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12.002 Physics and Chemistry of the Earth and Terrestrial Planets
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We can use this function to calculate heat loss through the surface of a planet if the planet simply cools statically with time (no convection or plate tectonics). The surface heat flow that would be predicted from the error function can be computed from:

$$q(z,t) = K \frac{\partial T}{\partial z}$$

where K is the thermal conductivity, and substituting the error function for T(z,t) gives the heatflow q:

$$q(z,t) = \left[\frac{2K(T_m - T_s)}{2\sqrt{\pi\kappa t}} \right] e^{-\left(\frac{z}{2\sqrt{\kappa t}}\right)^2}$$

Evaluating at the surface, z=0:

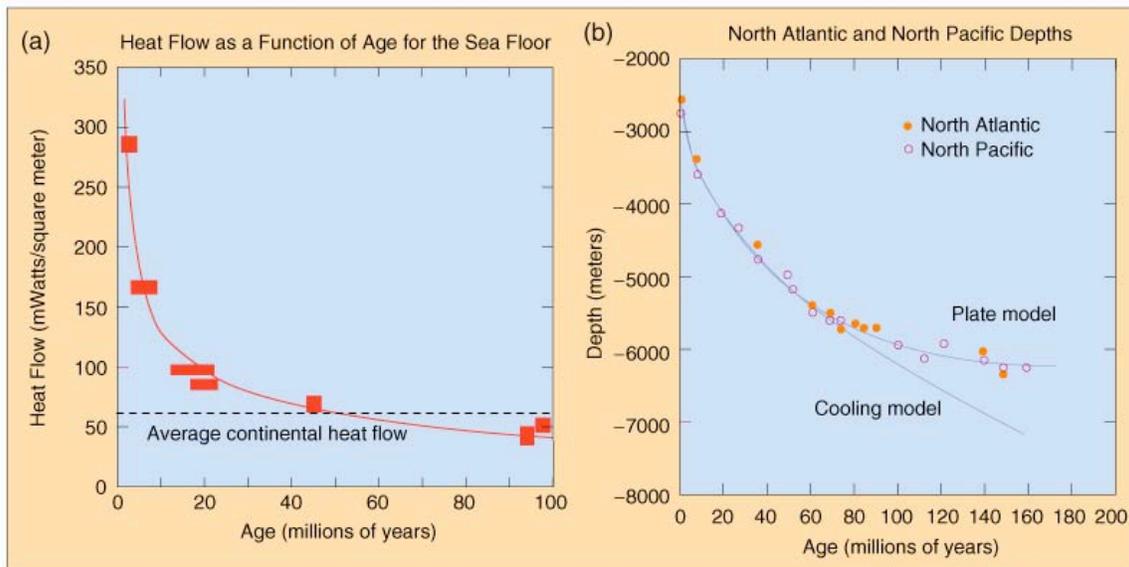
$$q(0,t) = \left[\frac{K(T_m - T_s)}{\sqrt{\pi\kappa t}} \right]$$

This implies that surface heat flow should decrease as the inverse of the square root of time in a conductively cooling half-space (or a very thick layer such that the base of the layer is not cooling yet). We can apply this, or see that it works, by looking at the oceanic lithosphere of the Earth.

Oceanic Subsidence and Heat Flow

The theory of sea-floor spreading says that new oceanic crust and lithosphere is created at mid-ocean ridges and that the new lithosphere moves away from the ridge crest with time. If this is correct, then the properties of the oceanic lithosphere, as a function of time, should be governed by the laws of heat conduction as this outer shell of the Earth cools through time. Understanding how this results in physical observables is important not just for our understanding of the Earth, but also for understanding how lithospheric age and cooling may be displayed on other planets.

We have just shown that surface heat flow for a cooling lithosphere goes as one over the square root of time. Let's look at the data from the oceans: Plotted against the observed heat flow data from the oceans the agreement between observed heat flow and square-root of time relationship looks pretty good for appropriate values of K , κ , etc.:

