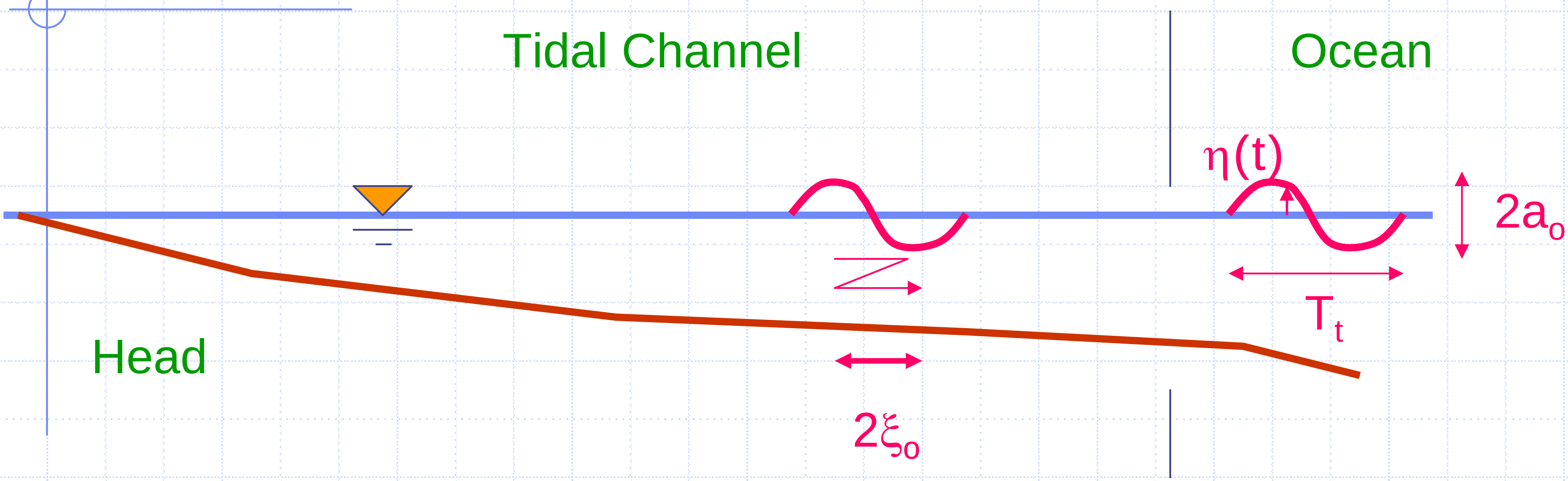
4 Estuarine Mixing

- Initial concepts: tides and salinity
- Tide-resolving models
- Tidal-average models
- Tracers for model calibration
- Mixing diagrams
- Residence time
- Dual tracers

What is an estuary?

- A semi-enclosed coastal body of water which has a free connection with the open sea and within which sea water is measurably diluted with fresh water derived from land drainage (Pritchard, 1952)
- Where the river meets the ocean
- Like a river but with tides and salinity gradients

Tidal motion



a_o = tidal amplitude

$2a_o$ = tidal range

T_t = tidal period

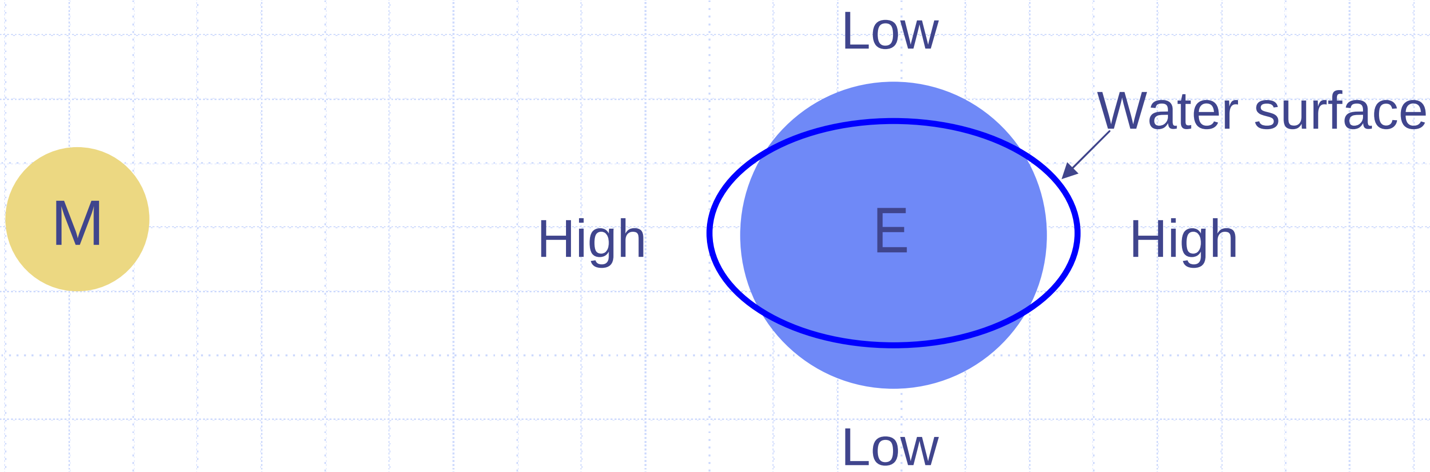
$2\xi_o$ = tidal excursion

Gravitational and centrifugal acceleration (E with M & S)

Ocean range ~ 0.5 m

Coastal waters may have much larger ranges

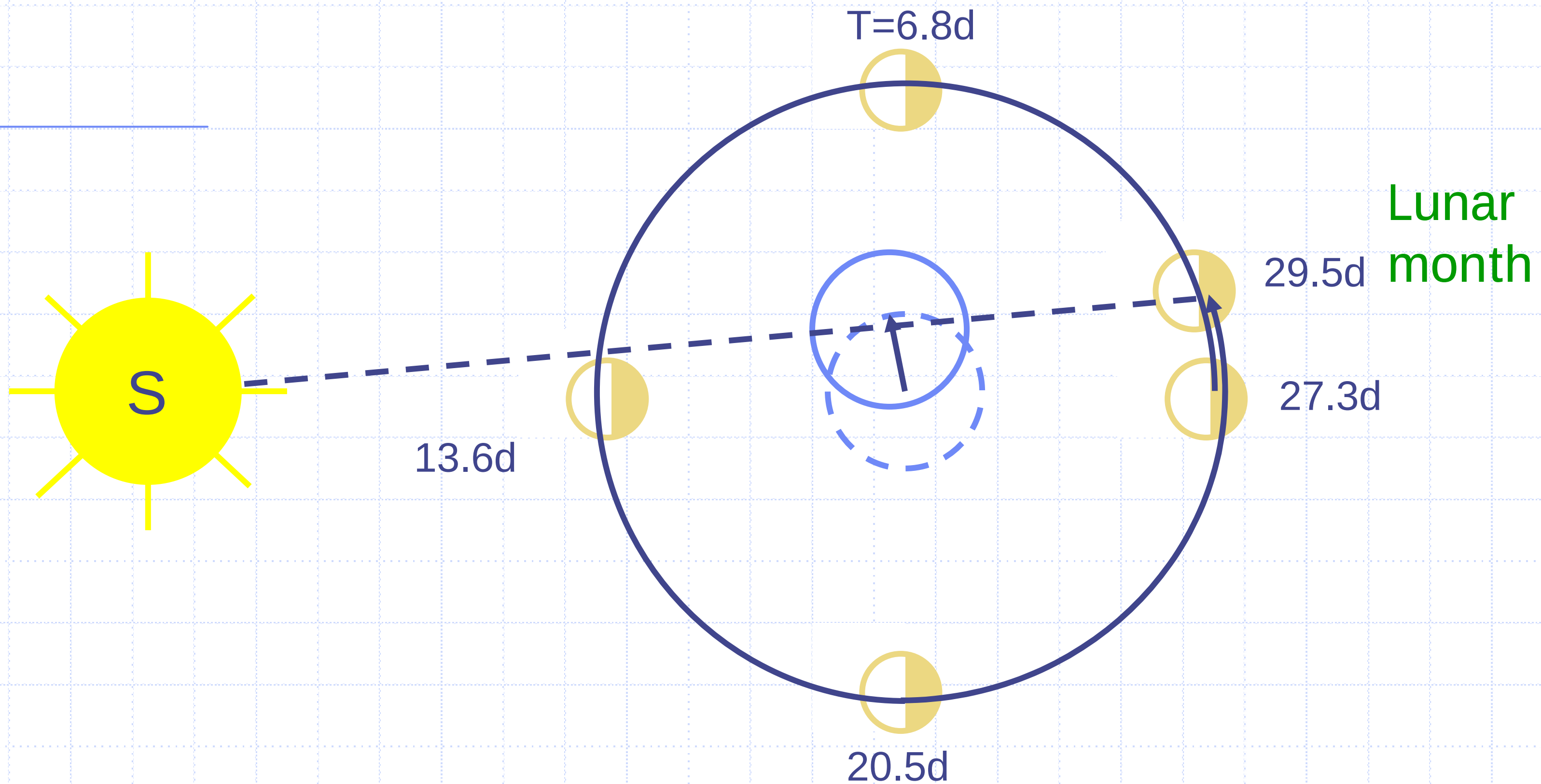
Equilibrium tide; moon only



At any time: 2 high and 2 low tides;

At any location: ~ 2 high and 2 low tides per day

Combined sun and moon

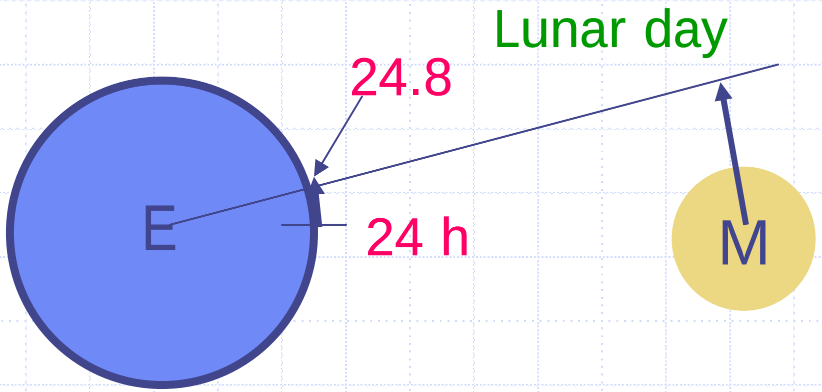


Sun and moon aligned (full and new moon) => spring tide;
Sun and moon opposed (1st and 3rd quarters) => neap tide

Because the earth revolves, period of spring-neap cycle =
365d/[(365/27.3)-1] = 29.5 days

Number of full moon's per year

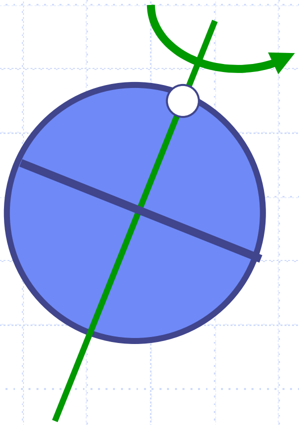
And because the moon revolves



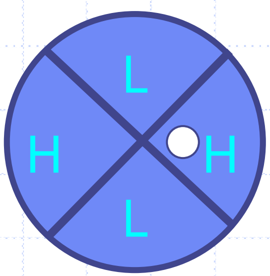
Lunar day = 29.5 d /(29.5 – 1) = 24.8 hours

Dominant (lunar semi-diurnal tidal) period is 12.4 h

Also a diurnal period



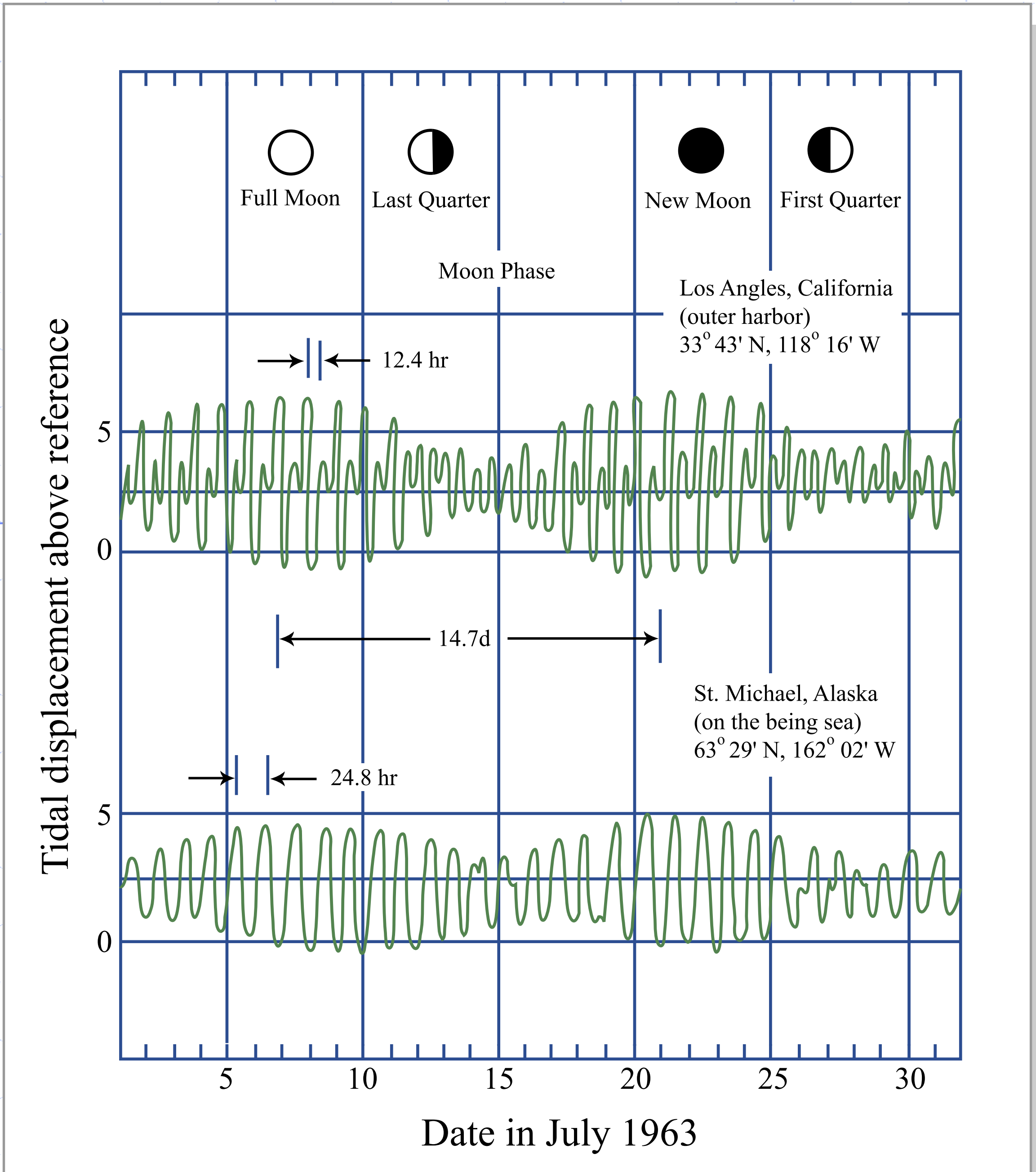
“Side” View



“Top” View

Because of the earth’s declination higher latitudes tend to experience a single (diurnal) cycle per rotation

In general a number of tidal constituents are required to compose an accurate tidal signal



Mixed tide (with strong semi-diurnal component; lower latitude)

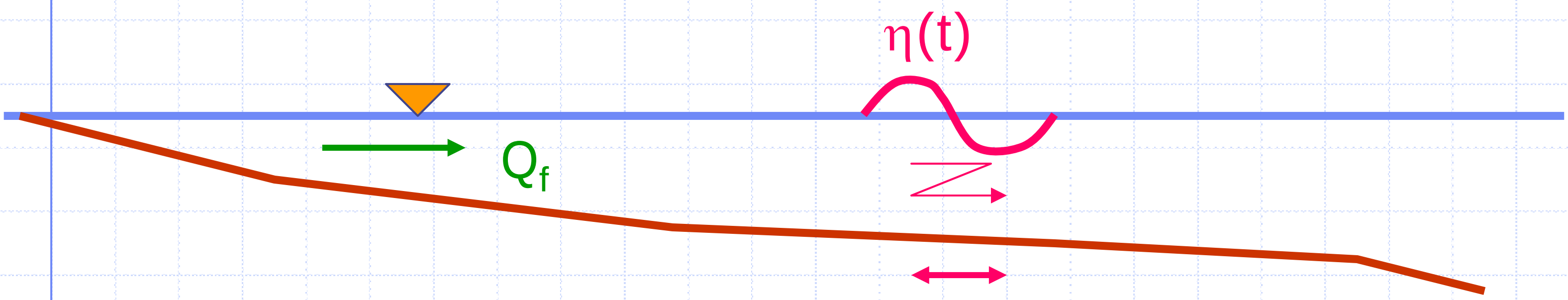
Diurnal tide (higher latitude)

Spring-neap cycle

Ippen, 1969

Figure by MIT OCW.

