12.002 Physics and Chemistry of the Earth and Terrestrial Planets
Fall 2008

12.002 Physics and Chemistry of the Terrestrial Planets
Fall 2008
Professors Leigh Royden and Benjamin Weiss

**Problem Set #2: Moment of inertia**
due Friday Oct 3 in class

**Problem 1. Moment of Inertia, Mass and Core Size**

The density of the earth's mantle, an iron-magnesium silicate, has a near-surface density of approximately 3400 kg/m$^3$, while the density or iron, also measured at near-surface pressure, is approximately 8000 kg/m$^3$. This indicates that for the smaller terrestrial planets and moons, where pressure-induced phase changes are not important, the ratio of core density to mantle density is approximately 2.5. Assuming that the core and the mantle have uniform densities with a density ratio of exactly 2.5, plot the moment of inertia $L=I/mR^2$ as a function of core radius (e.g. as a function of $r_c/R$). At what value of $r_c/R$ is L a minimum?

**Problem 2. Moment of Inertia, Mass and Core Size**

The radius, average density (easily determined from mass divided by volume) and moment of inertia for Mars, Mercury and the Moon are as follows.

| | radius (km) | average density (kg/m$^3$) | L |
|---|---|---|---|
| Mars | 3396 | 3933 | .376 |
| Mercury | 2440 | 5427 | .33 |
| Moon | 1738 | 3350 | .394 |

(a) Using the plot you have constructed for Problem 1, estimate the core size for each of these planetary bodies. You should have two possible values of $r_c/R$ for each planet.
(b) Knowing that the mantle density and core density should be in the general vicinity of 3400 kg/m$^3$ and 8000 kg/m$^3$, find the unique value of mantle density, core density and core radius for each of these bodies.

**Problem 3. Yield Stress Envelopes.**

(a) The Moon is known, from the few seismometers left there by the Apollo Mission, to have Moon quakes over the depth range of 50-200 km. Assuming that the lithosphere of the Moon deforms in a brittle manner over this depth range and ductilely below this depth range, sketch (but neatly and to scale) what the yield stress diagram might look like for the Moon. (Use an arbitrary scale for stress difference on the horizontal axis because we have not yet discussed stress magnitudes.)

Use the estimate that ductile deformation will take over from brittle deformation at about 650°C in mantle rocks, that the average temperature of the Moon's surface is -25°C, and that the base of the Moon's lithosphere is 1300°C to scale your yield stress diagram appropriately with depth. You will also want to assume that the thermal gradient is linear through the Moon's lithosphere.

(b) Assuming that the thermal conductivity of the Moon's lithosphere is the same as that for the Earth (5 W/mK), and that the thermal gradient is linear through the lithosphere, use your results from part (a) to compute the surface heat flow through the surface of the Moon. How does this compare to the heat flow through the Earth's surface?

**Problem 4. "Fun" Heat flow Fact.**

People have a heat flow, or heat loss, that we can compute if we assume that all the food we eat is converted to heat and lost through our skin to the outside (this is not actually quite right, but never mind.) Suppose that you eat 2000 Cal per day. These are actually kilocalories, or $2 \cdot 10^6$ cal. Suppose you also have a surface area of 2 $m^2$. What is the heat flow out of your body? How large an area on the surface of the earth would lose the same amount of heat per day as you do? Would you want to play soccer on something this size?

You will need to know that 1 cal = 4.18 J and that 1 W = 1 J/s Remember that heat flow is in units of energy (J) per unit time (s) per unit surface area ($m^2$) and be sure to put everything into the appropriate units.

**Problem 5. Seismic Wave Equation for Planar S-Waves.**

Using the same procedure that we used in class for the plane P-wave, derive the wave equation for a plane S-wave traveling in the x-direction and vibrating in the y-direction to obtain:

$$\frac{\partial^2 v}{\partial t^2} = \left(\frac{\mu}{\rho}\right)\frac{\partial^2 v}{\partial x^2}$$

and show that the S-wave has a velocity:

$$v_s = \sqrt{\frac{\mu}{\rho}}$$

**Problem 6. Seismic Waves**

Direct P arrivals for the earth are not seen beyond 103° beyond the seismic source (if the source is near the surface). If the Earth had a uniform velocity mantle, use this information to estimate the radius of the Earth's core? How does this compare with the actual radius of the core? Explain carefully why your simple estimate differed from the real value?

**Problem 7. Seismic Waves**

On a planetary mission, you leave 4 seismometers on the surface of planet Shakey-Shakey. One month later, a large near-surface quake gives the following data as a function of angular distance:

| D(°) | P or PKP arrival time (s) |
|---|---|
| 46 | 312 |
| 82 | 525 |
| 84 | not observed |
| 180 | 300 |

The radius of Shakey-Shakey is 2000 km. What is the mantle P-wave velocity? The core radius? The core P-wave velocity?

Do you think this planet has a liquid or a solid core? Explain.