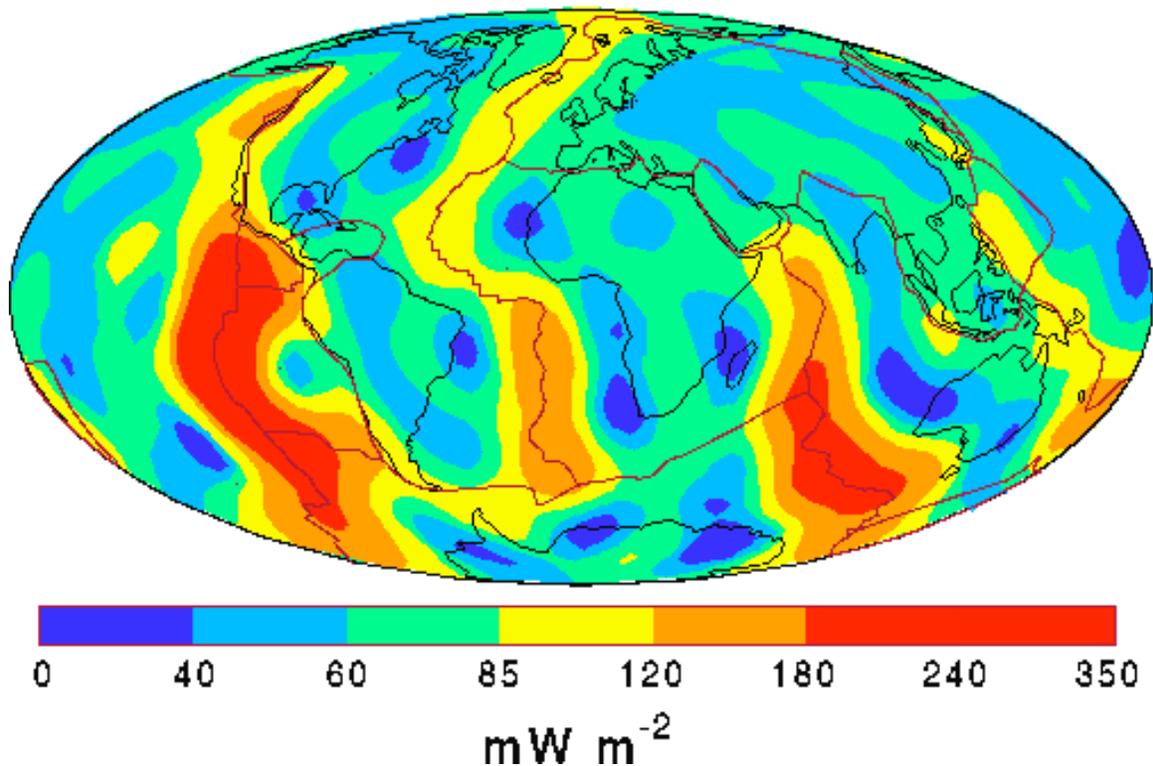


<http://www.earth.northwestern.edu/people/seth/107/Ridges/depthage.htm>

Courtesy of Seth Stein, Northwestern University. Used with permission.

However, as you can see from the map below, there is actually a huge scatter in heat flow observations, because heat flow measurements easily perturbed by water circulation in the sediments on the sea floor as well as through cracks in the underlying basement.

Heat Flow



We can't measure the temperatures down in the lithosphere below a few kilometers depth, but we can measure average temperature through the effects of thermal expansion and contraction of the cooling mantle.

First, let's calculate how much a column of mantle (and a little crust on top) contracts as it cools. (Because of conservation of mass, the amount of material stays the same, but the vertical, or radial, dimension of the material will decrease according to its temperature and the coefficient of thermal expansion:

Let's suppose, as before, that the initial temperature of the mantle is a uniform T_m and that later the temperature of the mantle is $T(z,t)$. The coefficient of thermal expansion, α , is defined as:

$$\partial l = \partial l_o [1 + \alpha(T - T_o)]$$

where ∂l_0 is the thickness of a column of rock at temperature T_0 , and ∂l is height of the same column of rock at temperature T . Rearranging, we can see that the height of a column of rock changes by an amount:

$$(\partial l - \partial l_0) = \partial l_0 \alpha (T - T_0)$$

as the temperature changes from T_0 to T . If we want to apply this to a column that goes from the surface to great depth within an infinite half space, we just need to integrate:

$$\Delta l = \int_0^{\infty} \alpha (T(z) - T_0) dz$$

There are multiple ways to proceed, but the easiest is to relate this to relate temperature to heat. Remembering that the change in temperature is related to the change in heat per unit volume by:

$$\Delta Q = \rho C_p \Delta T$$

We can now substitute in change in heat per unit volume for temperature, to find:

$$\Delta l = \frac{\alpha}{\rho C_p} \int_0^{\infty} \Delta Q(z) dz$$

The integral just gives the change in heat in a column with unit surface area extending from the surface downwards to infinity. If we take the time derivative of this with respect to time, we will have the rate at which heat is changing in a column per unit time per unit surface area. This has to be equal to the rate at which heat is leaving the column through the surface at $z=0$, or in otherwords has to be equal to the surface heat flow, q_s .