Idealized (linear) tidal motion



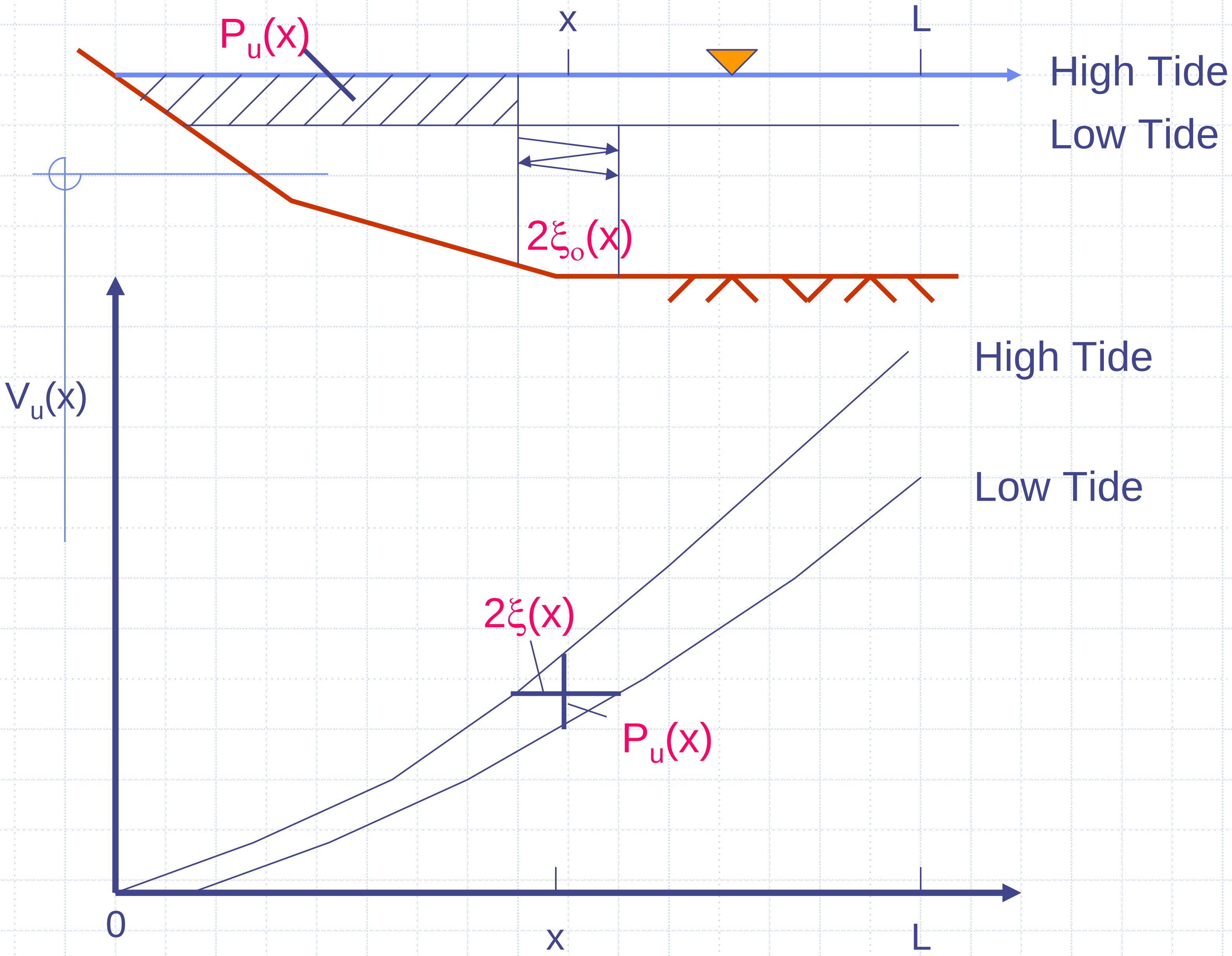
$$\eta(t) = a\cos(\omega t)$$ $\omega = 2\pi/T_t$ $2\xi_o$

$$u(t) = \frac{Q_f}{A} + u_{\max}\cos(\omega t + \phi)$$

$$\xi(t) = \overline{\overline{u}}\,t + \frac{u_{\max} T_t}{2\pi}\sin(\omega t + \phi)$$

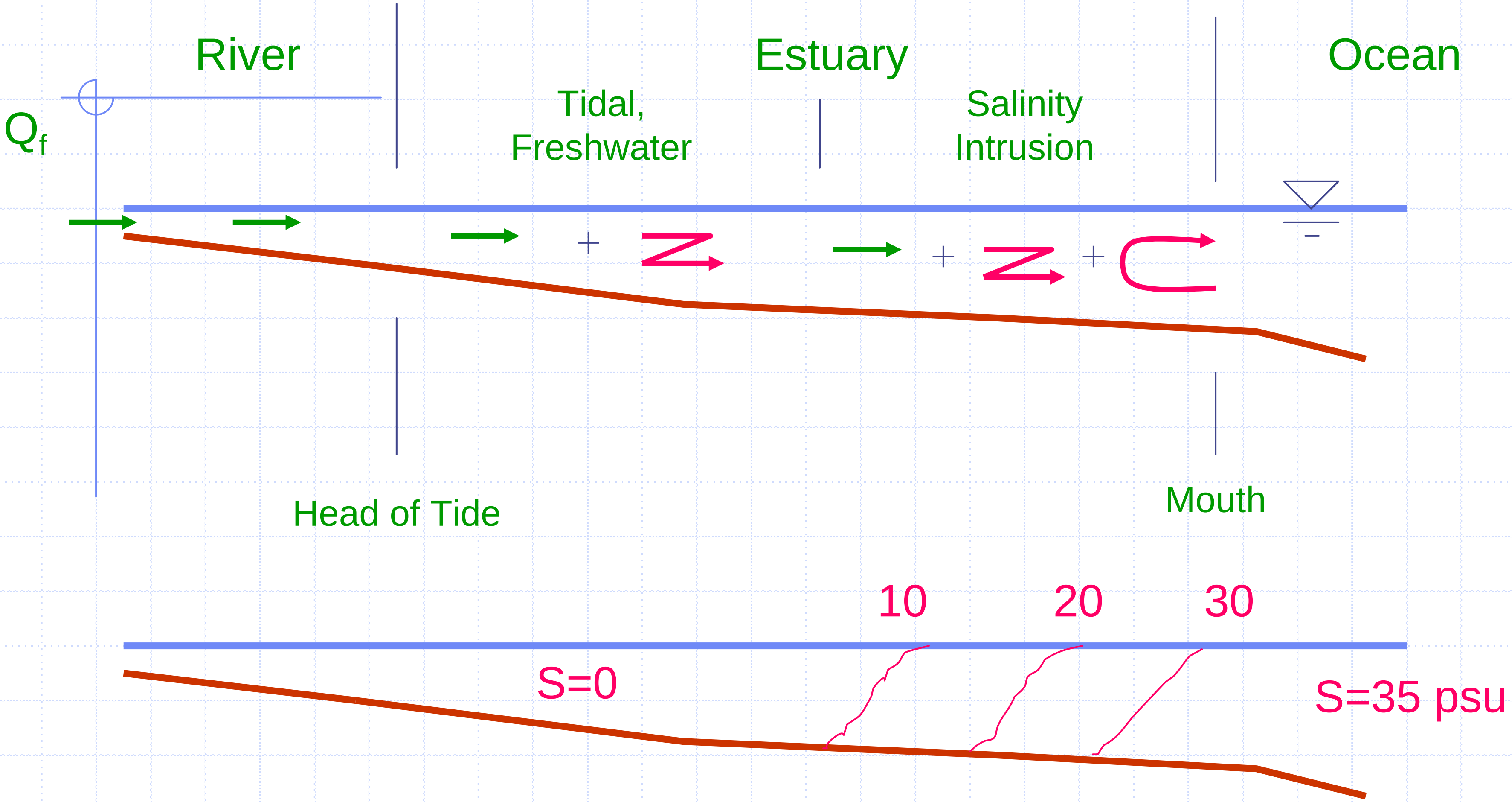
$$2\xi_o(x) = \frac{u_{\max} T_t}{\pi}$$ Tidal excursion

$$P_u(x) = V_{u,high}(x) - V_{u,lowh}$$ Upstream tidal prism



$P_u(x)$
x
L
High Tide
Low Tide
$2\xi_o(x)$
High Tide
$V_u(x)$
Low Tide
$2\xi(x)$
$P_u(x)$
0
x
L

Now introduce salinity



PSU = practical salinity unit,

an operational definition of salinity (mass fraction: ppt, $^{o}/_{oo}$ or g/kg)

Equation of State (Gill, 1982; ch 6)

$$\rho = \rho(T) + \Delta\rho(S) + \Delta\rho(TSS)$$

(Also pressure at deep depths)

$$\rho(T) = 1000\left[1 - \frac{T + 288.9414}{508929.2(T + 68.12963)}(T - 3.9863)^2\right]$$

$$\Delta\rho(S) = AS + BS^{3/2} + CS^2$$

$$A = 0.824493 - 4.0899x10^{-3}T + 7.6438x10^{-5}T^2 - 8.2467x10^{-7}T^3 + 5.3875x10^{-9}T$$

$$B = -5.72466x10^{-3} + 1.0227x10^{-4}T - 1.6546x10^{-6}T^2$$

$$C = 4.8314x10^{-4}$$

$$\Delta\rho(TSS) = TSS\left[1 - \frac{1}{SG}\right]x10^{-3}$$

ρ = kg/m^3, T in ^{o}C, S in PSU (g/kg), TSS in mg/L

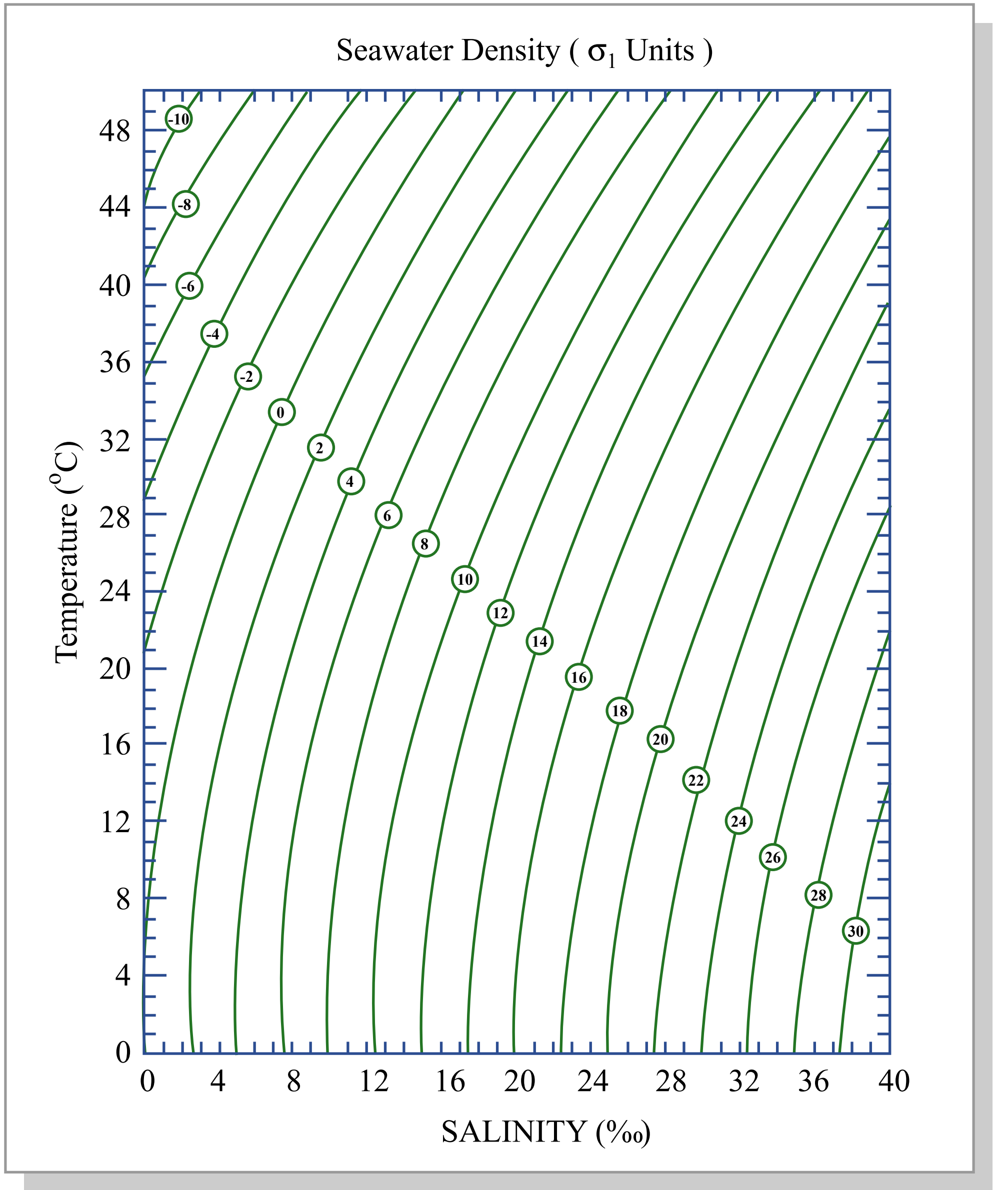


Figure by MIT OCW.

Fischer, et al. (1979)

$\sigma_t = 1000*(\rho-1)$

(ρ in g/cm^3)

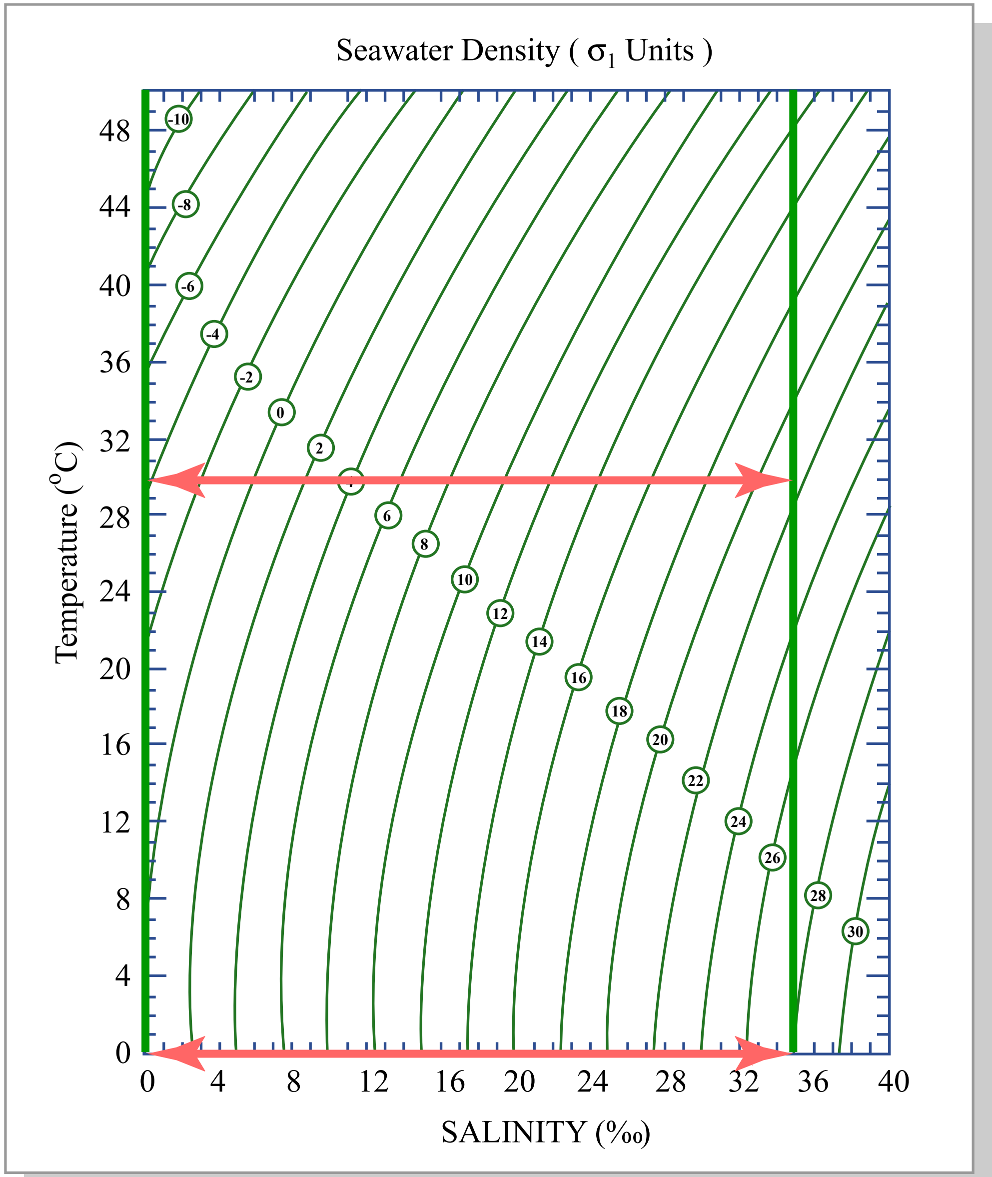


Figure by MIT OCW.

Example:

Salt water – Freshwater density difference $\Delta\rho_o/\rho$

Ocean salinity
~ 35 psu

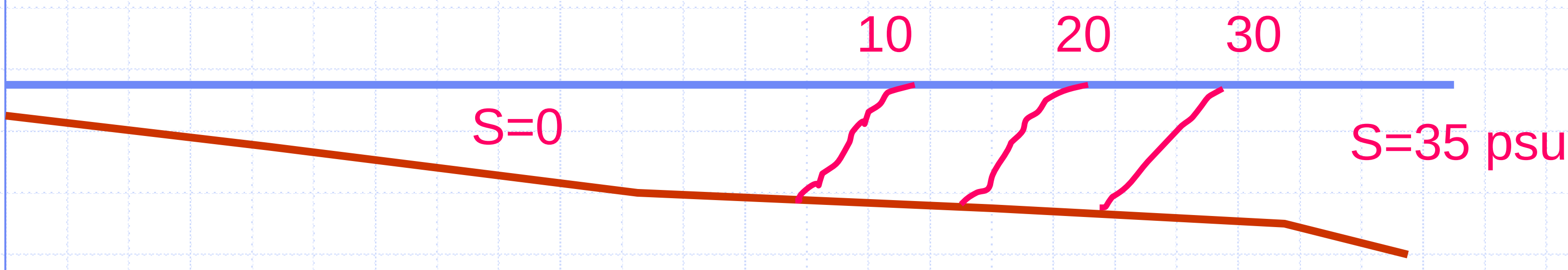
Freshwater salinity
0 psu

Temperatures 0 to 30C

$\Delta\rho_o/\rho$ = [28-0]/1000=0.028
(0C)

= [22-(-4)]/1000=0.026
(30C)

Estuary classification



Well mixed: isohaline lines approach vertical (Delaware R)

Partially mixed: isohaline lines slant

Vertically stratified (salt wedge): isohaline lines approach horizontal (Mississippi R.)

