12.002 Physics and Chemistry of the Earth and Terrestrial Planets
Fall 2008

<u>The Error Function</u>

We will use the conduction equation in a few more classes to relate lithospheric thickness to time and temperature so we might as well right down one very useful time-dependent solution, called the error function and written erf(y).

$$erf(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-y'^2} dy'$$

This function has the property that erf(0)=0, erf(+infinity)=1.

Consider now the function:

$$T(z,t) = T_s + (T_{in} - T_s) erf(z/2\sqrt{\kappa t}) = T_s + \frac{2(T_{in} - T_s)}{\sqrt{\pi}} \int_0^{z/2\sqrt{\kappa t}} e^{-y'^2} dy'$$

This function has the property that its value at z=0 is T_s at all times, its value at t=0 is T_{in} for all values of z>0, and it is a solution to the heat conduction equation. (We will leave it to the homework to show that it is a solution and to calculate the rate of conductive cooling of a planetary surface.) Later on we will use this expression to calculate the thermal behavior of the oceanic lithosphere on earth.

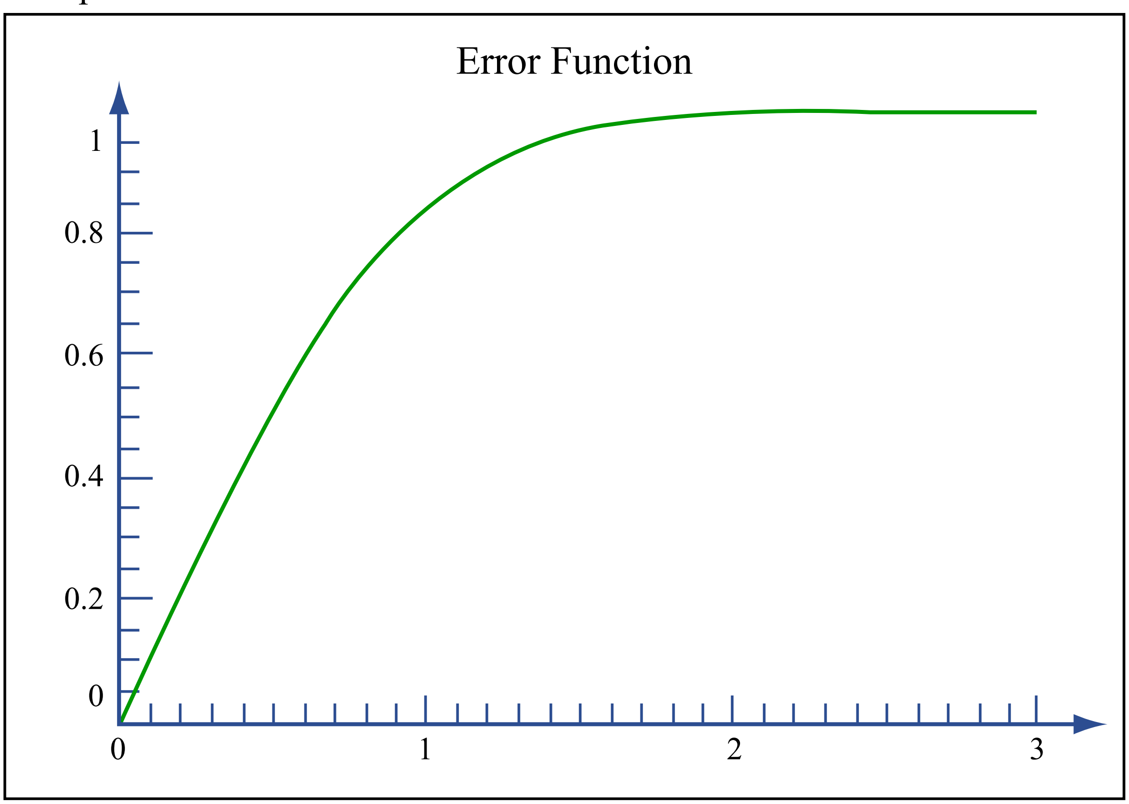

Figure by MIT OpenCourseWare.