$$\frac{\partial \Delta l}{\partial t} = \frac{\alpha}{\rho C_p} \frac{\partial}{\partial t} \left[\int_0^{\infty} \Delta Q(z) dz \right] = \frac{\alpha}{\rho C_p} q_s(t)$$

This is easy because it just says that the rate of change of thickness of the material is linearly proportional to the surface heat flow for

a conductively cooling half space. We have already calculated the surface heat flow, so we can just substitute that in:

$$\frac{\partial \Delta \ell}{\partial t} = \frac{\alpha}{\rho C_p} \left[\frac{K(T_m - T_s)}{\sqrt{\pi \kappa t}} \right]$$

Or, we can easily integrate this over time to find:

$$\Delta \ell = \left[\frac{2\alpha K(T_m - T_s)}{\rho C_p \sqrt{\pi \kappa}} \right] \sqrt{t}$$

Finally, remembering that $K = \kappa \rho C_p$:

$$\Delta \ell = \frac{\sqrt{\kappa}}{\sqrt{\pi}} 2\alpha(T_m - T_s) \sqrt{t}$$

This means that the amount of “shrinkage” during cooling should simply be proportional to the square root of time. It would be the amount of subsidence that we would expect to see in cooling oceanic lithosphere but, because there is water on top of the ocean floor, there is an amplification effect (the water adds a load) that we will learn how to calculate in a few weeks. For now we can just stick it in to get:

$$\Delta \ell = \frac{\sqrt{\kappa}}{\sqrt{\pi}} 2\alpha(T_m - T_s) \left(\frac{\rho_m}{(\rho_m - \rho_w)} \right) \sqrt{t}$$

where ρ_m and ρ_w are the densities of the mantle and seawater, respectively.

We can verify how well this works by looking at the depth of sea-floor of different ages. The figure above shows the square-root-time depth-age relationship is excellent out to about 60 m.y. or so. After that it doesn't work so well and the ocean floor does not subside as fast as the error function model suggests. This is usually attributed to processes that limit the cooling at depths between 100 and 125 km depth beneath the oceanic lithosphere, but they are not well understood. This could be quite important in terms of the evolution of other planetary lithospheres where the only way to keep the planet losing heat rapidly, if no plate tectonics or planetary

resurfacing, is by keeping the lithosphere thin by some process that acts on the base of the lithosphere and does not let it thicken beyond some point.

Concepts for Heat Convection

Density and Temperature

Mantle heat convection is driven by the presence of hotter rocks at great depth than are present at shallow depth. Of course, it is not the temperature itself, but rather the density of the rocks, that is important for convection. Density decreases as a function of temperature and, for a given package of rock, can be related to temperature through the coefficient of thermal expansion, α :

$$\rho = \rho_0 (1 - \alpha T) \quad \text{or} \quad d\rho = - \rho_0 \alpha dT$$

where ρ_0 is the density of the material at $T=0$. The coefficient of thermal expansion is generally pretty small, so the change in density due to temperature changes in the earth or planets is very small compared to the total density. For example, for the earth's upper mantle, α is around $3 \cdot 10^{-5} \text{ } ^\circ\text{C}^{-1}$. For a temperature change of 2000°C , the change in density would be around 3%.

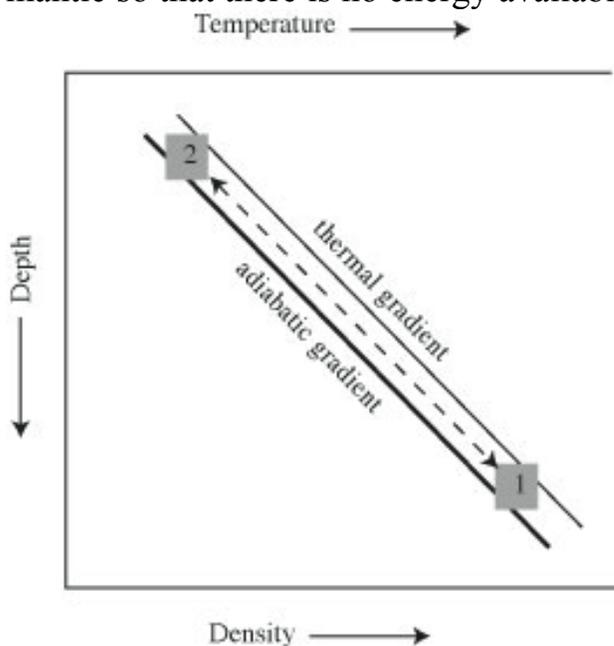
Density depends much more strongly on pressure than on temperature. In the earth's mantle, density increases downward because the increasing pressure has a much larger effect on density than does the increasing temperature. This leads rise to the concept of the "adiabatic gradient".