Desire to classify to know what type of model/analysis to use; several options available; none is perfect

Estuary classification, cont'd

Densimetric Estuary number (Harleman & Abraham, 1966; Thatcher & Harleman, 1972)

$$E_d = \frac{P_t F_d^{\,2}}{Q_f T_t}$$

P_t = tidal prism; Q_f = freshwater flow rate;
T_t = tidal period

$$F_d = \frac{u_o}{\sqrt{g(\Delta\rho_o / \rho)h}}$$

F_d is a densimetric Froude number

u_o = maximum tidal velocity; h = estuary depth;
$\Delta\rho_o/\rho$ = salt water – fresh water density difference

Estuary classification, cont'd

Estuary Richardson number (Fischer, 1972; 1979)

$$R = \frac{\Delta\rho_o g Q_f / W}{\rho u_t^3} \sim E_d^{-1}$$

W = estuary width;

u_t = RMS tidal velocity $\cong 0.71 u_o$

R ~ potential energy rate/kinetic energy rate

$R < 0.08$	well-mixed
$0.08 < R < 0.8$	partially stratified
$0.8 < R$	vertically stratified (salt wedge)

Example later

Estuary classification, cont'd

The definitions are related

$$E_d \sim R^{-1} \sim \frac{u_t^{\,3}}{u_f u_d^{\,2}}$$

Each involves 3 velocities:

u_t = RMS tidal velocity — Tends to mix estuary

u_f = fresh water velocity = Q_f/A — Tends to stratify estuary

u_d = density velocity = $\sqrt{g(\Delta\rho_o/\rho)h}$ — Tends to stratify estuary

Hanson-Rattray (1966)

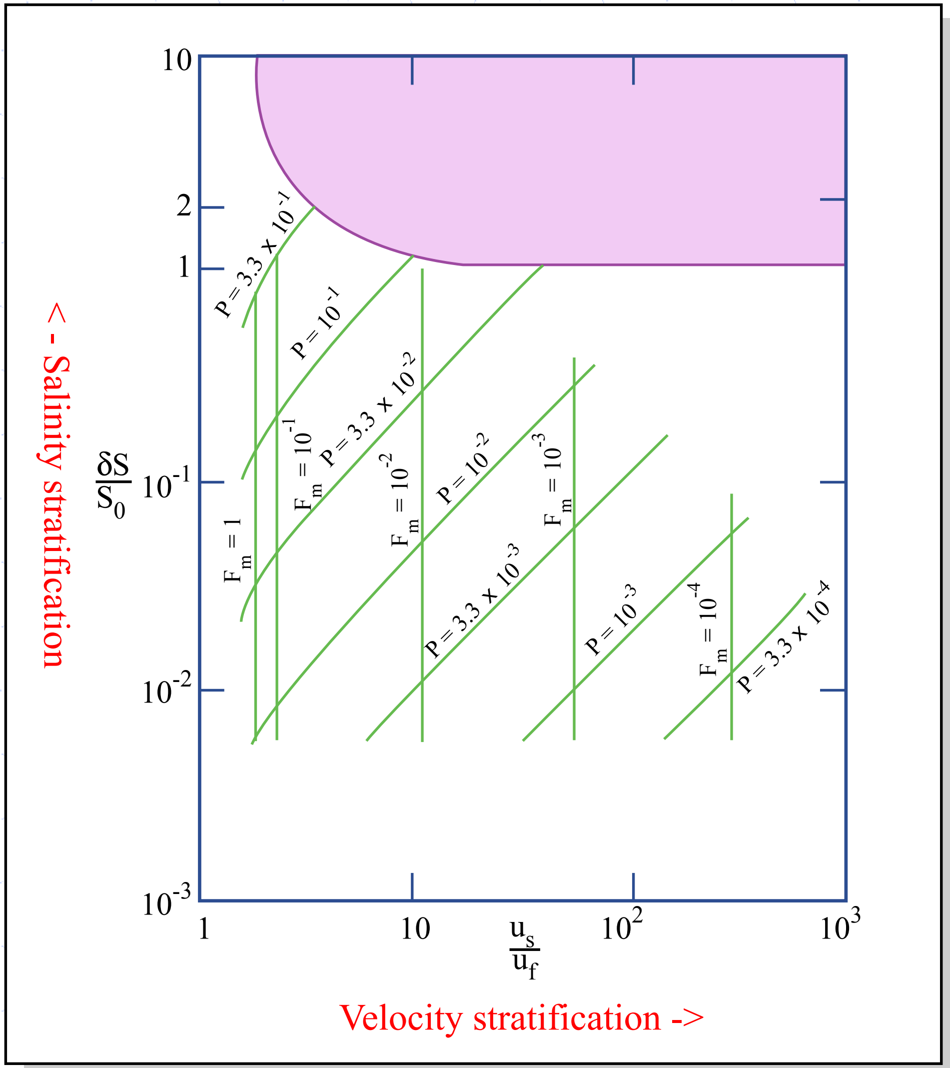


Figure by MIT OCW.

- Semi-empirical
- Predicts
  - salinity stratification

    $$\delta S/S_o = (S_b - S_s)/\overline{\overline{S}}$$

    Increases w/ P, decreases w/ F_m
  - Velocity stratification

    u_s/u_f =tidal average surf vel / tidal and depth aver vel

    Decreases w/ F_m

$$P = \frac{u_f}{u_t}; \quad F_m = \frac{u_f}{u_d}$$

Tide resolving models

Well-mixed (1-D) estuary

$$\frac{\partial c}{\partial t} + \underbrace{u(t)}\frac{\partial c}{\partial x} = \frac{1}{A}\frac{\partial}{\partial x}\left(A\underbrace{E_L(t)}\frac{\partial c}{\partial x}\right) + \frac{q_L(c_L - c)}{A} + \sum r_i + \sum r_e{}'$$

Major difference between river and well-mixed estuary are 1) u is time-varying, 2) E_L is constrained by reversing tide.

Look at 2) first

Characteristic dispersion time scales

(Fischer et al., 1979)

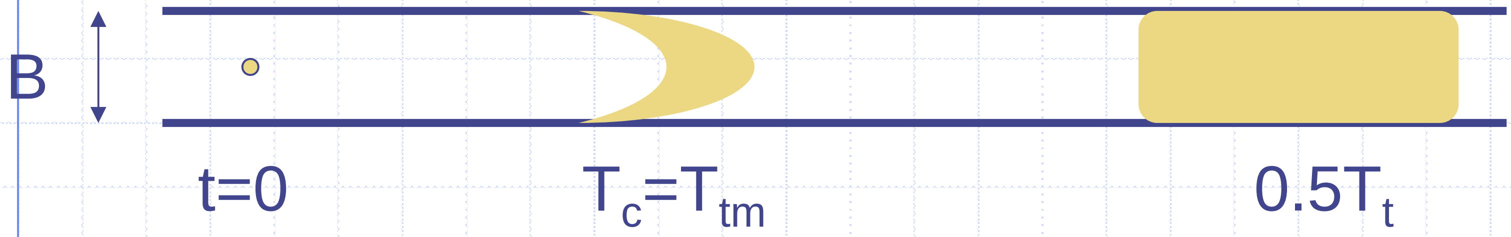
- $E_L \sim U_c^2 T_c \sim u_*^2 T_c$
- For rivers, two possible time scales, T_c:
  - $T_{tm} \sim B^2/E_T$ and $T_{vm} \sim h^2/E_z$
  - $T_{tm} >> T_{vm} => E_L \sim u_*^2\ T_{tm}$ (after transverse mixing)
- For estuaries, additional possibility: $T_c = T_t/2$
  - $T_{tm} >> T_t/2 \sim T_{vm} => E_L \sim u_*^2\ T_{tm}$ or $u_*^2\ T_t/2$

Previous example, B = 100 m, H = 5 m, u = 1 m/s

T_{vm} = 750 s, T_{tm} = 34000 s, $T_t/2$= 22000 s (6.2 h)

Dispersion in reversing flow

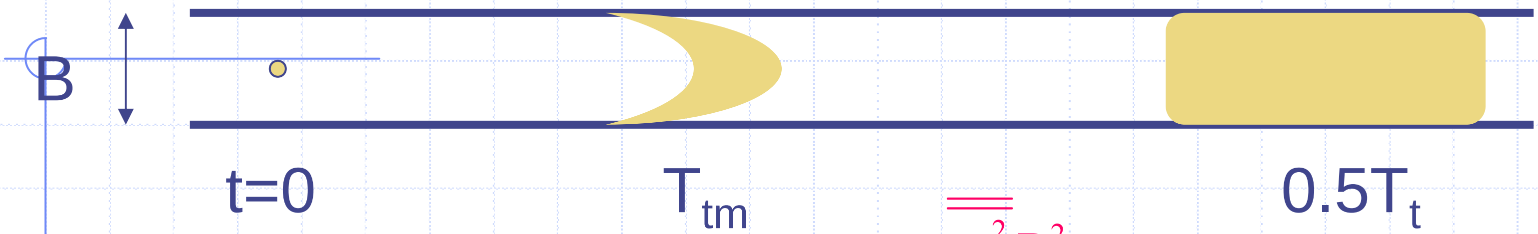
"narrow" channel



Dispersion governed by T_{tm}, $E_L \sim \dfrac{\overline{\overline{u_*^2}}\,B^2}{E_T}$

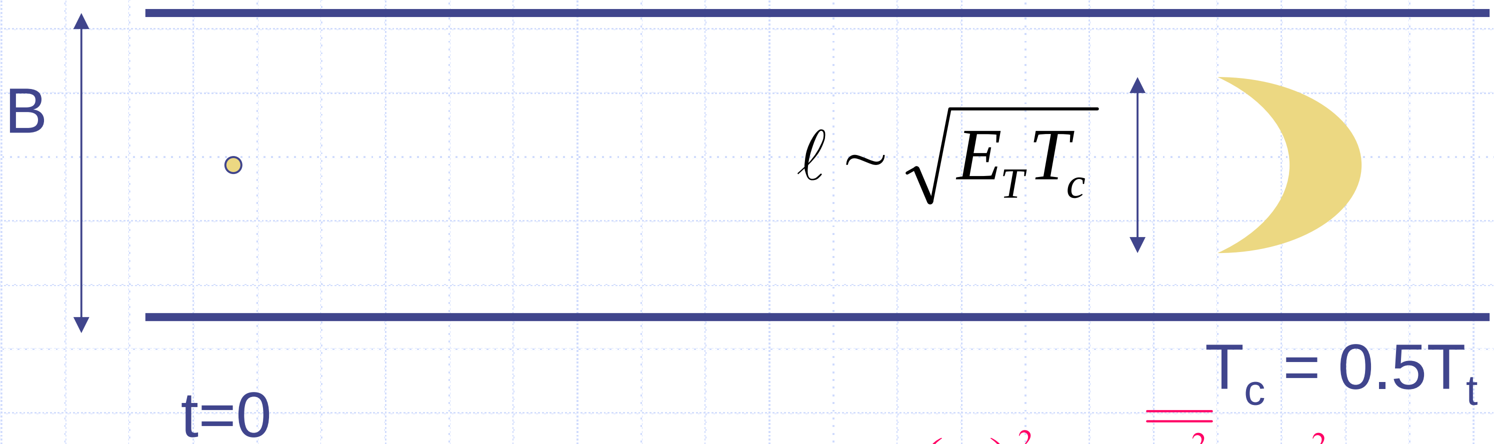
Dispersion in reversing flow, cont'd

"narrow" channel



Dispersion governed by T_{tm}, $E_L \sim \dfrac{\overline{\overline{u_*^{\,2}}} B^2}{E_T}$

"wide" channel

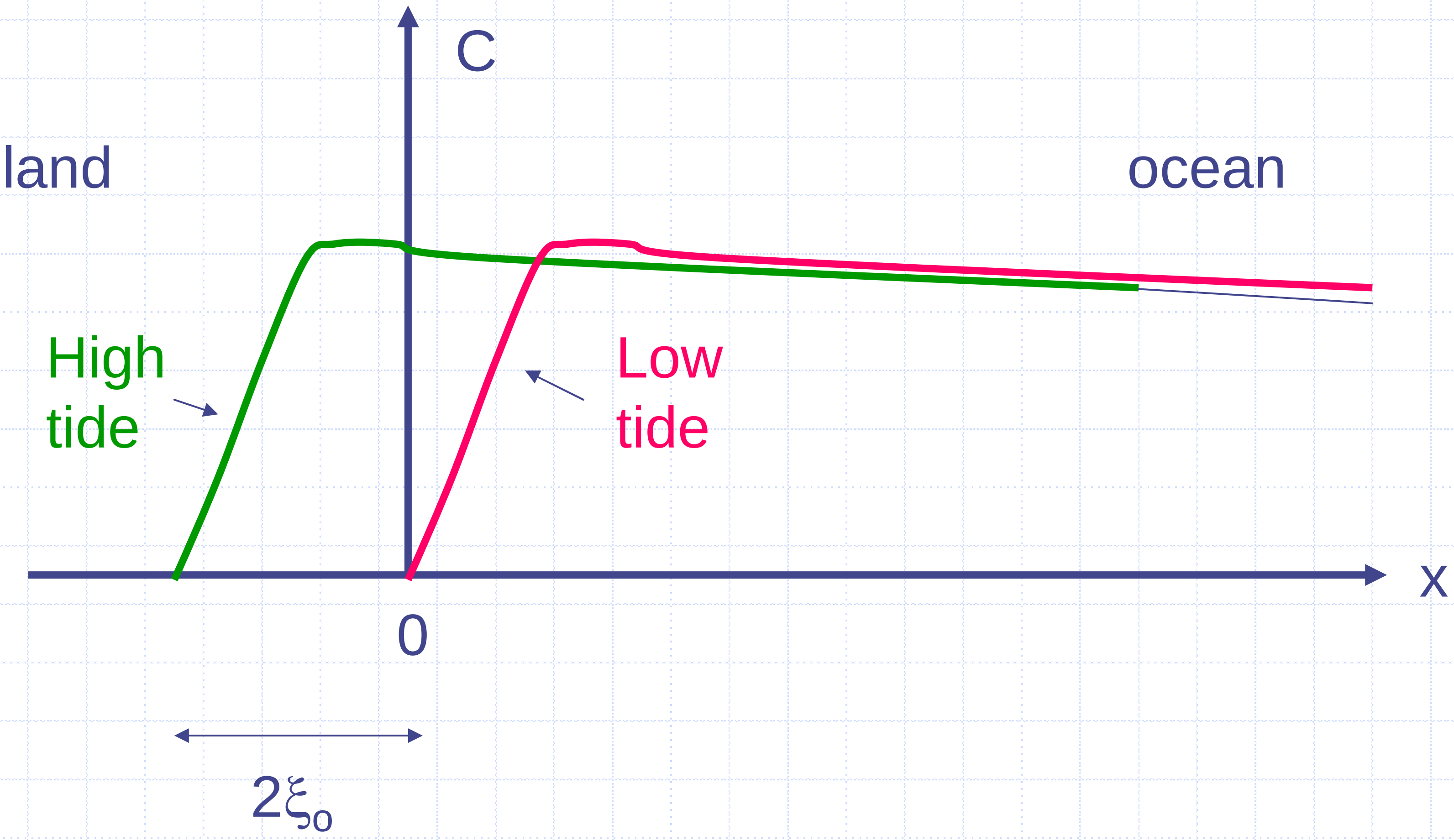


$$\ell \sim \sqrt{E_T T_c}$$

Dispersion governed by T_t, $E_L \sim \overline{\overline{u_*^{\,2}}} \left(\dfrac{\ell}{B}\right)^2 T_t \sim \dfrac{\overline{\overline{u_*^{\,2}}} E_T T_t^{\,2}}{B^2}$

Effects of reversing u(t)

Mass continuously injected at x = 0



An actual simulation

Harleman, 1971

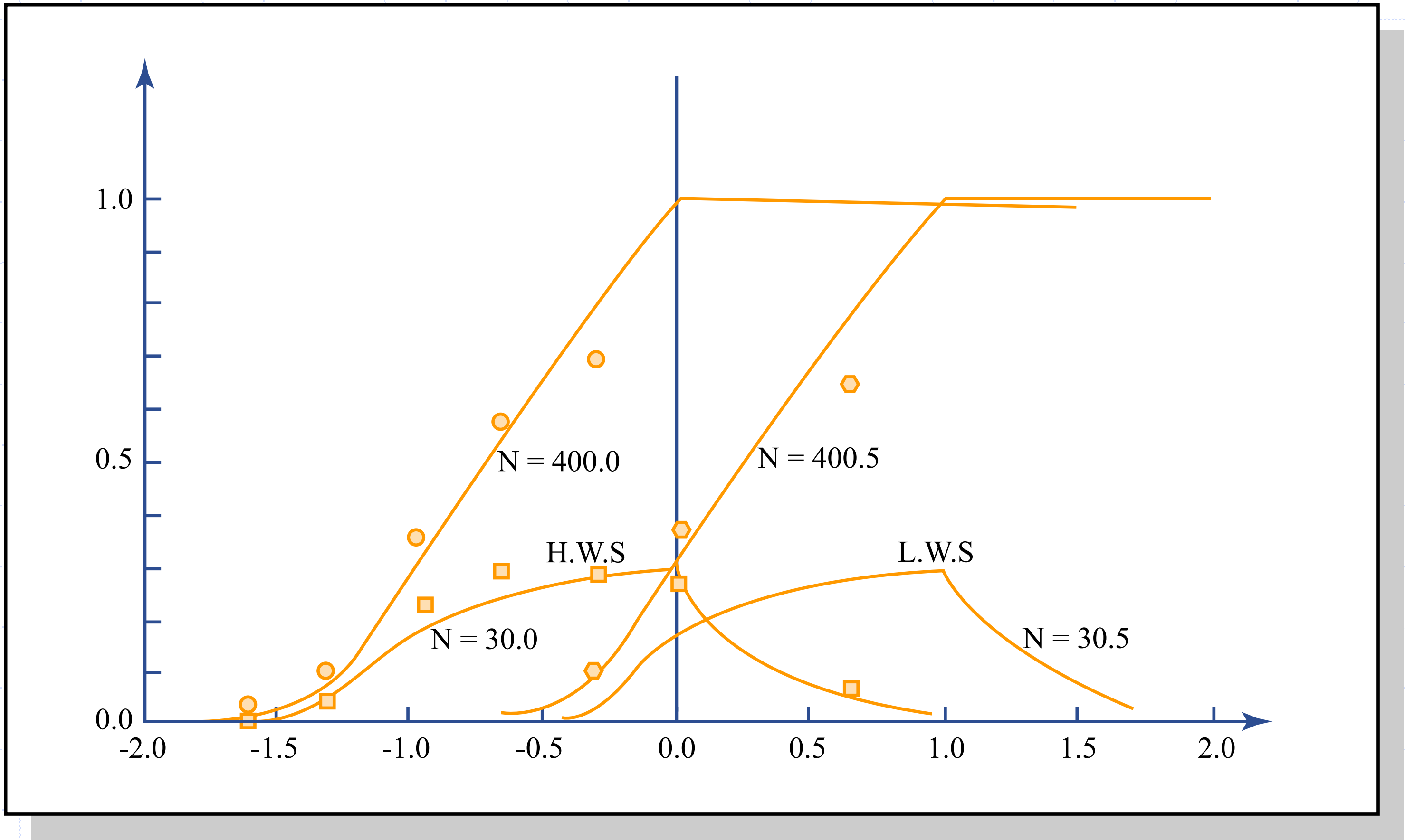


Figure by MIT OCW.

