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12.002 Physics and Chemistry of the Earth and Terrestrial Planets
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Gravity Anomalies

For a sphere with mass M , the gravitational potential field V_g outside of the sphere is:

$$V_g = -\frac{MG}{r}$$

where G is the universal gravitational constant ($6.67300 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$) and r is distance from the center of the sphere. The gravitational field is related to the potential field by:

$$\vec{g} = -\nabla V_g = -\frac{MG}{r^2} \hat{r}$$

again provided that the field is measured outside of the sphere.

Planetary Reference Gravity Field

Because planets are not spheres, but rather slightly oblate spheroids (because of their rotation around the spin axis), the gravitational field is not quite that of a sphere. For an oblate spheroid, the magnitude of the gravitational field has terms proportional to the sign of the latitude, or:

$$g = \frac{MG}{r^2} + A \sin^2 \lambda + B \sin^4 \lambda$$

This is called the “reference gravity field”. If we want to look at gravity due to mass anomalies within the crust and mantle, we need to first subtract off the reference gravity field from the observed gravity field to look at what is left over that is due to mass anomalies or dynamic processes that result in mass anomalies. On earth g at the surface is 9.81 m/s^2 , while the anomalies we want to look at are 4 or 5 orders of magnitude smaller. Generally we measure gravity anomalies in units of mgal, where $1 \text{ mgal} = 10^{-5} \text{ m/s}^2$.

Free-Air (Elevation) Corrections

Suppose we have a planetary gravity field, with the best-fitting reference gravity field subtracted off. Now we would like to interpret what is left

over. Another issue that we need to worry about is the fact that our gravity observations are probably made at different elevations relative to the best-fitting oblate spheroid for the planet of interest. This happens because of topography, if we make the measurements at ground level, or because our space craft's orbit changes elevations with time. We don't want to mix up measurements that change due to the elevation of measurement with measurements that change due to the presence of mass anomalies.

For simplicity, from now on we will assume our planet is a sphere, and that the reference gravity field can locally described by MG/r^2 where r is just the distance from the point of measurement to the center of mass of the planet. Suppose that the radius of the planet is R and the measurement is made at a height h above the best-fitting oblate spheroid. Also suppose that h is small compared to R . Then the gravity field measured at h will be:

$$g_h = \frac{MG}{r^2} = \frac{MG}{(R+h)^2} = \frac{MG}{R^2(1+h/R)^2} \approx \frac{MG(1-2h/R)}{R^2} = g_s \left(1 - \frac{2h}{R}\right)$$

where g_s is the reference gravity field at the surface of the planet (or rather at the surface of the oblate spheroid). The expected variation in the gravity field with height is thus:

$$g_h = g_s - \left(\frac{2h}{R} g_s\right)$$

The term in parentheses is called the “free-air” correction, meaning that it is a height correction that can be used to reduce gravity measurements to what they would be if made at sea-level (or to another oblate spheroid, in space for example, with a distance from the center of the planet of R). It is easy to evaluate this correction. For example, for Earth, $R=6400$ km, $g_o=9.81$ m/s². This gives $(2g_o/R)=3.06 \cdot 10^{-6} \text{ s}^{-2}=0.306$ mgal/m or 306 mgal/km.

Mass Anomalies and Gravity Anomalies

Now that we have taken care of the reference gravity field and the elevation corrections, we are ready to start computing gravity anomalies. For simplicity we will do everything in two-dimensions and in Cartesian coordinates, so that all mass anomalies are invariant in the y -direction. Suppose that we have a “line” anomaly, with a density $\Delta\rho$ with the

surrounding crust or mantle and a width dx_o and height dz_o . Suppose that the anomaly is at a depth b below the point where we make our measurements.

Remember that in class we showed that when we measure the absolute value of the gravitational field, all we are really able to measure is the vertical component of the gravity field created by the mass anomaly (because the mass anomaly field will be so much smaller than the total planetary field and our gravimeters measure only the magnitude of the field).

