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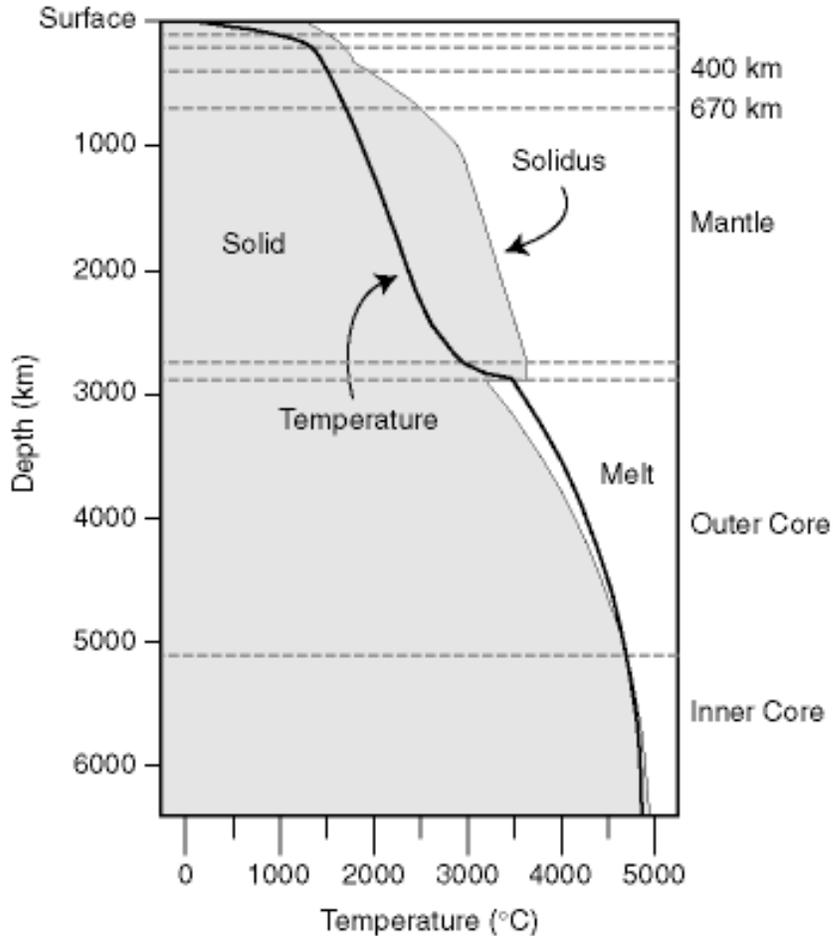
12.002 Physics and Chemistry of the Earth and Terrestrial Planets
Fall 2008

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Planets as Heat Engines

In the earliest days of the solar system, all the terrestrial planets and moons were exceedingly hot – most were probably entirely molten. Today they are much cooler, although only the internal temperature of the earth is well known. Sparse data are available for the mantles of the moon, Venus and Mars, largely from the chemistry of igneous rocks now at the surface.

Rocks within the interiors of planets also contain various amounts of radioactive elements, mainly uranium, thorium and potassium. The core probably contains rather little in the way of these heat producing elements, while the mantle contains much more. On earth, the crust is highly enriched in radiogenic elements compared to the mantle. Heat is also generated by freezing of the core, through the heat needed to turn fluid outer core into solid inner core. The moon and Mars probably have solid cores. The states of the cores of Mercury and Venus are unknown, although there are a few data that bear on the question (magnetic field strength).



http://eqseis.geosc.psu.edu/~cammon/HTML/Classes/IntroQuakes/Notes/Images_specific/Geotherm.gif

Courtesy of Prof. Charles J. Ammon, Penn State University. Used with permission.

Mechanisms for Planetary Heat Loss

Conduction (diffusion of heat through material)

Convection (advection is heat carried by movement of material, called convection when movement is thermally driven)

Radiation (loss of heat through generation of electromagnetic radiation at planetary surface)

Within planetary interiors, heat transfer occurs by convection and by convection. Which is more efficient, and which operates where?

