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12.002 Physics and Chemistry of the Earth and Terrestrial Planets
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Seismic Phases and Travel Times

Seismic body waves (P and S) travel through the crust, mantle and cores of planetary bodies. These waves are given different names when they travel through the core. P waves in the outer core are called K waves, although we still refer to the P wave velocity of the core. In the earth, there are no S waves propagating through the outer core because the outer core is a liquid and cannot sustain shear stresses. In this class we won't worry about the names for P and S waves propagating through the inner core.

Unlike petrology, which has random names for every mineral and rock imaginable, the travel paths and phases of seismic waves are simply named. For example, PKP is a P-wave that travels through the mantle, is refracted into the outer core as a P (K) wave, and then refracted back into the mantle as a P wave. Similarly, SKS is an S-wave that travels through the mantle, is refracted into the outer core as a P (K) wave, and then refracted back into the mantle as a S wave. At every large seismic velocity interface within a planetary interior, especially at the core/mantle boundary and inner core/outer core boundary, waves are refracted and or reflected. Thus an incoming P wave will, at a velocity interface, be reflected and/or refracted as another P phase and another S phase (although S will not propagate in a liquid, but it is still created at the interface, it just doesn't go anywhere).

The laws governing the reflection/refraction angles are exactly the same as the laws you learned for optical reflection and refraction of light, and can be derived from the same basic conservation laws of momentum and energy. This rule is called "Snell's law" and related the seismic velocity of a P or S wave to the angle at which it leaves and enters an interface. If an incoming seismic wave with velocity v_1 encounters an interface at an angle θ_1 relative to the perpendicular, then the refraction angle for a wave with seismic velocity v_2 on the other side of the interface is, by Snell's law:

$$\frac{\sin(\theta_1)}{v_1} = \frac{\sin(\theta_2)}{v_2}$$

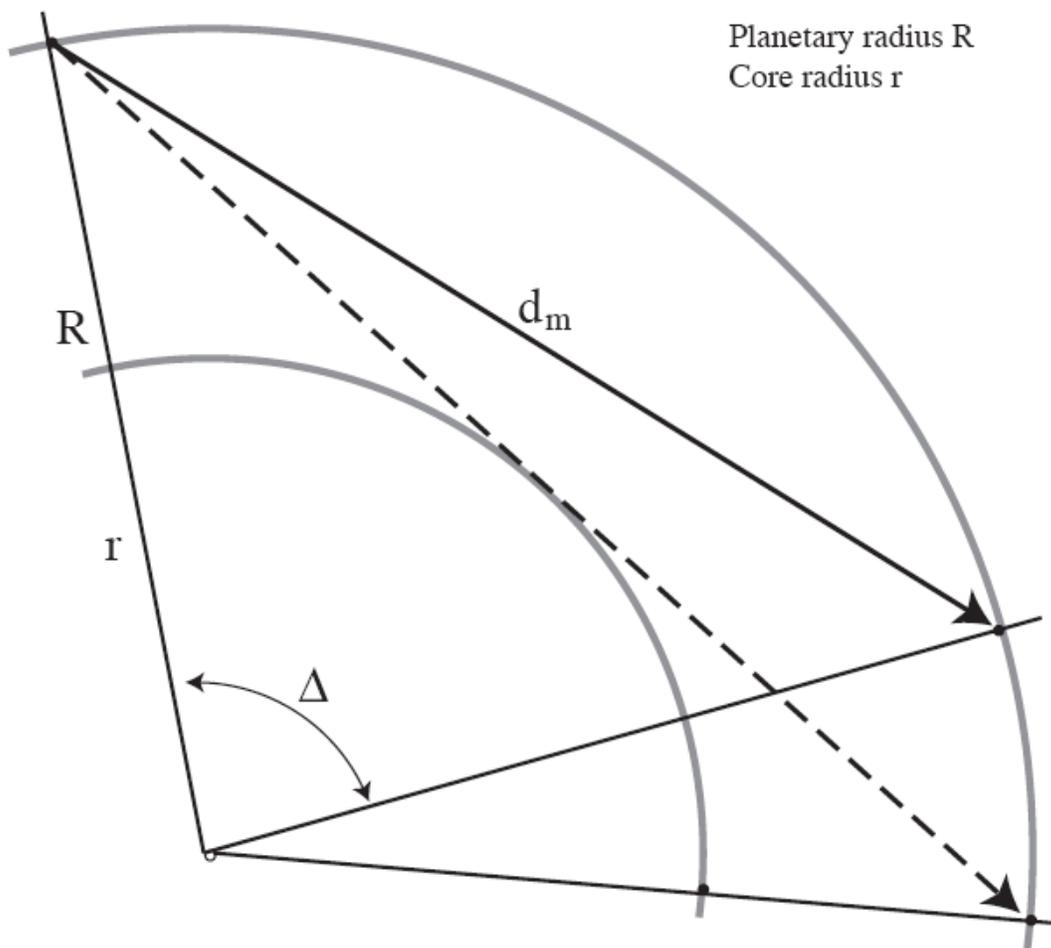
If v_2 is greater than v_1 , there is a critical incidence angle beyond which incoming waves will be reflected rather than refracted:

