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12.002 Physics and Chemistry of the Earth and Terrestrial Planets
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Mass and Moment of Inertia

The masses of the planets were not known until the 17th century, when Galileo, Kepler and Newton laid the groundwork for understanding the dynamics of planetary bodies. In 1686, using Kepler's laws of planetary orbits and his own laws of gravity, force and acceleration, Newton calculated that the mass of Jupiter was approximately 312 times larger than that of Earth (G was not known so it was not possible to calculate an absolute value for either mass). Modern observations show that this estimate is correct to better than ~3%.

The dimensions of the planets are more difficult to determine than the mass. They were first estimated from the occultation of stars by the planets and later by spacecraft data, radar-ranging and Doppler tracking from spacecraft. From the planetary mass and radius, the average densities of the planets are easily determined, but the distribution of mass, as a function of radius, is almost totally unconstrained.

The first estimates for the radial mass distribution within the planets were obtained from the moment of inertia of the planet rotating about its own axis. Typically, computing the moment of inertia depends on the fact that the planets are not spherical, but bulge outward at the equator due to the accelerations associated with rotation. In addition, the rotation axis of a planet is generally not perpendicular to the plane in which it rotates around the sun (or around a planet if it is a moon). This means that there is a torque on the non-spherically symmetric part of the rotating planet and the axis of rotation precesses like a gyroscope (only the motions are small). As for a gyroscope, the rate of precession of the rotation axis depends on the moment of inertia of the planetary body, so that the moment of inertia can be estimated from this precession rate. Moments of inertia can also be calculated from the quadrupole moment of the gravity field of a planet, but such data are only available for Earth, Moon, Mars and Venus.

The moment of inertia, together with the planetary mass, is useful for computing how mass is concentrated toward the center of a planet, and in particular in estimating the core size and/or density. The moment of inertia for a sphere of uniform density is $I = (2/5)MR^2$, where M is the mass of the sphere and R is the radius. If the moment of inertia is smaller than $(2/5)MR^2$, the planet is denser near the center than near the surface.

We can also write the moment of inertia in terms of density (ρ) rather than mass: $I=(8/15)\pi\rho R^5$. If instead of a uniform density sphere, we have a sphere with a uniform density spherical “core” with density ρ_χ and radius r_χ , and a uniform density mantle of density ρ_m , then the moment of inertia and the mass are:

$$I = \frac{8\pi}{15} \rho_m R^5 + \frac{8\pi}{15} (\rho_\chi - \rho_m) r_\chi^5$$

$$M = \frac{4\pi}{3} \rho_m R^3 + \frac{4\pi}{3} (\rho_\chi - \rho_m) r_\chi^3$$

It is typical for the moment of inertia of a planet to be expressed as a dimensionless number $L=I/MR^2$:

$$L = \frac{I}{MR^2} = \left(\frac{8\pi}{15} \right) \left[\frac{\rho_m R^5 + (\rho_\chi - \rho_m) r_\chi^5}{MR^2} \right]$$

This simplifies to $L=2/5$ for a uniform sphere (for $r_\chi=0$ or $r_\chi=R$). If the main density variations within the planet are due to a denser core and a less dense mantle, then L will be less than 0.4. And, if either the core density or the mantle density can be estimated, then the approximate radius of the planetary core can be determined from the mass and moment of inertia of the planet.

It can sometimes be useful to write L as:

$$L = \frac{I}{MR^2} = \left(\frac{2}{5R^2} \right) \frac{\rho_m R^5 + (\rho_\chi - \rho_m) r_\chi^5}{\rho_m R^3 + (\rho_\chi - \rho_m) r_\chi^3}$$

or

$$L = \frac{I}{MR^2} = \left(\frac{2}{5} \right) \left[\frac{1 + (\rho_\chi / \rho_m - 1)(r_\chi / R)^5}{1 + (\rho_\chi / \rho_m - 1)(r_\chi / R)^3} \right]$$

If we know the mass of the planet, then the minimum value of L will occur for a value of r_x that can be determined by setting the derivative of L , with respect to r_x , equal to zero.

	radius (km)	average density (kg/m^3)	L
Earth	6378	5515	.3308
Venus	6052	5243	.33
Mars	3396	3933	.376
Mercury	2440	5427	.33
Moon	1738	3350	.394

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<http://nssdc.gsfc.nasa.gov/planetary/factsheet/>