First, define heat flow (q): the amount of heat (joules) per unit time (seconds) through a unit area of surface (square meters). Heat flow in the earth is measured in units of mW/m^2 , noting that a watt=joule/sec. Typical values for the outermost layer of the earth are between about $q=40 mW/m^2$ and $q=100 mW/m^2$.

Conductive heat flow is always proportional to the thermal gradient in a material, with a proportionality constant called the thermal conductivity, \mathbf{K} , measured in units of W/mK:

$$q_{\text{conductive}} = \mathbf{K} (\partial T / \partial z)$$

Typical values for the thermal gradient in crustal rocks are between 10°C/km and 50°C/km.

Convective heat flow is computed as the velocity of the material moving through a unit surface area, \mathbf{v} , times the temperature of the material times the heat capacity of the material. Heat capacity per unit volume is equal to the heat capacity per unit mass (\mathbf{C}_p) times the material density, ρ . Thus:

$$q_{\text{convective}} = \mathbf{v} T \rho C_p$$

In general, convective heat loss in large bodies is much more efficient than conductive heat loss, especially over distances of more than 100 km. This is small compared to the radius of most terrestrial bodies, so it is difficult for the planets to cool much by conduction. But, in order for convection to occur, the interior of the planet has to be soft and weak enough that the “heat engine” of the planet can drive ductile flow within the interior. Planets are not hot enough in the upper tens to hundreds of kilometers for ductile flow to occur, which needs temperatures somewhere above 1300°C even in the shallow mantle. This means that all of the terrestrial planets have a strong outer shell that does not deform in a ductile fashion (it could have faults). Depending on the temperature of the planetary interior, there may or may not be a ductile region in the deeper part of the planet’s mantle. This strong outer layer is called the **lithosphere** (lithos means rock). It is not the same as the crust. It usually includes all of the planetary crust and some of the uppermost mantle. In the earth, the tectonic plates consist of lithosphere.

Mantle convection is driven by the density inversion in the mantle. All things being equal, hot material is less dense than cold material. Thus deeper and hotter parts of the mantle are more buoyant and tend to rise, while shallower and colder parts of the mantle are less buoyant and tend to sink. Although the details are a bit more complicated, this is the basic idea behind thermal convection.