$x / 2\xi_o$

Continuous injection at x = 0; output after 30, 400 tidal periods (high slack) and 30.5 and 400.5 tidal cycles (low slack)

Tidal-average models

- Perhaps we don’t care to resolve intra-tidal time-dependence
- Strong non-uniformities prevent resolution of intra-tidal variability
- Long term calculations more efficient with tidal-average time step
- However, averaging obscures physics

Tidal-average models, cont'd

Analogous, in principle, to time and cross-sectional averaging

$$u = \overline{\overline{\overline{u}}} + u'''$$

$$c = \overline{\overline{\overline{c}}} + c'''$$

Triple bars imply tidal average

Insert into GE and tidal-average

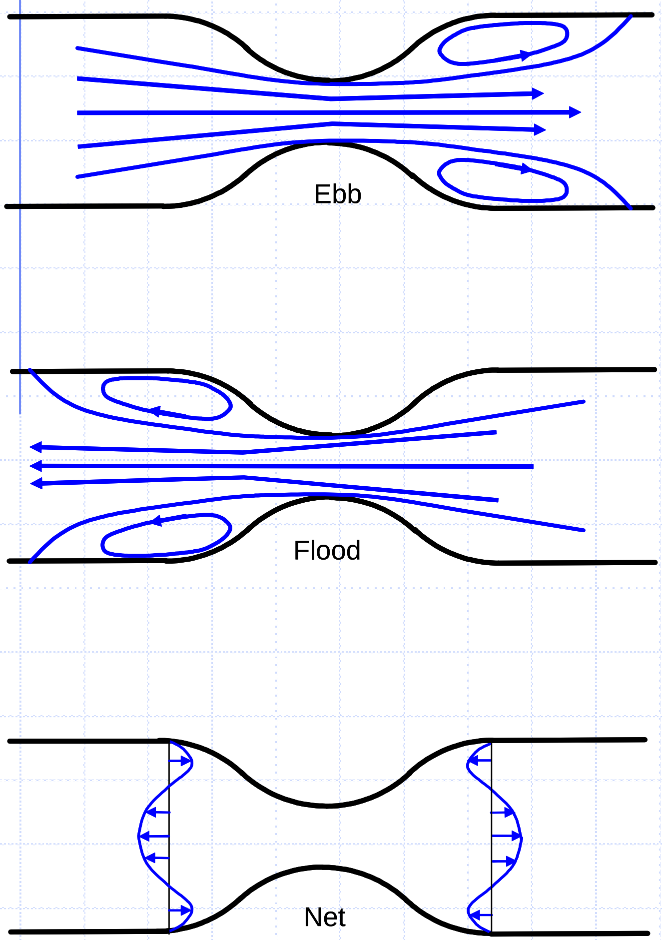
$$\frac{\partial c}{\partial t} + \underbrace{\overline{\overline{\overline{u}}}}\frac{\partial \overline{\overline{\overline{c}}}}{\partial x} = \frac{1}{A}\frac{\partial}{\partial x}\left(A\underbrace{\overline{\overline{\overline{E_L}}}}\frac{\partial \overline{\overline{\overline{c}}}}{\partial x} \right) + \sum r_i + \sum r_e$$

Tidal average velocity

Tidal average disp coef

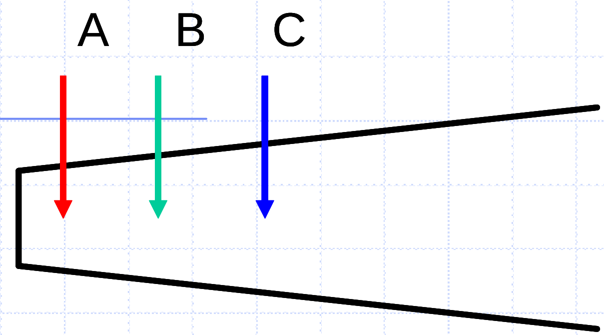
Structurally similar to equation for river transport => similar solutions

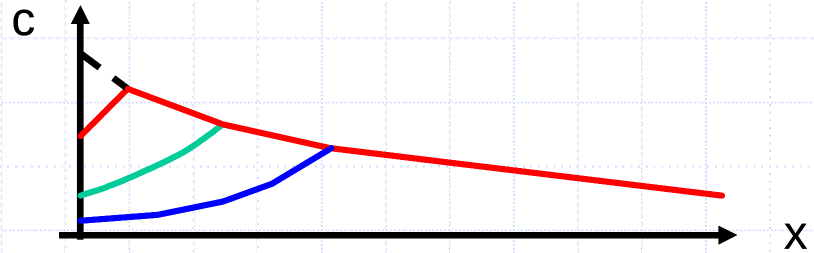
Tidal average dispersion



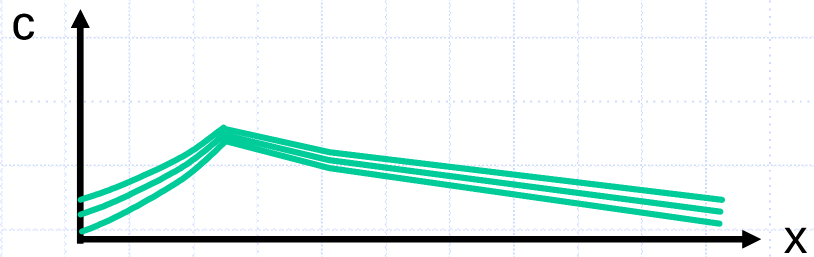
- Tidal pumping (shown)
  - Asymmetric ebb (a) & flood (b)
  - Tidal averaging => mean velocity (c)
  - Trans mixing + trans velocity gradients => dispersion!
- Similar drivers
  - Tidal trapping
  - Coriolis + density
  - Depth-dependent tidal reversal
- $E_L \sim (2\xi_o)^2/T_t$

General result

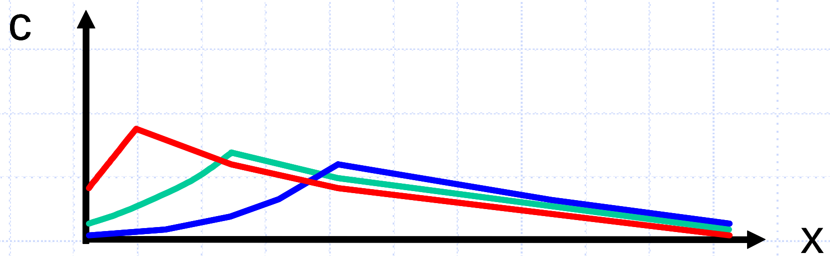




Conservative Tracer;
3 injection locations



Non-conservative
tracer; middle location

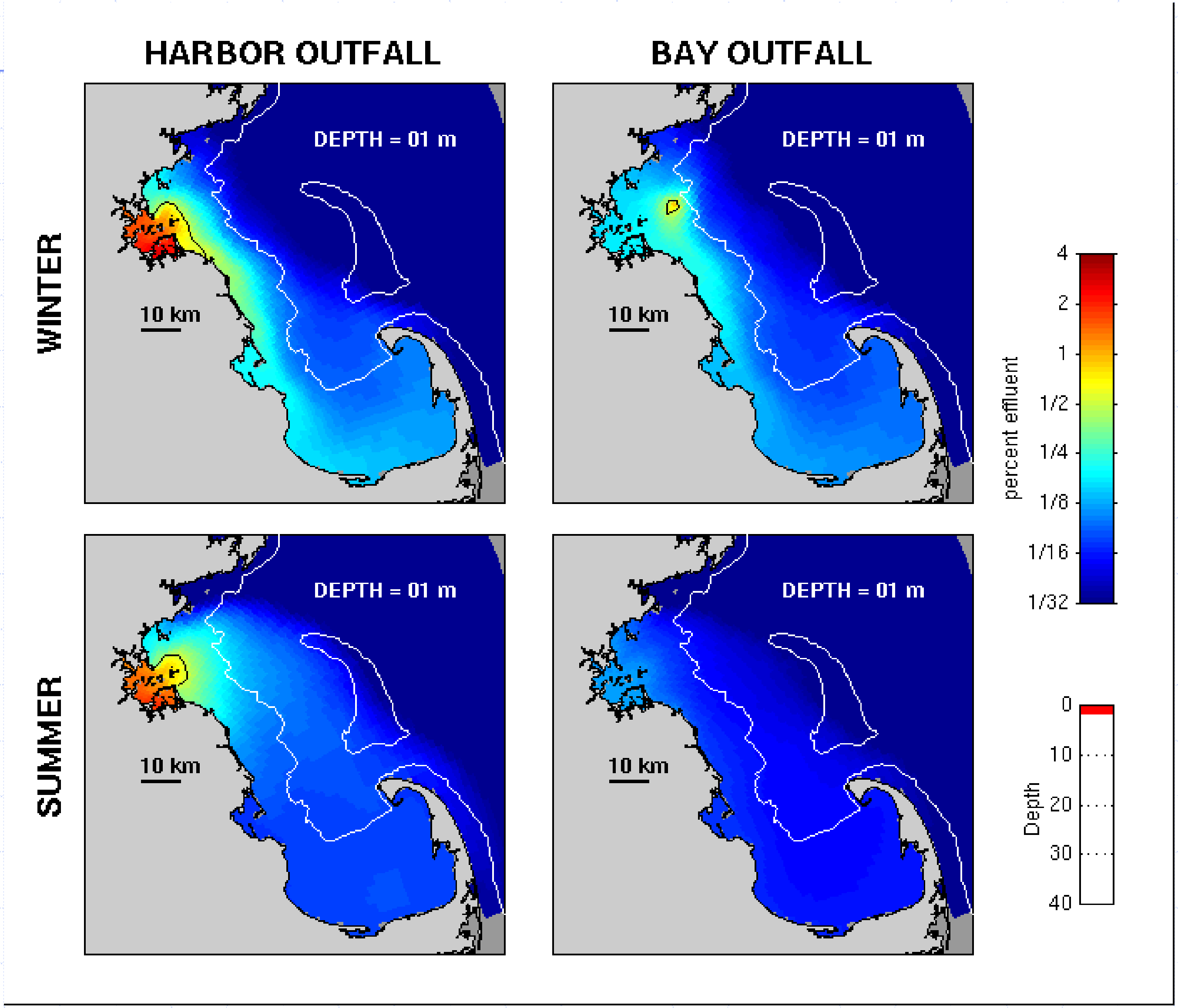


Non-conservative
tracer; 3 locations

Comments

- For conservative tracer, $c(x)$
  - Is independent of x_d for $x > x_d$
  - Decrease with x_d for $x < x_d$
- If you must pollute, do it downstream (more discussion later)
- Several specific solutions in notes

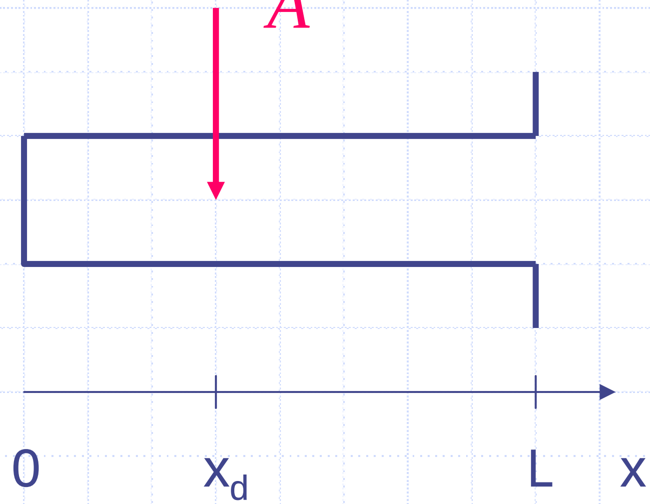
Conclusion applies loosely even if not 1-D



Signell, MWRA (1999)

One example

$$q'' = \frac{\dot{m}}{A}$$



Rectangular channel; no through flow

$$0 = \frac{d}{dx}\left(E_L \frac{dc}{dc}\right) - kc$$

$$E_L \sim (2\xi_o)^2 / T_t = \alpha x^2$$

$$0 = 2\alpha x \frac{dc}{dx} + \alpha x^2 \frac{d^2 c}{dx^2} - kc$$

Solution

$$c_+(x, x_d) - c_L = \frac{q''}{\alpha\kappa}\left[\frac{x^{-1/2-\kappa/2}}{x_d^{1/2-\kappa/2}} - \frac{x^{-1/2+\kappa/2}}{L^\kappa x_d^{1/2-\kappa/2}}\right]$$

$x > x_d$

$$\kappa = \sqrt{1 + 4k/\alpha}$$

$$c\_(x, x_d) - c_L = \frac{q''}{\alpha\kappa}\left[\frac{x^{-1/2+\kappa/2}}{x_d^{1/2+\kappa/2}} - \frac{x^{-1/2+\kappa/2}}{L^\kappa x_d^{1/2-\kappa/2}}\right]$$

$x < x_d$

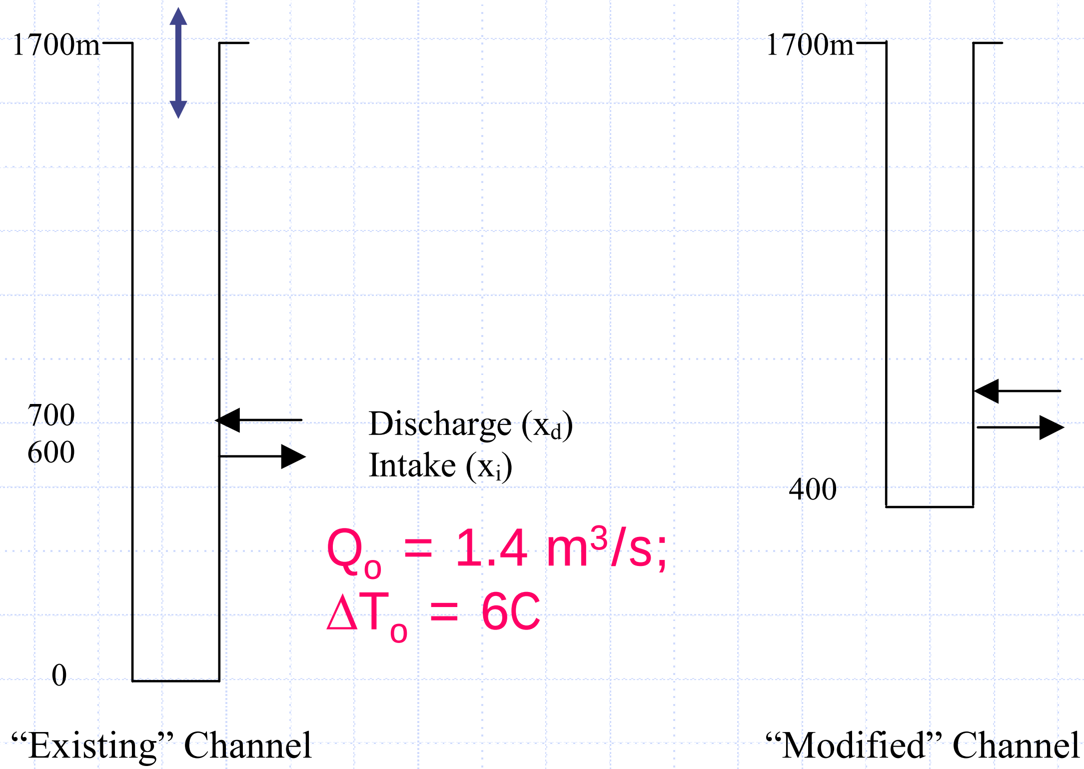
WE4-1 Proposed relocation of Gillette's Intake

- Proposal to shorten Fort Point Channel as part of the Big Dig threatened to limit Gillette's cooling water source

Details

Boston Harbor

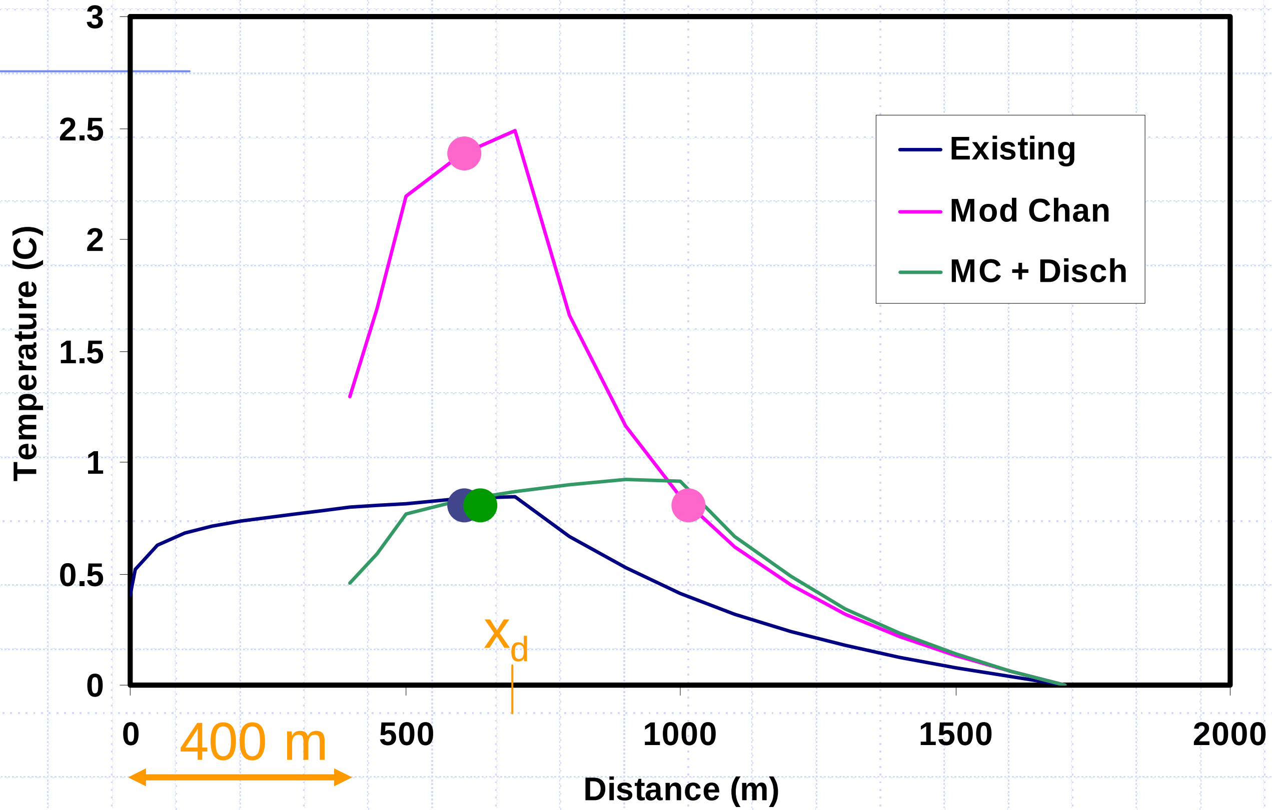
$2a_o = 2.9$ m;
$h = 6$ m;
$k = 0.1$ day^{-1}



$Q_o = 1.4\ m^3/s$;
$\Delta T_o = 6C$

Proposed remedies: move discharge and/or intake downstream. How far?