First, let's consider a only small segment of the line anomaly, of length dy_o located at position $(x_o, y_o, -b)$ and compute the vertical component of the anomaly field at position $(x, y, 0)$. This is easy:

$$\Delta g_z = \left(\frac{G\Delta\rho dx_o dy_o dz_o}{(x_o - x)^2 + (y_o - y)^2 + b^2} \right) \left(\frac{b}{((x_o - x)^2 + (y_o - y)^2 + b^2)^{1/2}} \right)$$

where the first term in parentheses is the total magnitude of the anomaly field at the observation point and the second term is the sine of the angle from the observation point to the mass anomaly. We can find the gravity anomaly due to the total line load by integrating this expression over y_o and letting the ends of the line go to \pm infinity. So:

$$\Delta g_{line} = \int_{-\infty}^{\infty} \frac{bG\Delta\rho dx_o dz_o}{((x_o - x)^2 + (y_o - y)^2 + b^2)^{3/2}} dy_o$$

$$\Delta g_{line} = \left[\frac{bG\Delta\rho dx_o dz_o}{((x_o - x)^2 + b^2)} \right] \frac{y_o}{((x_o - x)^2 + (y_o - y)^2 + b^2)^{1/2}} \Bigg|_{y_o=-\infty}^{y_o=\infty}$$

or:

$$\Delta g_{line} = \frac{2bG\Delta\rho dx_o dz_o}{((x_o - x)^2 + b^2)}$$

This expression is generally useful for computing predicted gravity anomalies from any distribution of masses at a depth b . If the depth range of the mass anomalies is not too different from b , for example variations of Moho (crust-mantle boundary) that oscillate around b , then we “flatten” all the mass onto a depth b . If the variation is too big for this, we have to do something more complicated, but for now we will just assume that we can treat all the anomalous mass as being concentrated onto a surface at depth b .

One useful distribution of mass to solve for is a strip of mass of thickness dz_o extending from $x_o=x_1$ to $x_o=x_2$. This is easy to solve for by integration.

$$\Delta g_{strip} = \int_{x_1}^{x_2} \frac{2bG\Delta\rho dz_o}{((x_o - x)^2 + b^2)} dx_o$$

or:

$$\Delta g_{strip} = 2G\Delta\rho dz_o \left[\tan^{-1}\left(\frac{(x_2 - x)}{b}\right) - \tan^{-1}\left(\frac{(x_1 - x)}{b}\right) \right]$$

If the load is very narrow, so that (x_1-x_2) is very small compared to b , then the gravity anomaly at the surface looks just like that of a line load:

$$\Delta g = \frac{2bG\Delta\rho(x_2 - x_1)dz_o}{\left[\left[(x_1/2 + x_2/2) - x\right]^2 + b^2\right]}$$

If the strip is very wide compared to the depth and the observer is located above the anomaly and far from the edges, then the mass anomaly looks like that of an infinite sheet of material:

$$\Delta g_{sheet} = 2\pi G\Delta\rho dz$$

If the strip is very wide compared to the depth but the observer is near enough to one edge, say at x_1 , to “see” it in the gravity, then:

$$\Delta g_{1/2sheet} = G\Delta\rho dz_o \left[\pi + 2 \tan^{-1}\left(\frac{(x - x_1)}{b}\right) \right]$$

Now we have enough information to begin to ask how well we can “see” mass anomalies at depth using gravity measured at the surface or from space. Let’s start by thinking about gravity measured at the surface of a planet before we go on to measurements made from space and ask how well we can distinguish a strip of mass from a line-mass, assuming that the total mass anomaly is the same in both cases and that the center of the strip and the center of the line-mass coincide. Let the line mass and the strip both be centered at $x_0=0$ and let the strip have a halfwidth a , so that the total mass anomaly in cross-section is $(2a dz_0 \Delta\rho)$.

The gravity anomaly from the line-mass is:

$$\Delta g_{line} = \frac{4baG\Delta\rho dz_0}{(x^2 + b^2)}$$

The gravity anomaly from the strip is:

$$\Delta g_{strip} = 2G\Delta\rho dz_0 \left[\tan^{-1}\left(\frac{x+a}{b}\right) - \tan^{-1}\left(\frac{x-a}{b}\right) \right]$$

or

$$\Delta g_{strip} = 2G\Delta\rho dz_0 \left[\tan^{-1}\left(\frac{x}{b} + \frac{a}{b}\right) - \tan^{-1}\left(\frac{x}{b} - \frac{a}{b}\right) \right]$$