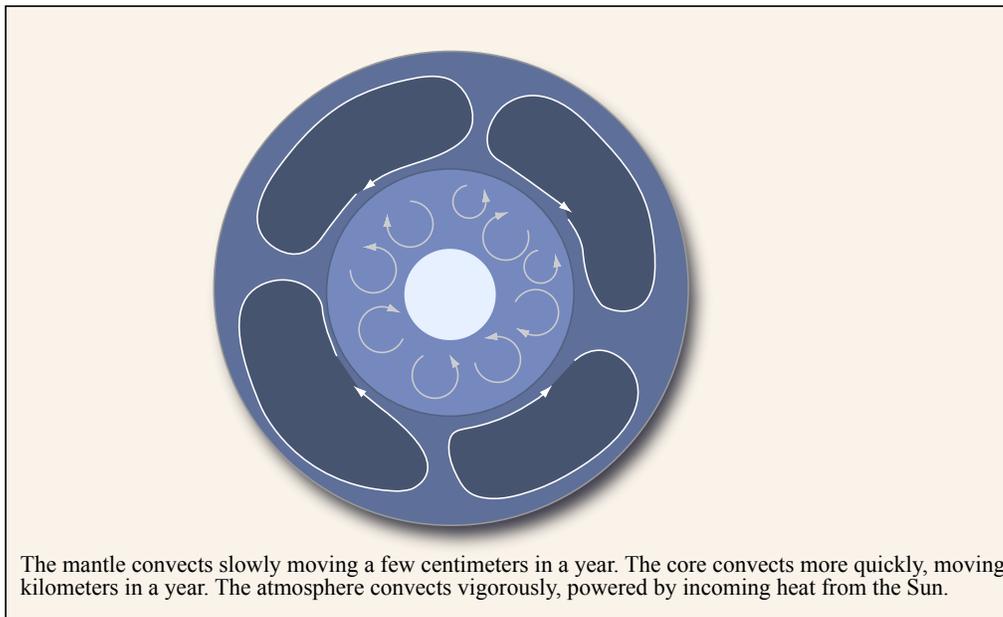
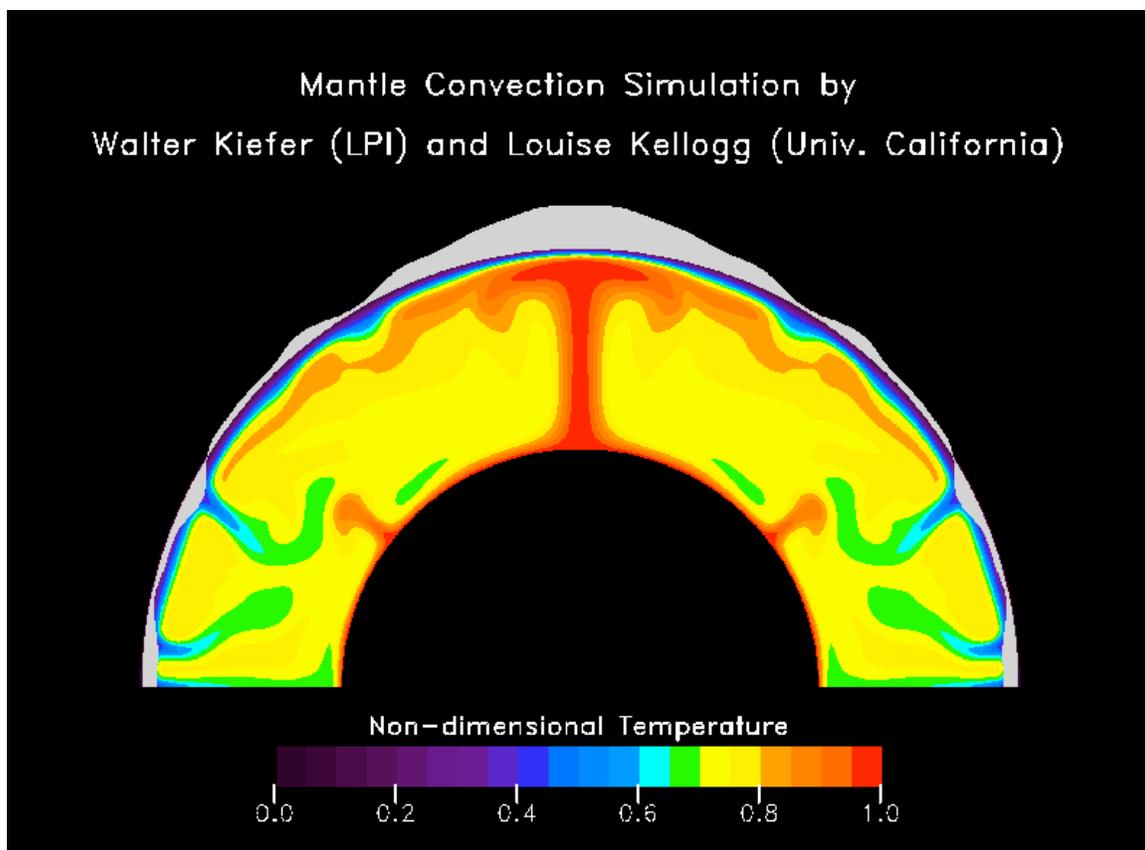


Figure by MIT OpenCourseWare.

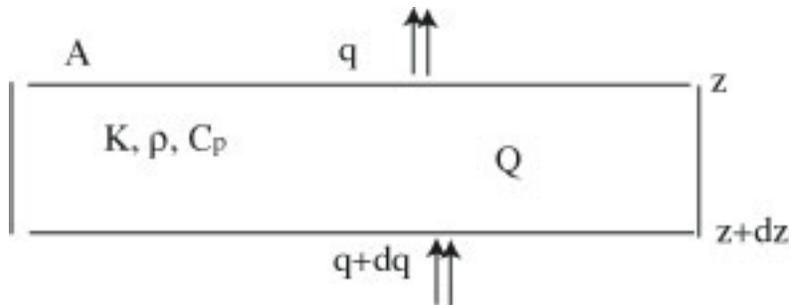


Courtesy of Walter S. Kiefer, Lunar and Planetary Institute. Used with permission.
<http://www.lpi.usra.edu/science/kiefer/Research/convect4FS.gif>

Math for Heat Conduction

Heat conduction, in one dimension, is governed by a partial differential equation that has derivatives in time and depth. We are going to ignore the effect of radiogenic heat production for our class work, although it can be very important in crustal rocks.