Results of analysis



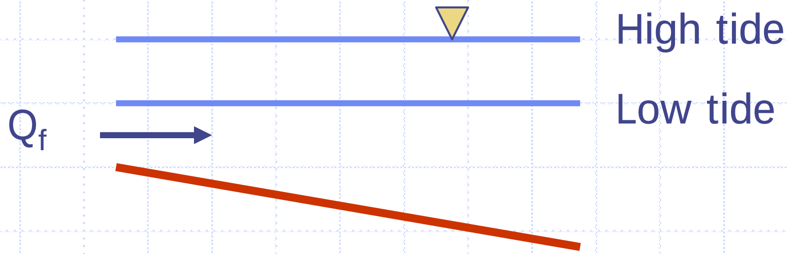
Existing: T_i (x=600) ~ 0.8C; Modified: T_i ~ 2.4C

Moving intake 400 m downstream (x=600) yields T_i ~ 0.8C

Moving discharge 300 m downstream (x=900) also yields T_i ~ 0.8C

Tidal Prism Method

- Treats whole channel as single well-mixed box
- Mass that leaves on ebb does not returns
- Except for harbors/short channels, this overestimates flushing; underestimates c.
- Hence common to “discount” P by defining the effective volume P’ of “clean” water. E.g., P’ = 0.5 P
- Formal ways to compute return factor using phase of circulation outside harbor



$$\frac{Pf}{T_t} = Q_f$$

$$\frac{Pc_{tp}}{T_t} = \dot{m}$$

$$c_{tp} = \frac{\dot{m}T_t}{P}$$

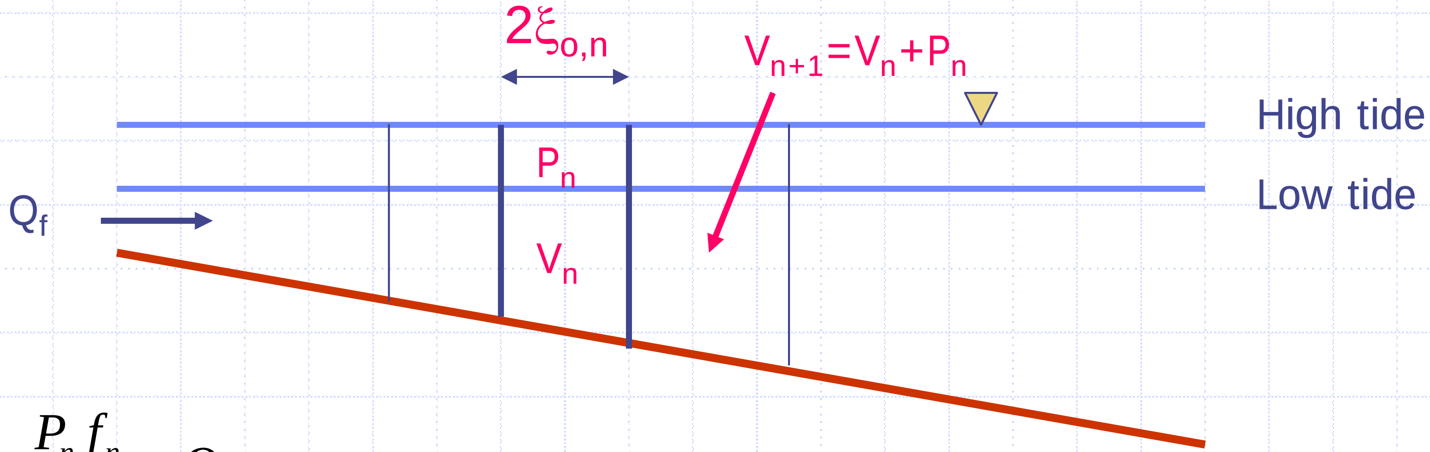
P = total tidal prism

f = “freshness” = $(S_o - S_n)/S_o$

Modified Tidal Prism Method

Divides channel into segments of length $2\xi_o$

Assumes $E_L = (2\xi_o)^2/T_f$ <=> net ds transport during T_t is P_n



$$\frac{P_n f_n}{T_t} = Q_f$$

f_n = "freshness" $=(S_o - S_n)/S_o$

$$\frac{P_n c_n}{T_t} = \dot{m}$$

mass injected continuously upstream of section n (behaves like freshwater)

$$c_n = \frac{\dot{m} T_t}{P_n}$$

Comments

- Modified Tidal Prism Method has been modified and re-modified many times
- Ad-hoc assumption => not always agreement with data
- Non-conservative contaminates reduced in concentration by χ

$$\chi = \frac{r}{1-(1-r)e^{-kT_t}}$$

$$r = 2a/h$$

Salinity as tracer to measure E_L

Steady, tidal average flow

$$\frac{d}{dx}\left(u_f AS\right) = \frac{d}{dx}\left(AE_L \frac{dS}{ds}\right)$$

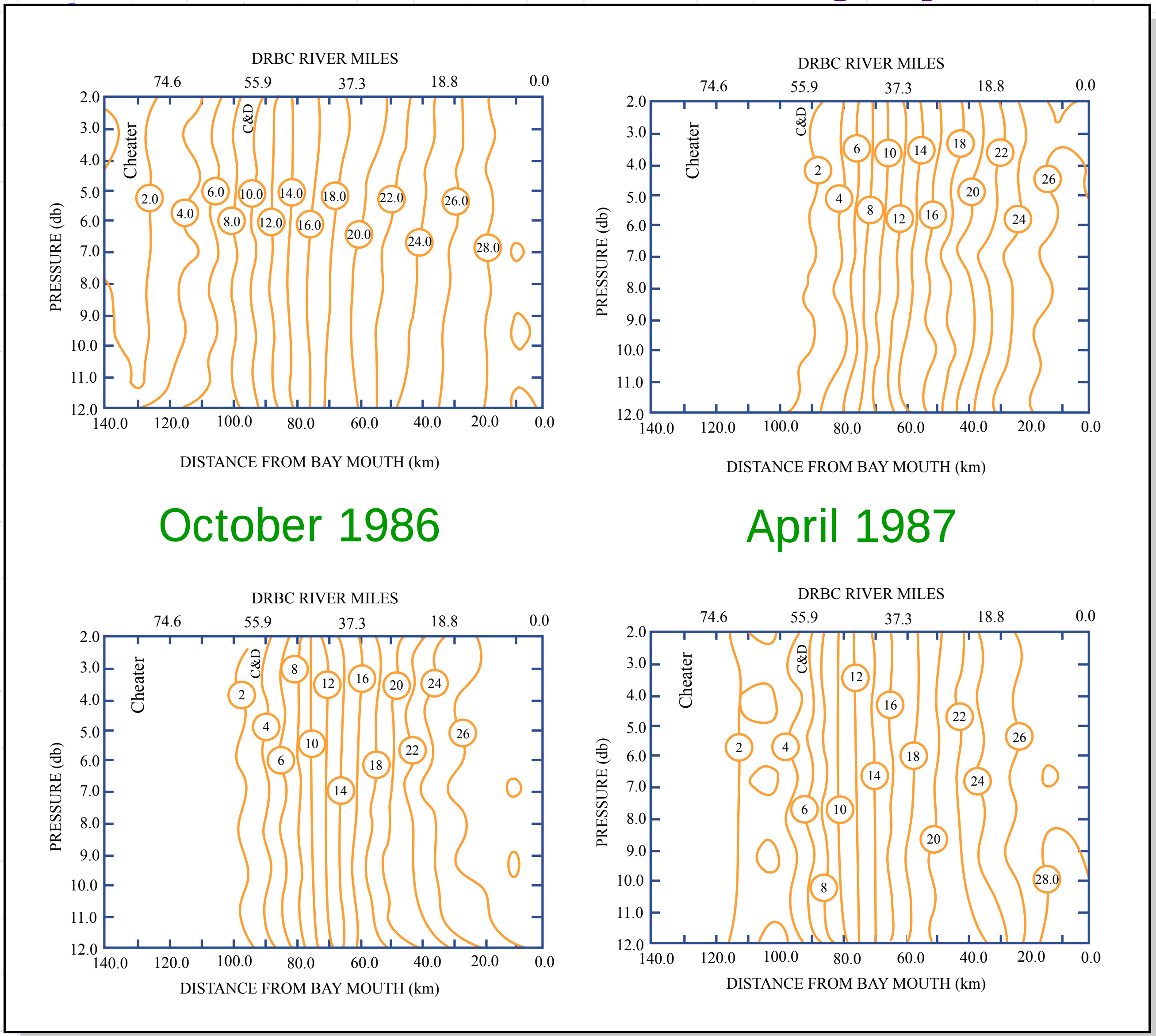
Integrate with

S = dS/dx at head, x=0

$$E_L = \frac{u_f S}{dS/dx}$$

Example: Delaware R (WE 4-2)

Measured salinity profiles



November 1987

April 1988

Salinity profiles show river to be well-mixed.

Should it be?

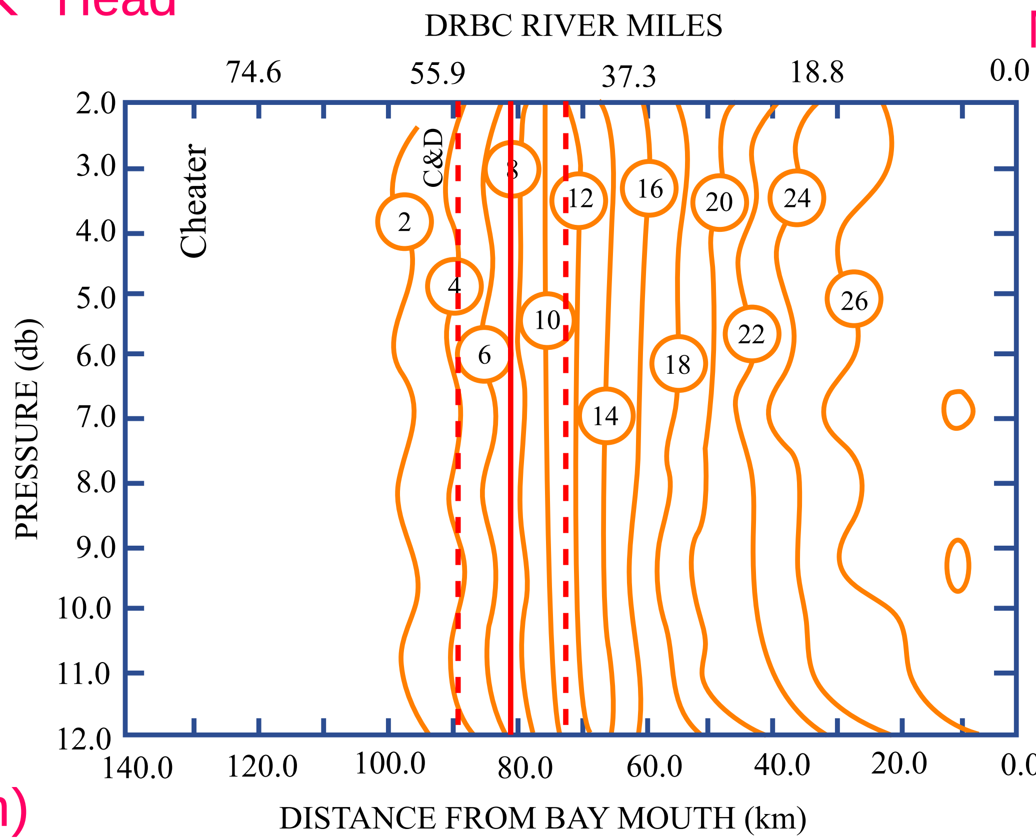
What is E_L?

Figure by MIT OCW.

Kawabe et al. (1990)

← Head

Mouth (ocean)



November 1987

Figure by MIT OCW.

~ h (m)

$$E_L = \frac{u_f S}{dS/dx} \cong \frac{(Q_f / A) S}{\Delta S / \Delta x}$$

$$\cong \frac{(260)(8)}{(1.5x10^4)(8)/20000)}$$

$$\cong 350\ m^2/s$$

Q_f = 260 m^3/s;   A = $1.5x10^4$ m^2;

S = 8 psu (80 km);

$\Delta S/\Delta x$ = (12-4) psu/20 km   (70-90 km)

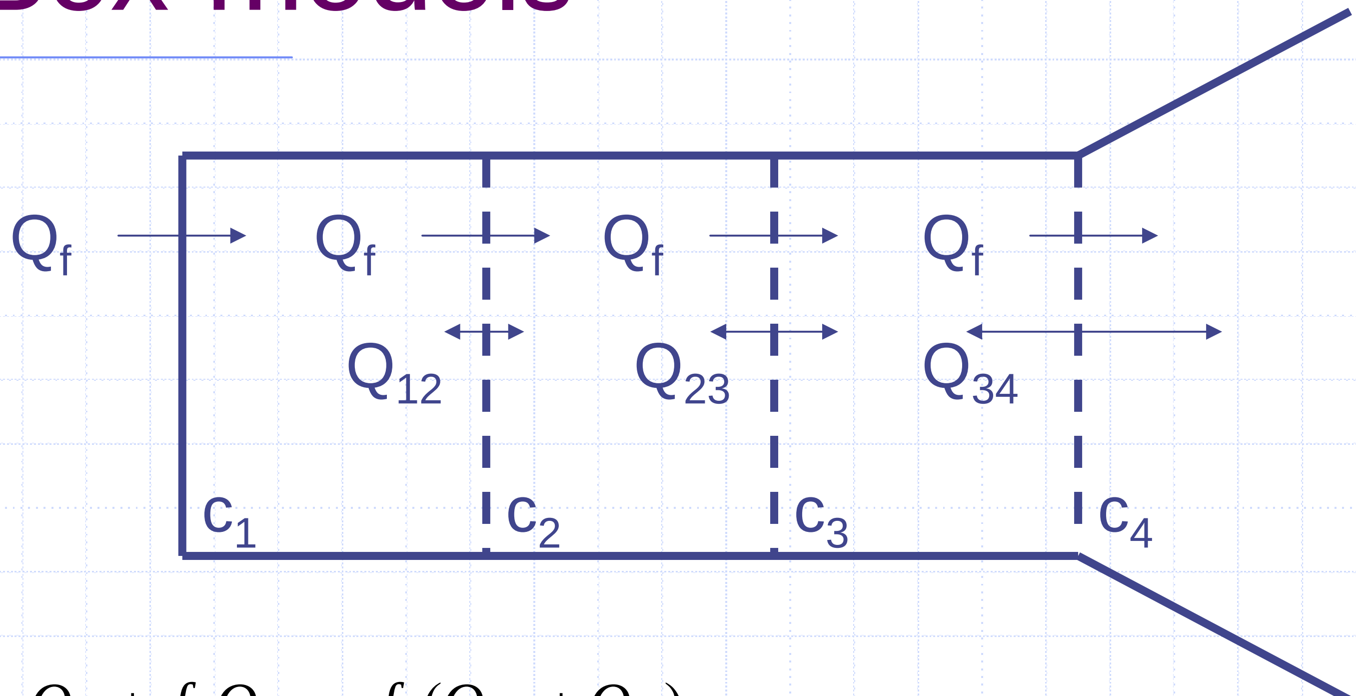
Should river be well-mixed?

$$R = \frac{\frac{\Delta\rho}{\rho} g \frac{Q_f}{W}}{u_t^{\,3}}$$

$$\cong \frac{(0.025)(10)(260)/4000}{1^3} \cong 0.02 < 0.08$$

Yes!

