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12.002 Physics and Chemistry of the Earth and Terrestrial Planets  
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$d_m$  is the distance that the wave travels in the mantle,  $d_c$  is the distance that it travels in the core.

Using Snell's law:

$$\frac{\sin(90^\circ - \delta/2)}{v_c} = \frac{\cos(\delta/2)}{v_c} = \frac{\sin(\beta)}{v_m}$$

the sine law for triangles:

$$\frac{\sin(\alpha)}{r} = \frac{\sin(180^\circ - \beta)}{R} = \frac{\sin(\beta)}{R}$$

and the fact that all angles in a triangle sum to  $180^\circ$ :

$$\alpha + (180^\circ - \beta) + \frac{\Delta}{2} - \frac{\delta}{2} = 180^\circ$$

or

$$\frac{\Delta}{2} = \frac{\delta}{2} + \alpha - \beta$$

and substituting in for everything, we can solve for  $\Delta$  in terms of  $\delta$ .

$$\Delta = \delta + 2 \sin^{-1} \left[ \frac{v_m \cos(\delta/2)}{v_c} \right] - 2 \sin^{-1} \left[ \frac{v_m r \cos(\delta/2)}{v_c R} \right]$$

The distance traveled by the wave in the core is:

$$d_c = 2r \sin(\delta/2)$$

and the distance traveled by the wave in the mantle (both legs) is:

$$d_m = 2 \left[ R^2 + r^2 - 2rR \cos(\Delta/2 - \delta/2) \right]^{1/2}$$

yielding the total travel time:

$$t = \frac{d_c}{v_c} + \frac{d_m}{v_m} = \frac{2r \sin(\delta/2)}{v_c} + \frac{2[R^2 + r^2 - 2rR \cos(\Delta/2 - \delta/2)]^{1/2}}{v_m}$$

The smallest value of  $\delta$  at which PKP arrivals occur can be determined by looking at this equation:

$$\Delta = \delta + 2 \sin^{-1} \left[ \left( \frac{v_m}{v_c} \right) \cos(\delta/2) \right] - 2 \sin^{-1} \left[ \left( \frac{rv_m}{Rv_c} \right) \cos(\delta/2) \right]$$

The argument of  $\sin^{-1}$  has to be less than or equal to one, so the smallest value that  $\cos(\delta/2)$  can have is  $v_c/v_m$  if  $v_m > v_c$ . and  $\delta = 0$  if  $v_c > v_m$ .

In the first case,  $v_m > v_c$ , this gives a minimum value of  $\Delta$ :

$$\Delta_{\min} = 2 \cos^{-1} \left( \frac{v_c}{v_m} \right) + 180^\circ - 2 \sin^{-1} \left( \frac{r}{R} \right)$$

In the second case, it is more difficult to compute a minimum value of  $\Delta$  because it does not necessarily, or even usually, correspond to the minimum value of  $\delta$ .

For  $v_m > v_c$  there will be a gap where neither P nor PKP is observed and for  $v_c > v_m$  there will be a zone of overlap where both P and PKP are observed.

We can extend this kind of analysis to three spherical shells to include an inner core. Skipping the geometry and the math (doing basically the same thing as we did for two shells) we find:

$$\Delta = \phi + 2 \sin^{-1} \left[ \frac{v_m s \cos(\phi/2)}{v_i r} \right] + 2 \sin^{-1} \left[ \frac{v_c \cos(\phi/2)}{v_i} \right] - 2 \sin^{-1} \left[ \frac{sv_m \cos(\phi/2)}{Rv_i} \right] - 2 \sin^{-1} \left[ \frac{v_c s \cos(\phi/2)}{rv_i} \right]$$

where  $\phi$  is the angular distance that the i-wave travels in the inner core,  $v_i$  is the velocity of the inner core and  $s$  is the radius of the inner core. It is also

useful to define  $\delta$ , as before, as the angular distance traveled by the K-wave in the outer core:

$$\delta = \phi + 2 \sin^{-1} \left[ \frac{v_c \cos(\phi/2)}{v_i} \right] - 2 \sin^{-1} \left[ \frac{v_c s \cos(\phi/2)}{v_i r} \right]$$

The distances traveled by the wave in the mantle (both legs), outer core and inner core, are:

$$\text{mantle} \quad d_m = 2 \left[ R^2 + r^2 - 2rR \cos(\Delta/2 - \delta/2) \right]^{1/2}$$

$$\text{outer core:} \quad d_c = 2 \left[ r^2 + s^2 - 2rs \cos(\delta/2 - \phi/2) \right]^{1/2}$$

$$\text{inner core:} \quad d_i = 2s \sin(\phi/2)$$

yielding a total travel time for PKiKP:

$$t = \frac{d_c}{v_c} + \frac{d_m}{v_m} + \frac{d_i}{v_i}$$

The PREM (Preliminary Reference Earth Model) shows the average P and S wave velocities for the Earth as a function of radial distance. In general, velocities increase downward with obvious discontinuities at the core-mantle and inner core-outer core boundaries. There are also important discontinuities within the upper mantle (at and above 660 km depth) and at the crust-mantle boundary (called the Moho). The general increase in seismic velocities downward is due to increasing pressure as a function of depth in the earth. This increases the density, which should lower seismic velocity, except that the elastic moduli increase with pressure faster than density increases with pressure.

In general, temperature increases downward in the earth, and this acts to partially counterbalance the effects of increasing pressure because hotter materials generally have slower seismic velocities than colder materials. One interval where the temperature effect on seismic velocity becomes quite important is just beneath the lithosphere, in the depth range of about 200-300 km. This is often called the “low velocity zone” because

seismic velocities here are commonly slower than in the material above or below.

The discontinuity in velocity between the inner and outer core is due to the change in the state of matter (liquid to solid) at this boundary. The smaller discontinuities that occur in the lower mantle between 410 and 660 km depth are due to mineral phase transitions, primarily in the mineral olivine. In general, the uppermost mantle is composed of olivine and pyroxine (together forming a rock called peridotite). The phase transitions in olivine occurs first at 410 km where olivine is transformed to spinel, a mineral that is more dense than olivine, and then finally at 660 km where the last transformation occurs to a mineral called perovskite, which has a very high density. The lower mantle is thought to be about 80% perovskite and 20% a somewhat less dense mineral called magnesiowustite.

Shallow Mantle: olivine + pyroxine  
 $(\text{Mg,Fe})\text{SiO}_4$  +  $(\text{Mg,Fe})\text{SiO}_3$

410 km depth  $\beta$ -spinel (Wadsleyite)

500 km depth  $\gamma$ -spinel (Ringwoodite)

660 km depth olivine goes to perovskite (80%)+magnesiowustite (20%)  
 $(\text{Mg,Fe})\text{SiO}_3$  +  $(\text{Mg,Fe})\text{O}$

At zero pressure and 270°C, the densities of olivine and perovskite are 3400 kg/m<sup>3</sup> and 4110 kg/m<sup>3</sup>, respectively.