Consider a layer of material conducting heat, with a heat flow out the top of q and a heat flow in the bottom of $q+dq$.



The total heat in the box, Q , is equal to the temperature in the box, T , times the heat capacity per unit volume, ρC_p , times the volume ($A dz$):

$$Q = T \rho C_p A dz$$

Because everything is a constant except T , the rate of change of heat in the box is just:

$$(\partial Q / \partial t) = (\partial T / \partial t) \rho C_p A dz$$

The rate of change of heat in the box is also equal to the heat flowing into the bottom of the box, $(q+dq)A$, minus the rate of heat flowing out of the top of the box, qA . So:

$$(\partial Q / \partial t) = (q+dq)A - qA$$

Setting the right hand sides of these equations equal we get:

$$(\partial T / \partial t) \rho C_p A dz = (q+dq)A - qA$$

or:

$$(\partial T / \partial t) \rho C_p = [(q+dq) - q] / dz = \partial q / \partial z$$

Substituting the relationship between conductive heat flow, thermal conductivity and heat flow:

$$q = K (\partial T / \partial z)$$

gives:

$$(\partial T / \partial t) \rho C_p = \partial / \partial z [K (\partial T / \partial z)] = K (\partial^2 T / \partial z^2)$$

or:

$$(\partial T / \partial t) = \kappa (\partial^2 T / \partial z^2)$$

where $\kappa = K / \rho C_p$ is the thermal diffusivity of the material, in units of m^2/s . This is the general equation for the conduction of heat in a stationary medium without heat production and with uniform thermal diffusivity. In the earth's crust and mantle, a ballpark value for thermal diffusivity is $\kappa = 10^{-6} \text{ m}^2/\text{s}$, at least enough to make some approximate calculations.

A quick way of estimating how far, ζ , a significant change in temperature can propagate in a given amount of time, τ , can be estimated by writing:

$$(dT/\tau) = \kappa (dT/\zeta^2) \quad \text{or} \quad \zeta = (\kappa\tau)^{1/2}$$

putting in a variety of values for time (τ) and $\kappa = 10^{-6} \text{ m}^2/\text{s}$ gives characteristic conduction distances of:

τ	ζ
1 m.y.	5.6 km
100 m.y.	56 km
10 Gy	560 km

Since the planets are about 4.5 Gy old, cooling via thermal conduction can not have penetrated more than a few hundred kilometers downward from the surface. This means that if the planetary interiors have cooled at greater depth, they must have done so by convection. If convection were to be inhibited in the terrestrial planets, they could not cool significantly and

would remain near the same temperature, at least on the timescale of the solar system.

Steady-State Conductive Solution

One simple solution to the heat conduction equation, which is obvious from inspection, and is the “steady-state” or “equilibrium” solution when $\partial T/\partial t = 0$. This gives:

$$(\partial T/\partial t) = 0 = \kappa (\partial^2 T/\partial z^2)$$

or:

$$T = T_0 + bz$$

The steady-state solution is just a linear function of z and corresponds to a heat flow $q = Kb$. For now, we can make a table of the time it takes for a given thickness of rock (mantle) to approach a steady-state linear geotherm if we don't perturb it in any way except to let it cool conductively.

<u>time to steady-state</u>	<u>layer thickness</u>
1 m.y.	3 km
100 m.y.	30 km
10 Gy	300 km

The Error Function

We will use the conduction equation in a few more classes to relate lithospheric thickness to time and temperature so we might as well right down one very useful time-dependent solution, called the error function and written $\text{erf}(y)$.

$$\text{erf}(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-y'^2} dy'$$

This function has the property that $\text{erf}(0)=0$, $\text{erf}(\text{infinity})=1$, and it is a solution to the heat conduction equation. We will leave it to the homework to show that it is a solution and to calculate the rate of conductive cooling of a planetary surface. Later on we will use this expression to calculate the thermal behavior of the oceanic lithosphere on earth.