Box models



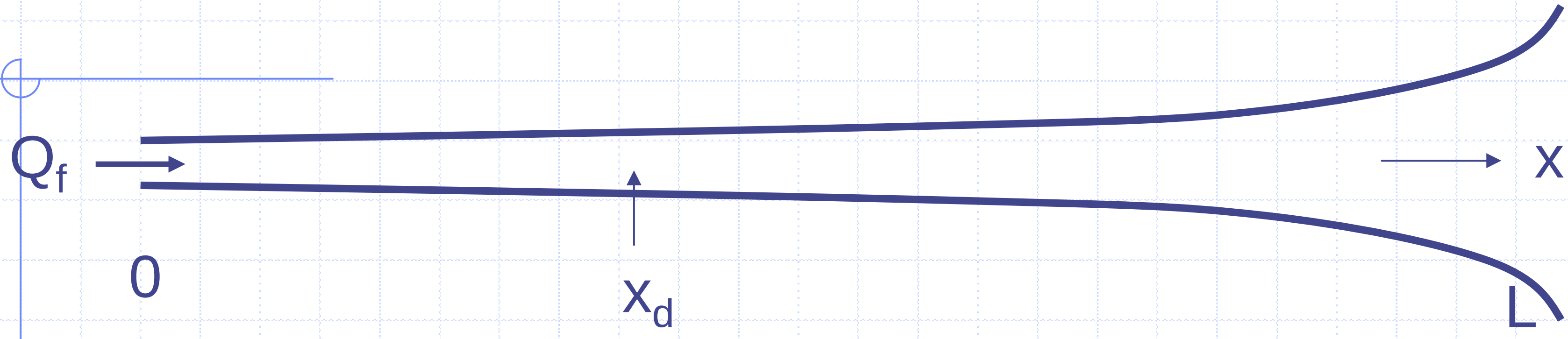
$$Q_f + f_2 Q_{1,2} = f_1 (Q_{1,2} + Q_f)$$

$$f_1 (Q_{1,2} + Q_f) + f_3 Q_{2,3} = f_2 (Q_{1,2} + Q_{2,3} + Q_f)$$

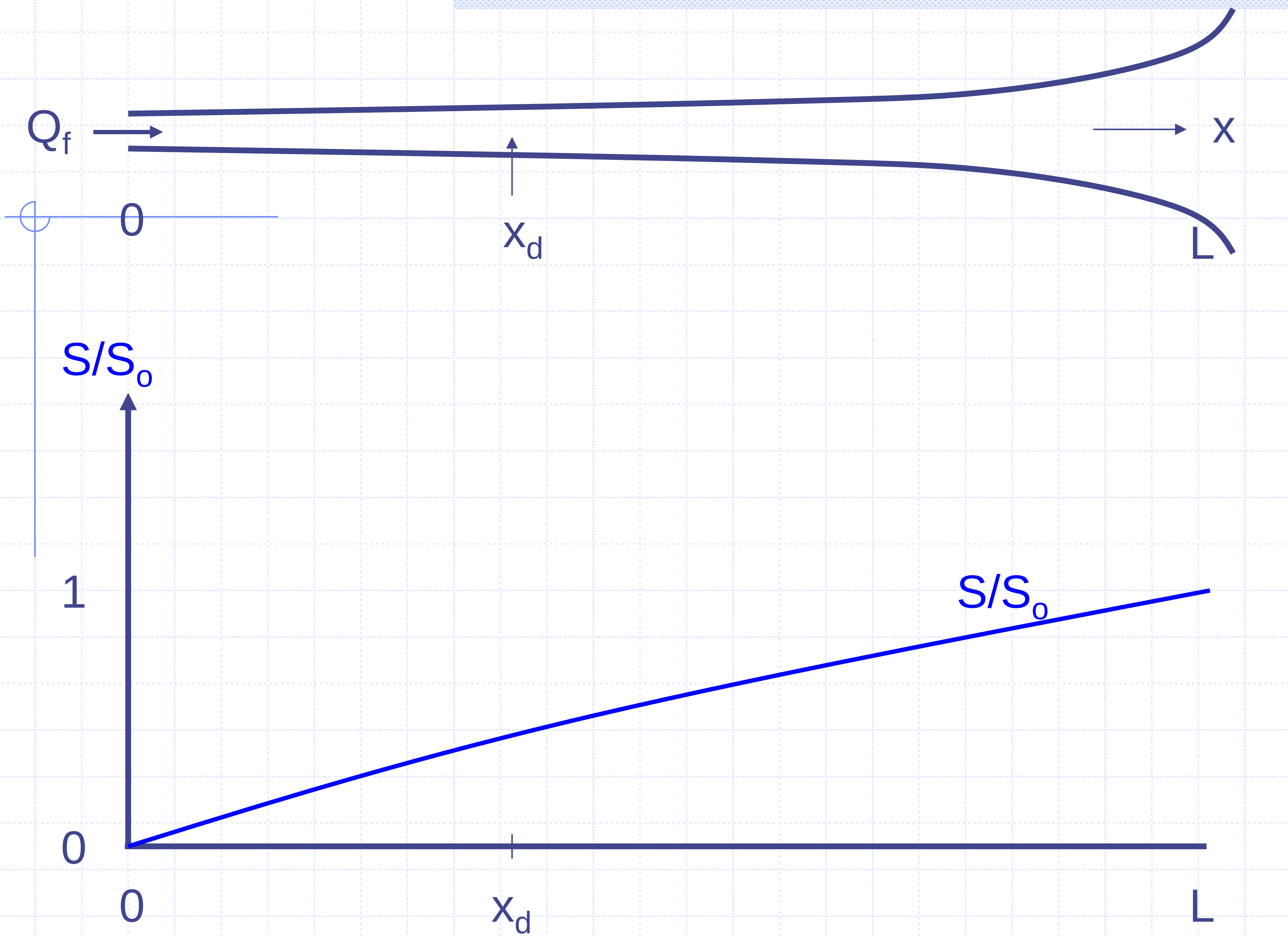
$$f_2 (Q_{2,3} + Q_f) + f_4 Q_{2,3} = f_3 (Q_{2,3} + Q_{3,4} + Q_f)$$

n equations in n unknowns; boxes dictated by geometry

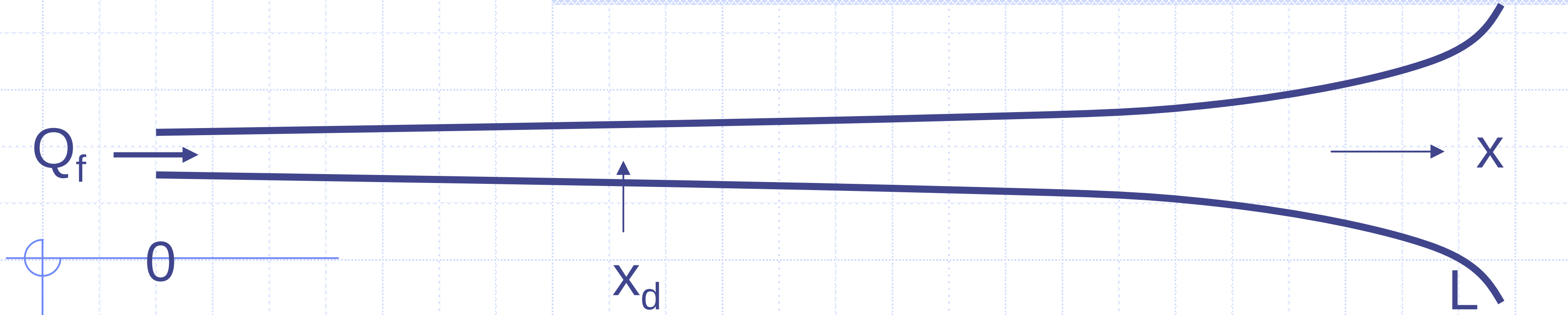
Salinity as direct measure of c



Use measured salinity distribution S(x) resulting from river discharge Q_f entering at head (x=0) to infer concentration distribution c(x) of mass entering continuously at downstream location x_d.

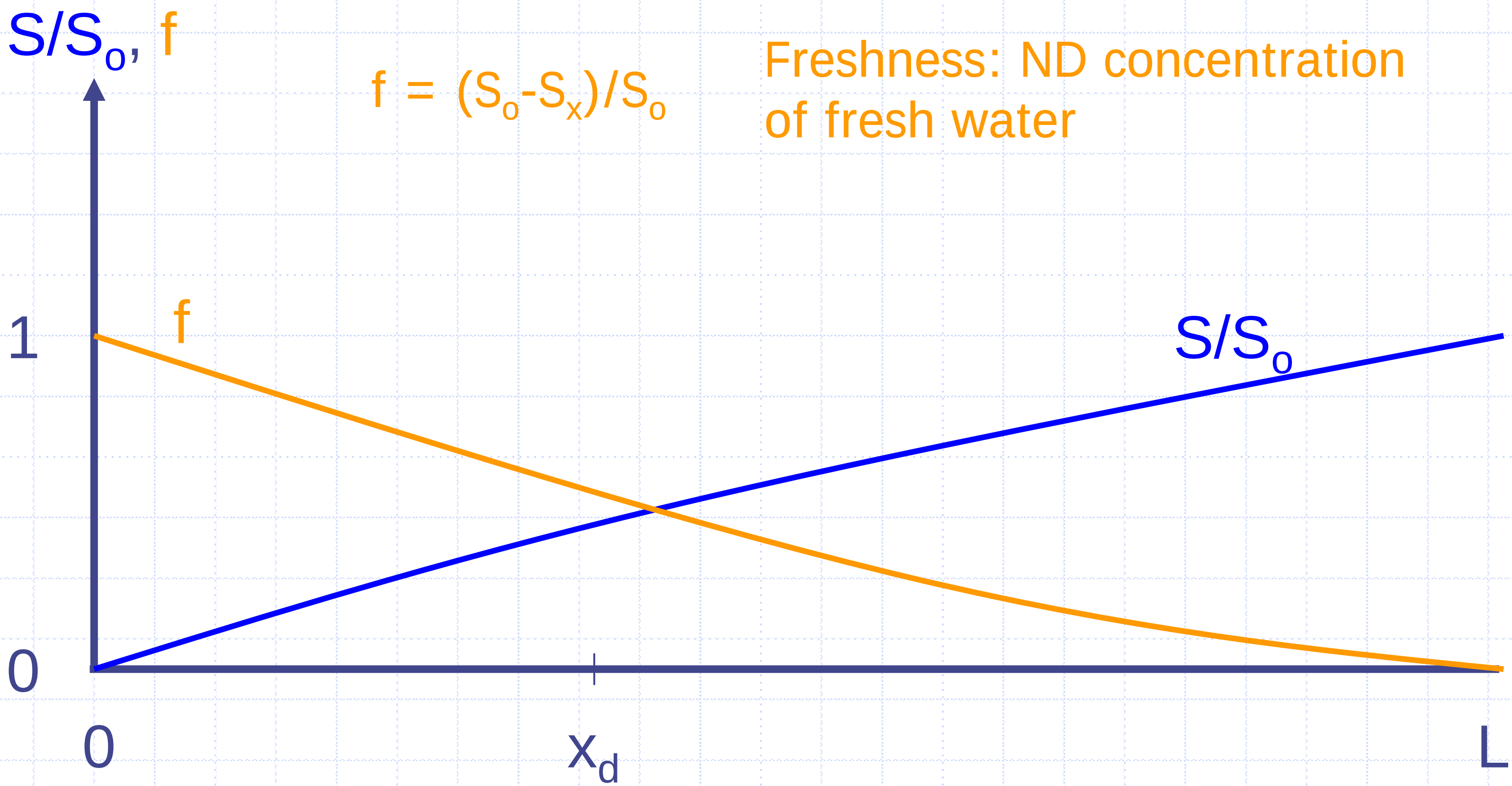


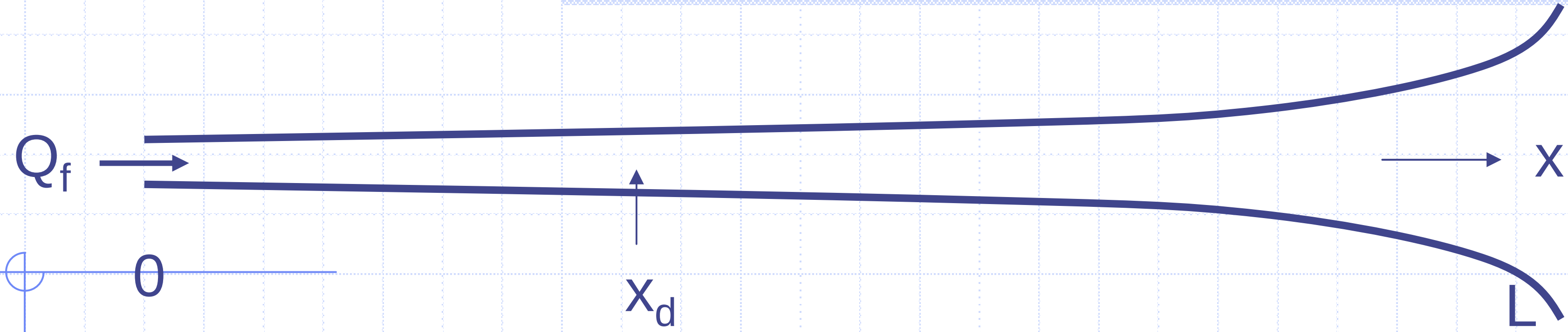
Q_f
x
0
x_d
L
S/S_o
1
S/S_o
0
0
x_d
L



$f = (S_o - S_x)/S_o$

Freshness: ND concentration of fresh water





Effective downestuary transport rate, Q_{eff}

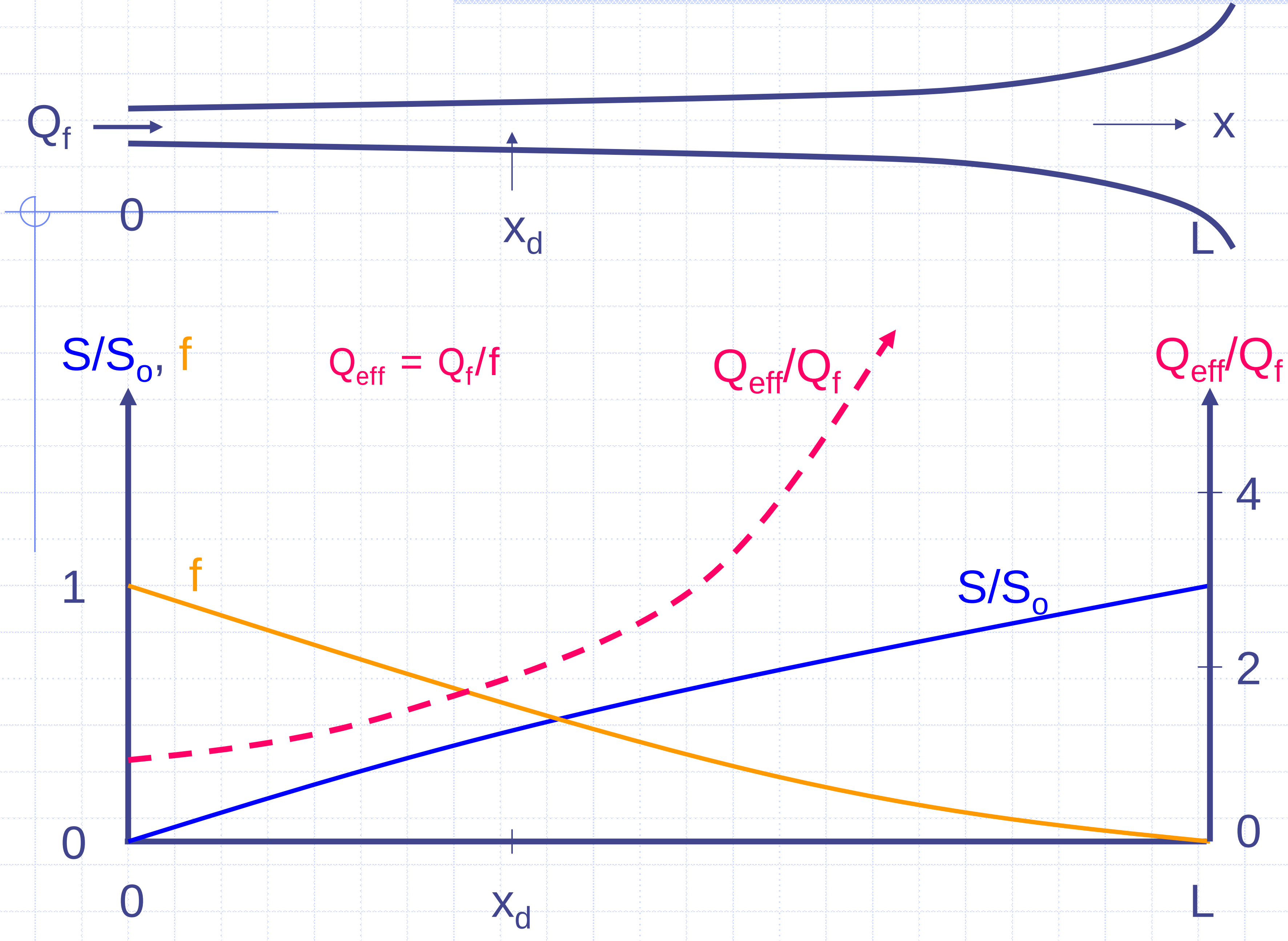
$$Q_{eff} = \frac{Q_f}{f_x}$$

Hypothetical flow rate necessary to transport freshness downstream by advection only (no tidal dispersion)

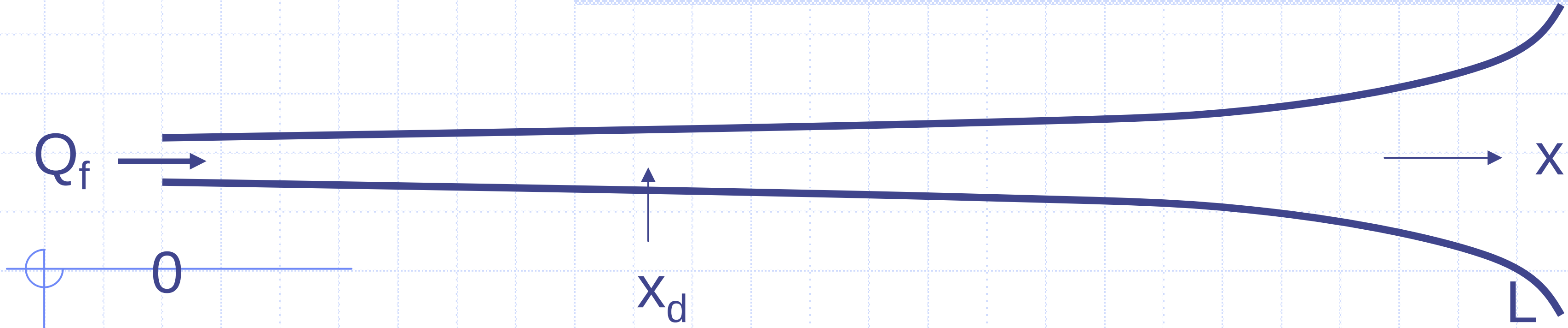
$$Q_{eff}\, f = Q_f = Q_f\, f - E_L A \frac{df}{dx}$$

$$Q_{eff} = Q_f - \frac{E_L A \, {}^{df}\!/_{dx}}{f}$$

Q_{eff} really accounts for both advection and dispersion



Q_f
x
0
x_d
L
S/S_o, f
$Q_{eff} = Q_f/f$
Q_{eff}/Q_f
Q_{eff}/Q_f
4
2
0
1
f
S/S_o
0
0
x_d
L