The Mantle Adiat

Suppose that we take a package of hot, high pressure mantle from deep in the mantle (1) and bring it to a shallower level (2) without letting it exchange any heat with the surrounding rocks. Because the pressure drops as the rock is uplifted through the mantle, the volume expands and its temperature and its density decrease. This change in temperature as a function of depth is the adiabatic gradient.

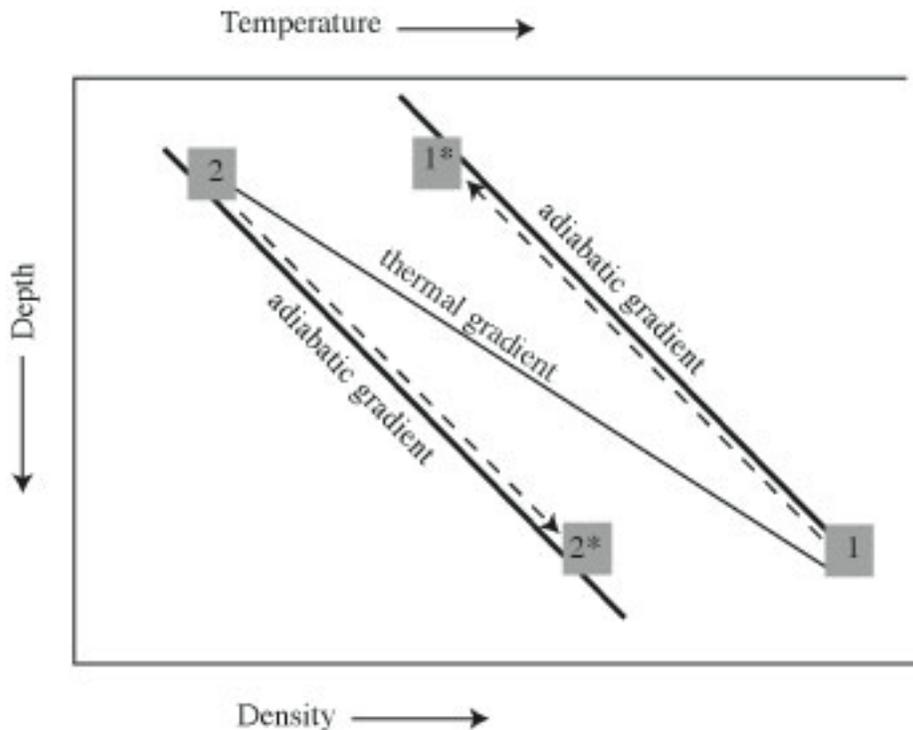
If the adiabatic gradient and the geothermal gradient are identical, then the uplifted rock at (2) will be at the same temperature as its surroundings before

and after uplift. Now suppose we take a second package of rock at (2) and bring it deeper into the mantle (1), also without letting it exchange heat with the surroundings (this is called adiabatic transport). Its temperature and its density will likewise increase along the same adiabatic gradient. If the adiabatic gradient and the geothermal gradient are identical, then this rock will also be at the same temperature as its surroundings having been carried to a greater depth. This means that for a geothermal gradient that is equal to the adiabatic gradient, moving material around in the mantle (without letting it gain or lose heat) does not change the temperature or density of the mantle as a function of depth. Thus it does not change the potential energy of the mantle so that there is no energy available to drive motion in the mantle.