$$\sin(\theta_{1crit}) = \frac{v_1}{v_2}$$

If v_1 is greater than v_2 , there is a maximum angle that the refracted waves can obtain:

$$\sin(\theta_{2max}) = \frac{v_2}{v_1}$$

Determining the seismic velocities of layers within a planetary interior is a matter of having enough distributed seismic sources (usually earthquakes, but can also be nuclear explosions or for local studies dynamite). Let's consider the simple case where the planetary mantle has a uniform seismic velocity (and ignore the crust).



Then the geometry of the P wave traveling through the mantle, and the distance d_m traveled by the wave, will be related to the angular distance Δ from source to receiver by:

$$d_m = 2R \sin(\Delta/2)$$

and the travel time for the wave will be:

$$t_p = \frac{d_m}{v_m} = \frac{2R \sin(\Delta/2)}{v_m}$$

There will be a maximum angular distance over which direct P arrivals can occur. Beyond this angle the core will be in the way of potential P phases traveling only within the mantle. This critical angle is $\sim 100^\circ$ for the earth. For a uniform velocity mantle, the critical angular distance is:

$$\frac{r}{R} = \cos(\Delta_{crit}/2)$$

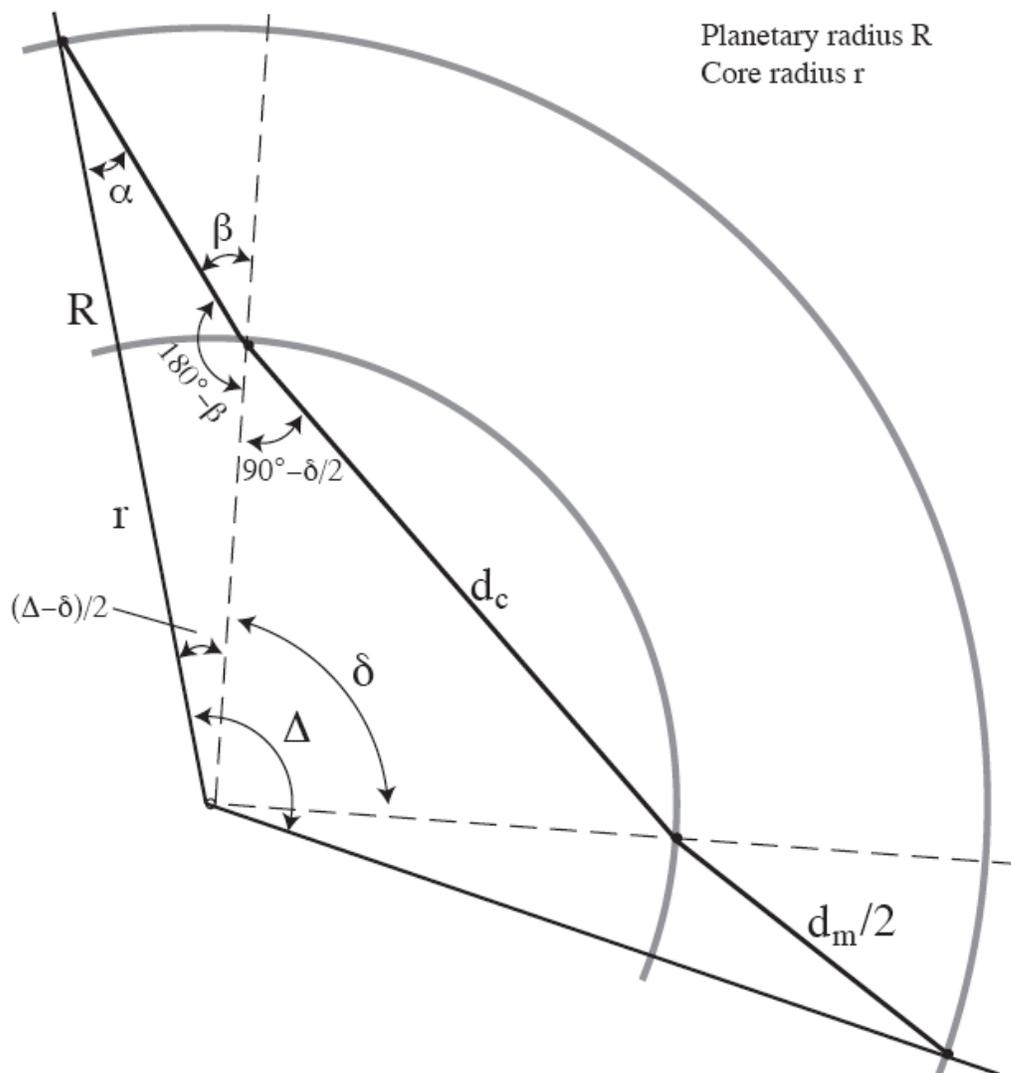
Beyond this angle, P waves will either be reflected off the core/mantle interface or refracted into the core, depending on the incident angle of the P wave and the relative magnitudes of seismic velocity in the core and mantle (ie Snell's law). The same relationships are correct for S waves incident on the core.