Downstream from x_d, mass is transported like freshness

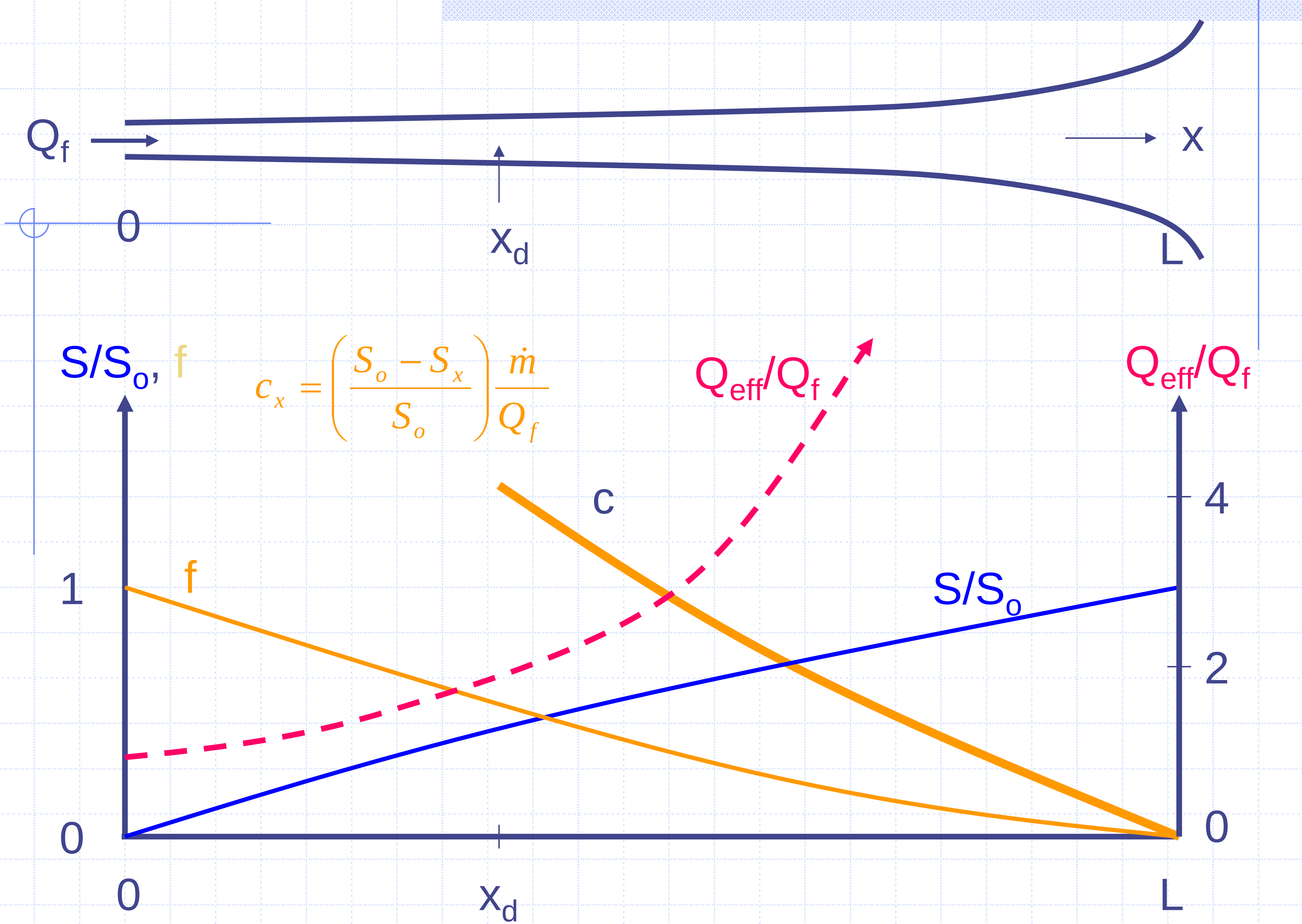
$$Q_{eff}\, f = Q_f \qquad Q_{eff}\, c = \dot{m}$$

$$\frac{c_x}{f_x} = \frac{\dot{m}}{Q_f}$$

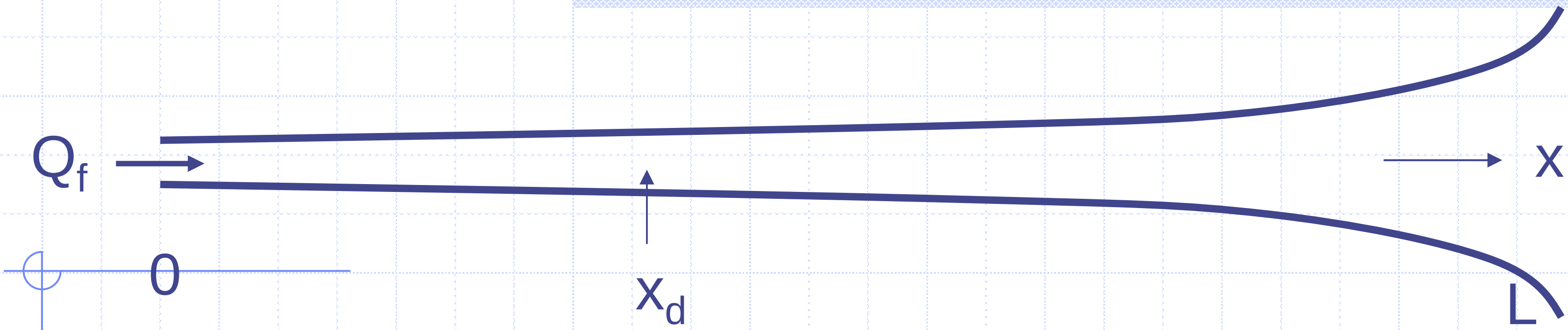
$$c_x = \left(\frac{S_o - S_x}{S_o}\right)\frac{\dot{m}}{Q_f}$$

Concentration at x_d

$$c_d = \left(\frac{S_o - S_d}{S_o}\right)\frac{\dot{m}}{Q_f}$$



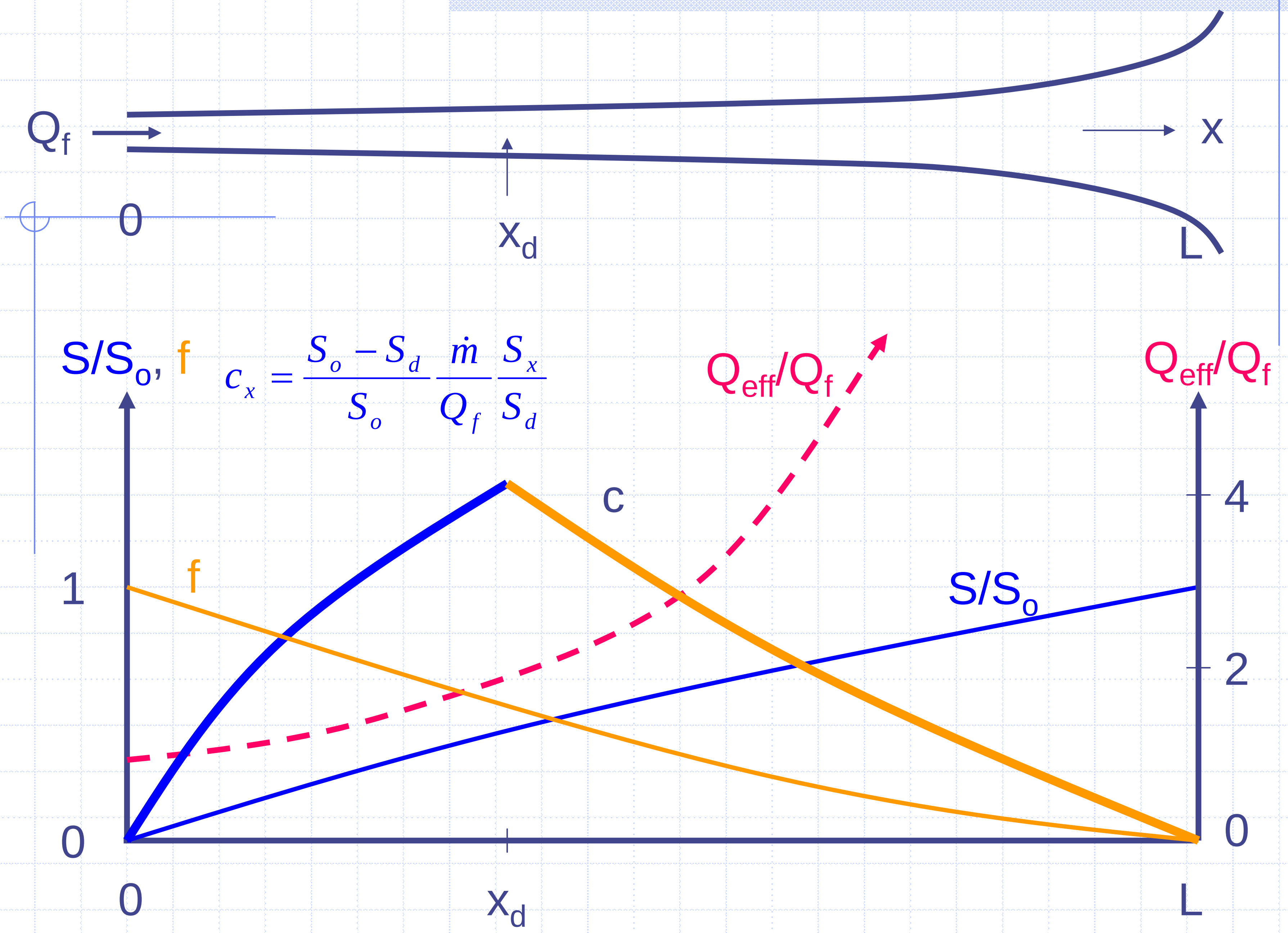
Q_f
x
0
x_d
L
S/S_o, f
$c_x = \left(\frac{S_o - S_x}{S_o}\right)\frac{\dot{m}}{Q_f}$
Q_{eff}/Q_f
Q_{eff}/Q_f
c
4
1
f
S/S_o
2
0
0
0
x_d
L



Upstream from x_d, mass is transported like salinity

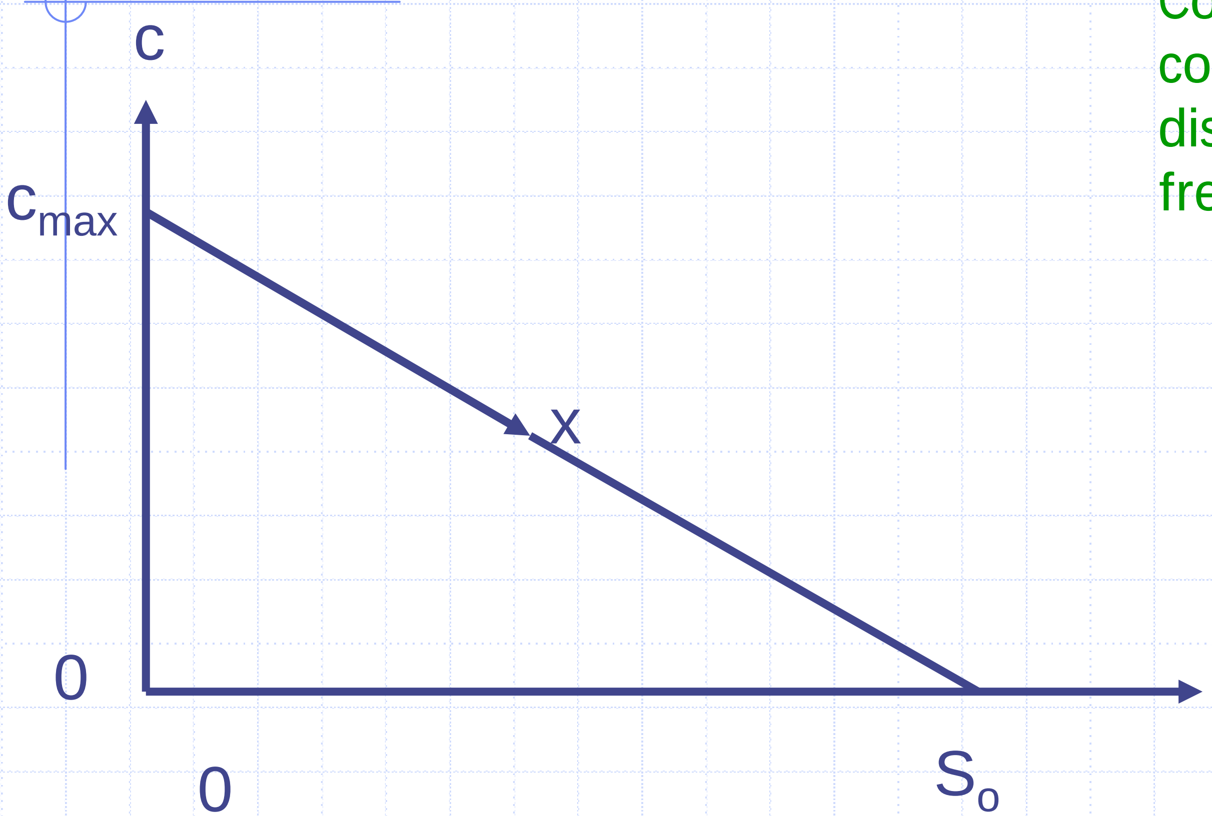
$$\frac{c_x}{c_d} = \frac{S_x}{S_d}$$

$$c_x = \underbrace{\frac{S_o - S_d}{S_o}\frac{\dot{m}}{Q_f}}_{c_d}\frac{S_x}{S_d}$$



Q_f
x
0
x_d
L
S/S_o, f
$c_x = \frac{S_o - S_d}{S_o} \frac{\dot{m}}{Q_f} \frac{S_x}{S_d}$
Q_{eff}/Q_f
Q_{eff}/Q_f
c
f
S/S_o
1
0
4
2
0
0
x_d
L

(Conservative) Mixing Diagrams



Concentration of conservative contaminant discharged at head (using freshness as tracer)

$$c_x = \left(\frac{S_o - S_x}{S_o}\right)\frac{\dot{m}}{Q_f}$$

$$c_x = a - bS_x$$

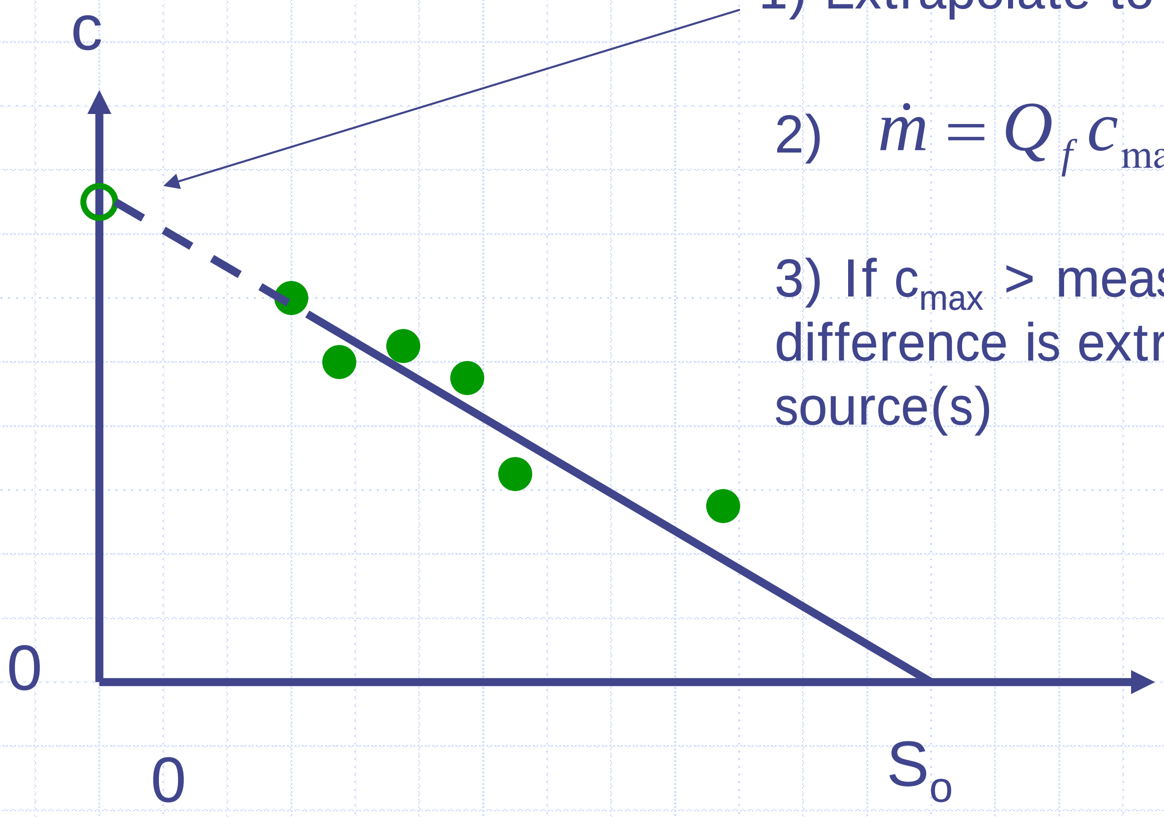
$$a = \frac{\dot{m}}{Q_f} = c_{\max}$$

aka C-S (or T-S, etc.) diagram, or property-salinity diagram

Uses for Property-S diagrams

- Determine end-member concentration and loading (S_o, Q_f known, but not $\dot{m}$)
- Identify extraneous sources (we think we know $\dot{m} = Q_f c_o$ but $c_{max} > c_o$)
- Distinguish different water masses
- Predict quality of mixed water masses
- Detect non-conservative behavior