Now consider a geothermal gradient in the mantle that is greater than the adiabatic gradient. A rock uplifted from (1) to (1*) via adiabatic transport will decrease in temperature and density, as before. However, after transport, it will be hotter than the surrounding rocks, but is at the same pressure as its surroundings so it will have a lower density. Likewise a rock transported adiabatically to greater depth, from (2) to (2*), will increase in temperature and density. But, it will be colder and denser than surrounding rocks. Thus vertical motion of rock packages in a mantle where the thermal gradient is greater than the adiabatic will create a mantle colder at the bottom and hotter near the top. It will also change the density structure so that rocks near the bottom are denser and rocks near the top are less dense. In doing this it decreases the potential energy of the mantle. (Remember

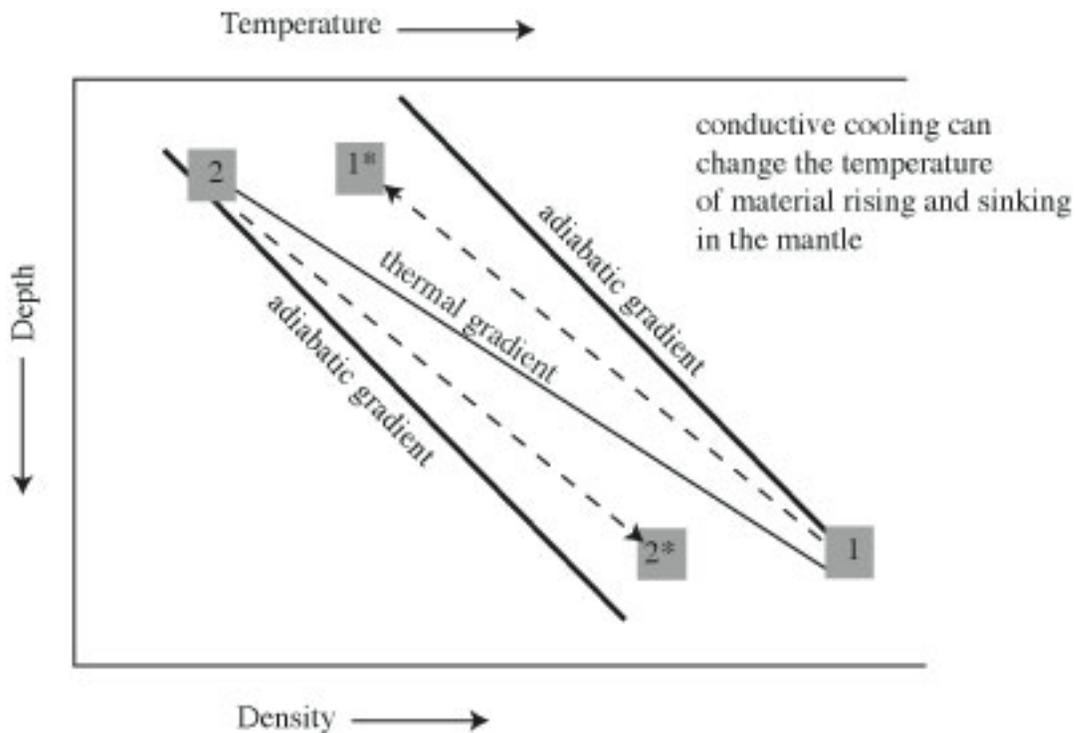
$PE = \rho gh \Delta z$ for a slice of material of thickness Δz and at height h above some reference horizon.)



This means that if the thermal gradient is greater than the adiabatic gradient, there is energy available to drive vertical motion of rock packages in the mantle. This thermally-driven process of overturn is what is known as thermal convection. It is important to notice that it is only the difference in temperature between the geotherm (temperature profile in the earth as a function of depth) and the adiabat (the curve whose slope is the adiabatic gradient at all depths) that can be used to drive convection. A mantle with a geotherm at or below the adiabat cannot convect and can only lose heat by the slow process of conduction.

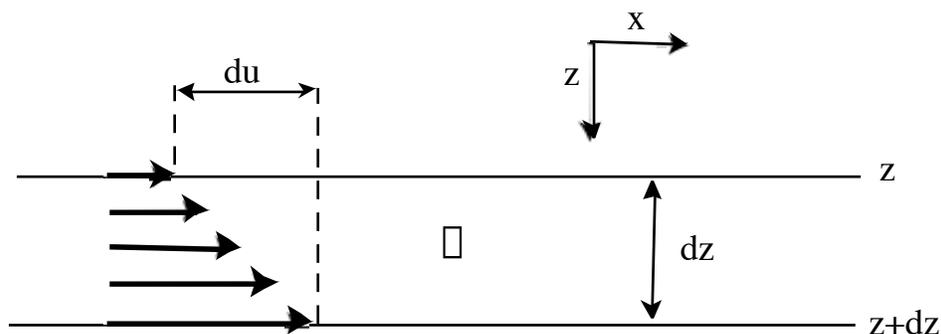
Finally, if rising or falling of a rock package in the mantle is not instantaneous, a rising package will lose some of its excess heat to the surrounding mantle during ascent, and vice versa. Thus its temperature will follow a path somewhere between the adiabat and the mantle geotherm. If the package rises quickly or if the diffusion of heat is slow (meaning that the thermal diffusivity of the material, κ , is small) then it will lose only a little heat and its path will be close to the adiabat. If it rises very slowly or has

rapid heat diffusion, its path will lie closer to the geotherm. This means that rapid rates of thermal conduction/diffusion in the mantle act to hinder convection.



