The gas constant R_v for water vapor is 461.5 J/kg K .

A useful approximation for the saturation vapor pressure (in hPa) as a function of temperature is

$$\log e_s = 9.4041 - \frac{2354}{T} \quad (4)$$

where the logarithm is to the base 10.

You may make the following assumptions:

- (a) An air parcel at a temperature of 30°C is saturated at cloud base (500 hPa).
- (b) This parcel ascends adiabatically to the tropopause (300 hPa) without condensation (i.e. the parcel becomes supersaturated).
- (c) Then excess water vapor condenses onto the nuclei that were in the parcel at the beginning of its ascent.
- (d) Each nucleus acquires the same amount of water.
- (e) Initially, the concentration of nuclei in the parcel is 100/cm³, which corresponds to very clean air.

Hint: Don't make this problem harder than it need be. You are only trying to show that cloud droplets cannot grow into raindrops by condensation of water vapor.

Briefly explain why these assumptions, although they are reasonable (a cloud base temperature of 1000°C will give larger droplets, but this temperature is absurd), lead to a good estimate of the maximum cloud droplet size.

4. Consider the tropical Hadley circulation in northern winter, as shown schematically in Fig.1. The circulation rises at 10°S, moves northward across the equator in the upper troposphere, and sinks at 20°N. Assuming that the circulation, outside the near- surface boundary layer, is zonally symmetric (independent of x) and inviscid (and thus conserves absolute angular momentum about the Earth's rotation axis), and that it leaves the boundary layer at 10°S with zonal velocity $u = 0$, calculate the zonal wind in the upper troposphere at (a) the equator, (b) at 10°N, and (c) at 20°N.