In the Earth, and probably also the other terrestrial bodies, the seismic velocity is not uniform through the core or mantle, but increases downward due to the increased pressure acting on the material through which the seismic waves propagate. Increasing density actually acts to reduce seismic wavespeeds (see equations above) but the elastic moduli (λ and μ) increase more rapidly downwards than the density, so that the wavespeeds increase with depth except at compositional interfaces or those where there is a change of state (outer to inner core) or crystal structure (at the 660 km discontinuity in the mantle).

In practice, this means that, for example, P waves do not travel through the mantle in a straight line but are concave upwards because of Snell's law. We will not worry about the details of this in this class, and assume that we have simple, uniform velocity (outer) cores and mantles in our planets.

core (similar to having a zero velocity). Even in the inner core the S velocity is slower than the mantle S velocity.

Computing the core velocity can be messy if you do it by hand, even for a simple planetary structure with uniform core and mantle velocities. It involves some annoying geometry of triangles and so on. It is most easily done by formulating everything in terms of an angle δ , which is the angular distance traveled by the P (K) wave in the core. We can also use the fact that everything is symmetric on the down-going and up-going part of the path. Some useful angles are labeled in the diagram below.



d_m is the distance that the wave travels in the mantle, d_c is the distance that it travels in the core.

Using Snell's law:

$$\frac{\sin(90^\circ - \delta/2)}{v_c} = \frac{\cos(\delta/2)}{v_c} = \frac{\sin(\beta)}{v_m}$$

the sine law for triangles:

$$\frac{\sin(\alpha)}{r} = \frac{\sin(180^\circ - \beta)}{R} = \frac{\sin(\beta)}{R}$$

and the fact that all angles in a triangle sum to 180° :

$$\alpha + (180^\circ - \beta) + \frac{\Delta}{2} - \frac{\delta}{2} = 180^\circ$$

or

$$\frac{\Delta}{2} = \frac{\delta}{2} + \alpha - \beta$$

and substituting in for everything, we can solve for Δ in terms of δ .

$$\Delta = \delta + 2 \sin^{-1}[\phi \cos(\delta/2)] - 2 \sin^{-1}[r^* \phi \cos(\delta/2)]$$

where $\phi = v_m/v_c$ and $r^* = r/R$.

The distance traveled by the wave in the core is:

$$d_c = 2r \sin(\delta/2)$$

and the distance traveled by the wave in the mantle (both legs) is:

$$d_m = 2 \left[R^2 - r^2 \phi^2 \cos^2(\delta/2) \right]^{1/2} - 2 \left[r^2 - r^2 \phi^2 \cos^2(\delta/2) \right]^{1/2}$$

yielding the total travel time:

$$t = \frac{d_c}{v_c} + \frac{d_m}{v_m} = \frac{2r \sin(\delta/2)}{v_c} + \frac{2[R^2 - r^2 \phi^2 \cos^2(\delta/2)]^{1/2}}{v_m} - \frac{2[r^2 - r^2 \phi^2 \cos^2(\delta/2)]^{1/2}}{v_m}$$

The smallest value of Δ at which PKP arrivals occur can be determined by looking at this equation:

$$\Delta = \delta + 2 \sin^{-1} \left[\left(\frac{v_m}{v_c} \right) \cos(\delta/2) \right] - 2 \sin^{-1} \left[\left(\frac{rv_m}{Rv_c} \right) \cos(\delta/2) \right]$$

The argument of \sin^{-1} has to be less than or equal to one, so the smallest value that $\cos(\delta/2)$ can have is v_c/v_m if $v_m > v_c$. and $\delta = 0$ if $v_c > v_m$.

In the first case, $v_m > v_c$, this gives a minimum value of Δ :

$$\Delta_{\min} = 2 \cos^{-1} \left(\frac{v_c}{v_m} \right) + 180^\circ - 2 \sin^{-1} \left(\frac{r}{R} \right)$$

In the second case, $v_c > v_m$, this gives a minimum value of Δ :

$$\Delta_{\min} = 2 \sin^{-1} \left(\frac{v_m}{v_c} \right) - 2 \sin^{-1} \left(\frac{v_m r}{v_c R} \right)$$

In the first case there will be a gap where neither P nor PKP is observed and in the second case there will be a zone of overlap where both P and PKP are observed.