Viscous Deformation

At the high temperatures needed for convection, planetary mantle material deforms ductilely. The mathematically simplest form of ductile deformation, which we will use here, involves “linear” or “Newtonian” viscosity. For a layer of material with viscosity μ , undergoing simple shear with velocity \mathbf{u} as shown below, the stress (force per unit area, typically measured in units of Pascals, or Pa, defined as N/m^2 or kg/ms^2) is linearly proportional to the spatial gradient of velocity.



If the velocity does not change along the length of the channel (ie $\partial u/\partial x = 0$) then stress along a horizontal plane is related to the velocity gradient by:

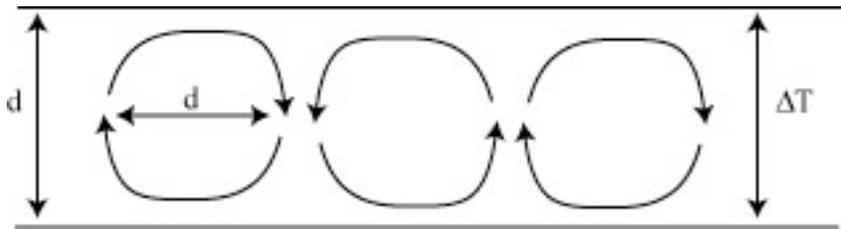
$$\sigma_{xz} = \mu \partial u/\partial z$$

This relationship shows that the stress required to drive flow increases with linearly with the viscosity of the material. (For non-Newtonian viscosity, there is a power-law relationship between stress and strain.)

The Rayleigh Number

The Rayleigh number is a dimensionless number that describes the vigor of convection within a convecting layer. It is essentially the ratio of the parameters that increase the vigor of convection to those that inhibit convection.

Consider a potentially convecting layer that looks something like this:



where the rolls show upwelling and downwelling limbs of a convective system. The thickness of the layer is d and the temperature difference across the layer (only that in excess of the adiabat) is ΔT . The parameters that affect subduction are:

κ	thermal diffusivity (m^2/s)
α	coefficient of thermal expansion ($^{\circ}\text{C}^{-1}$)
ΔT	temperature difference across the layer ($^{\circ}\text{C}$)
μ	viscosity ($\text{Pa s} = \text{Ns}/\text{m}^2 = \text{kg}/\text{ms}$)
d	layer thickness and approx. dimension of convection cell (m)
ρ	density (kg/m^3)

Helping Convection

Convection is aided by the temperature difference from top to bottom of the slab (after subtracting out the adiabat). The resulting density difference that can be used to drive convection is therefore: $\alpha\rho\Delta T$.

Increased layer thickness d enhances convection because: (a) the potential energy available to drive flow increases with d^2 (P.E.= $\rho g d^2/2$). (b) For a fixed flow geometry, the spatial gradient of flow is proportional to stress. Thus the flow velocity caused by an applied stress will scale linearly with the dimensions of the system, giving another factor of d .

The potential energy available for convection scales linearly with the magnitude of the gravitational field, g .

Hindering Convection

Larger viscosities mean that larger stresses are needed to cause flow at a given rate, so convection is hindered by larger viscosity, μ .

Rapid heat diffusion hinders convection because packets lose heat conductively during ascent (and vice versa for sinking), reducing the density contrast and energy to drive convection. Thus convection is hindered by larger thermal diffusivity, κ .

Dividing the enhancing factors by the retarding factors for convection gives the Rayleigh number, Ra :

$$Ra = d^3 \alpha \rho g \Delta T / \mu \kappa$$

We will leave it to the homework to show that this number is dimensionless. Experiment (laboratory and computer) show that convection will occur only when the Rayleigh number is in excess of the range 200-1000. The exact value of Ra at which convection will begin depends on the exact geometry and boundary conditions of the system, e.g rigid or stress-free upper boundary, whether heat is generated internally or input from below, etc. Because the geophysical equations that govern fluid flow are very complicated (fourth order PDE) the Rayleigh number will give us a useful insight into whether convection can occur and how vigorous it might be in planetary interiors.