Determining end member c

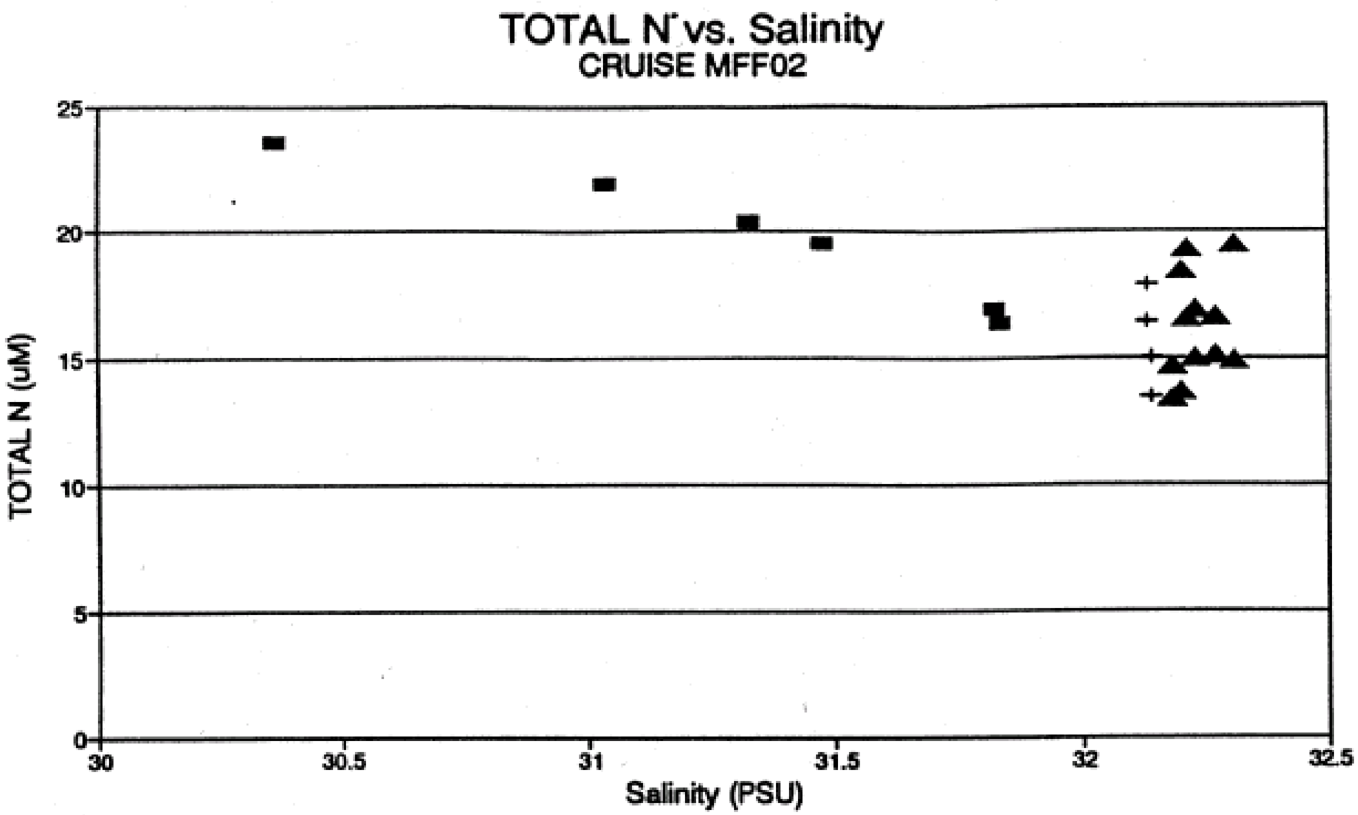


1) Extrapolate to get c_{max}

2) $\dot{m} = Q_f c_{max}$

3) If c_{max} > measured c_o, difference is extraneous source(s)

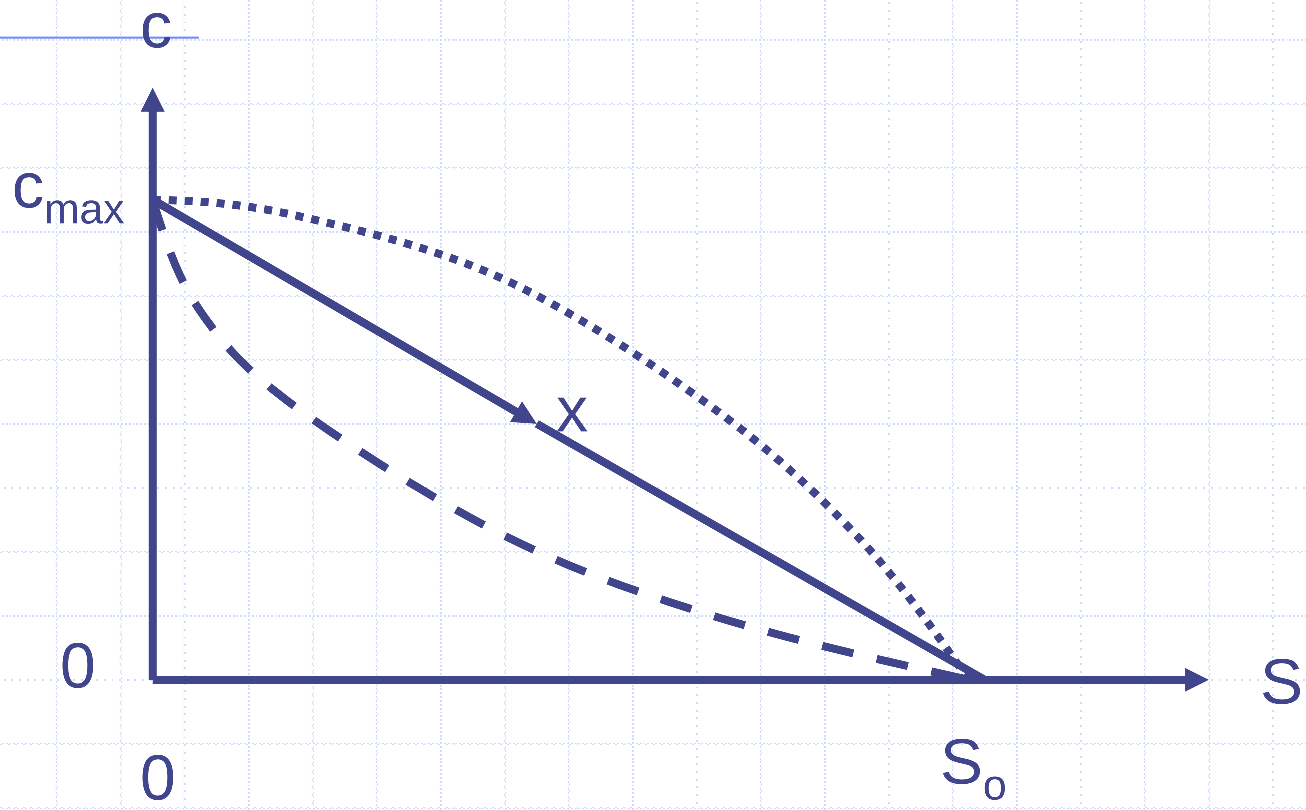
Distinguishing water masses



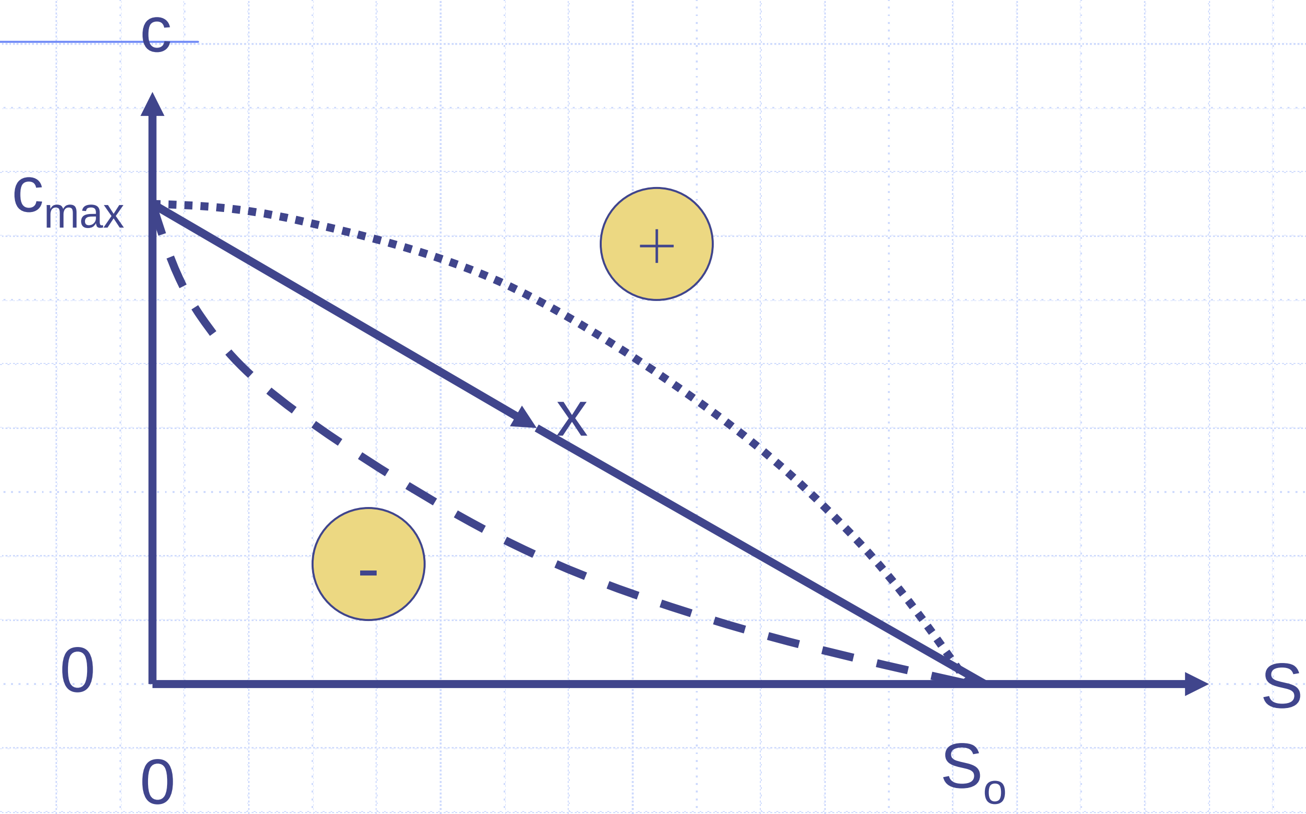
N-S diagram for Massachusetts Bay, Kelly (1993)

Used to identify coastal water vs offshore waters

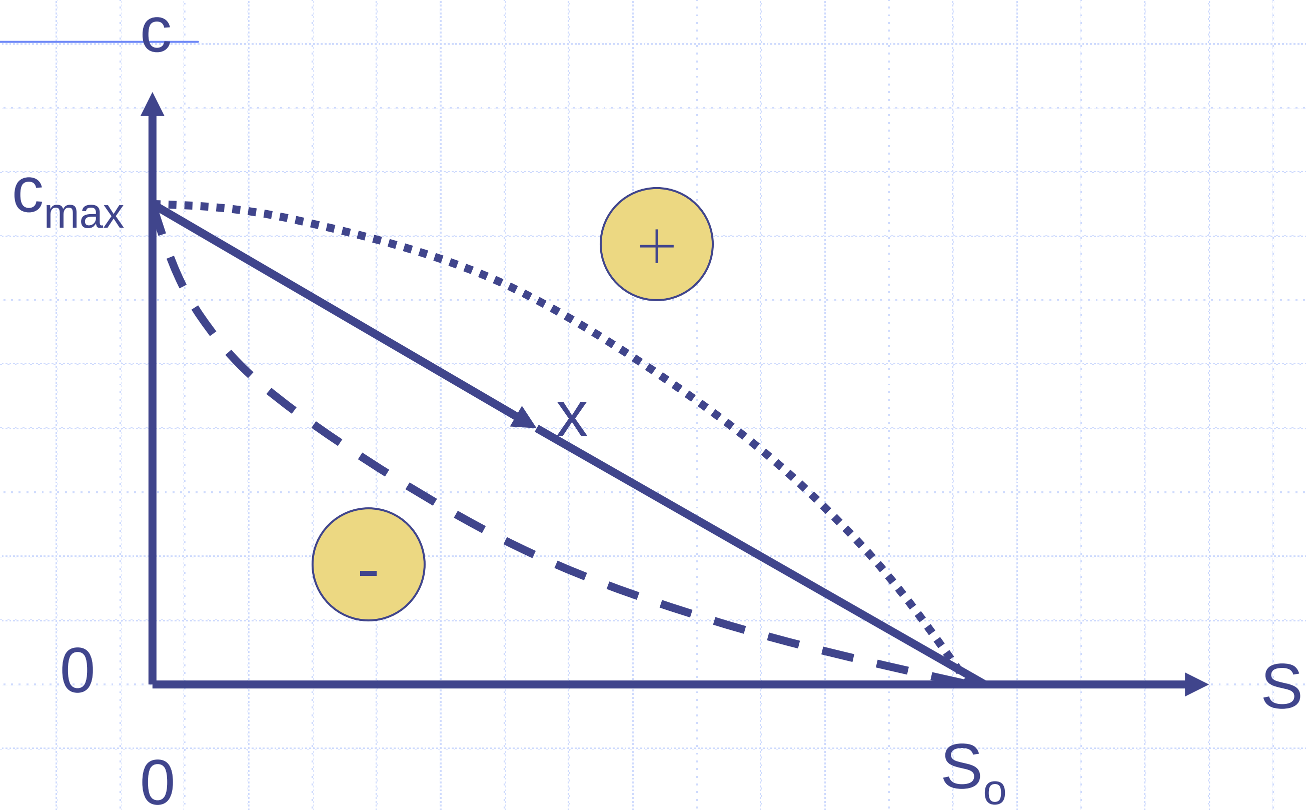
Non-conservative behavior



Non-conservative behavior

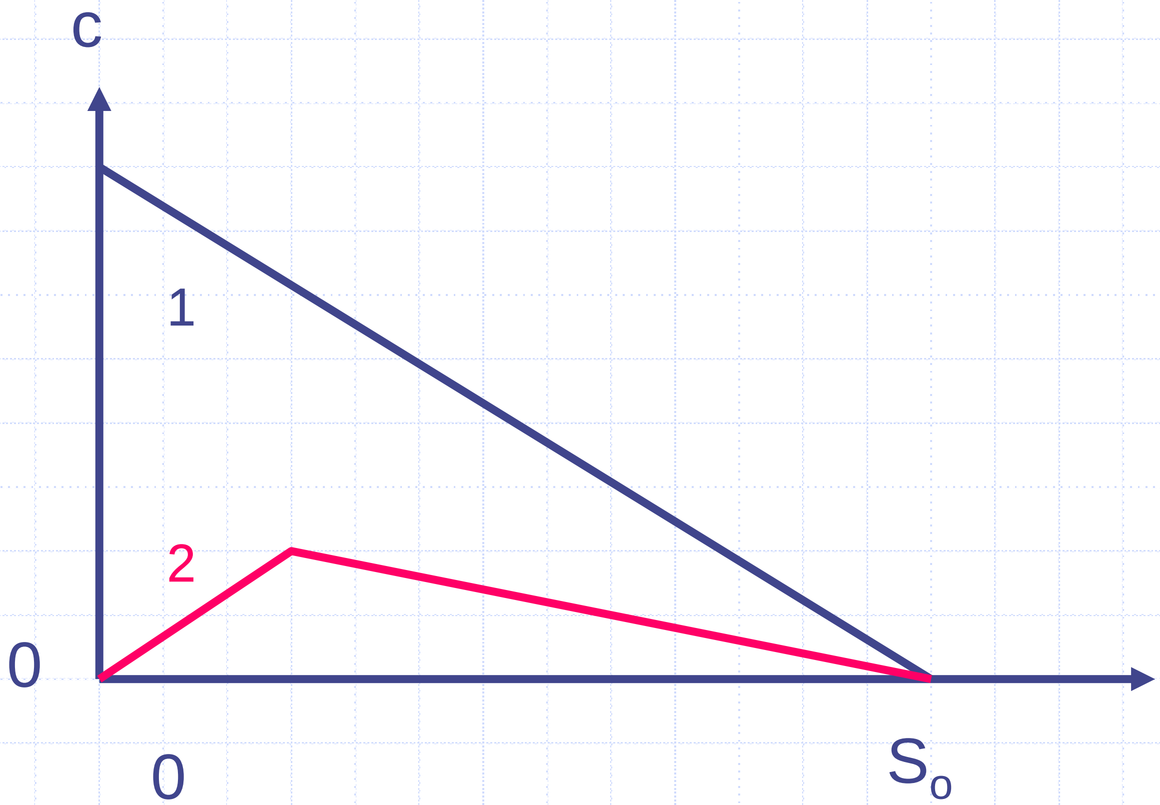


Non-conservative behavior

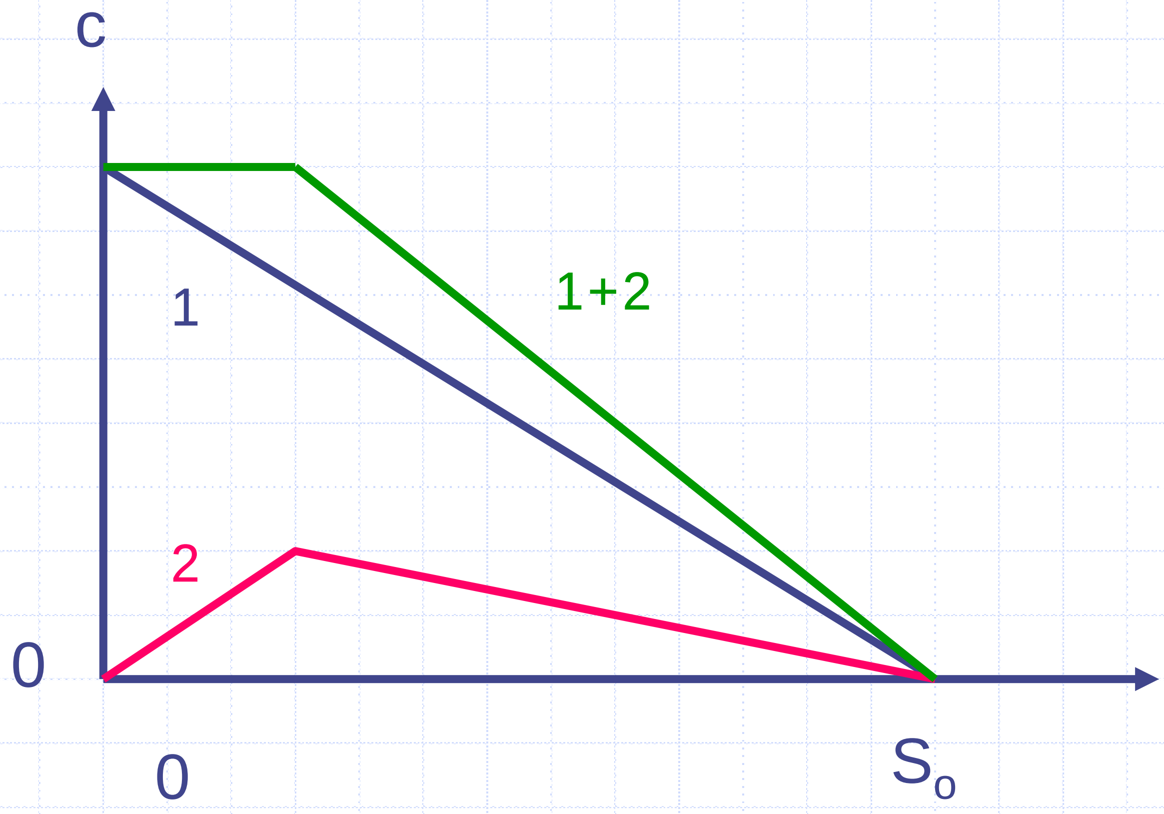


Note that conservative mixing curve is only linear if conditions are steady and there is a single source

Two conservative sources look like one NC source



Two conservative sources look like one NC source



Transient Conditions

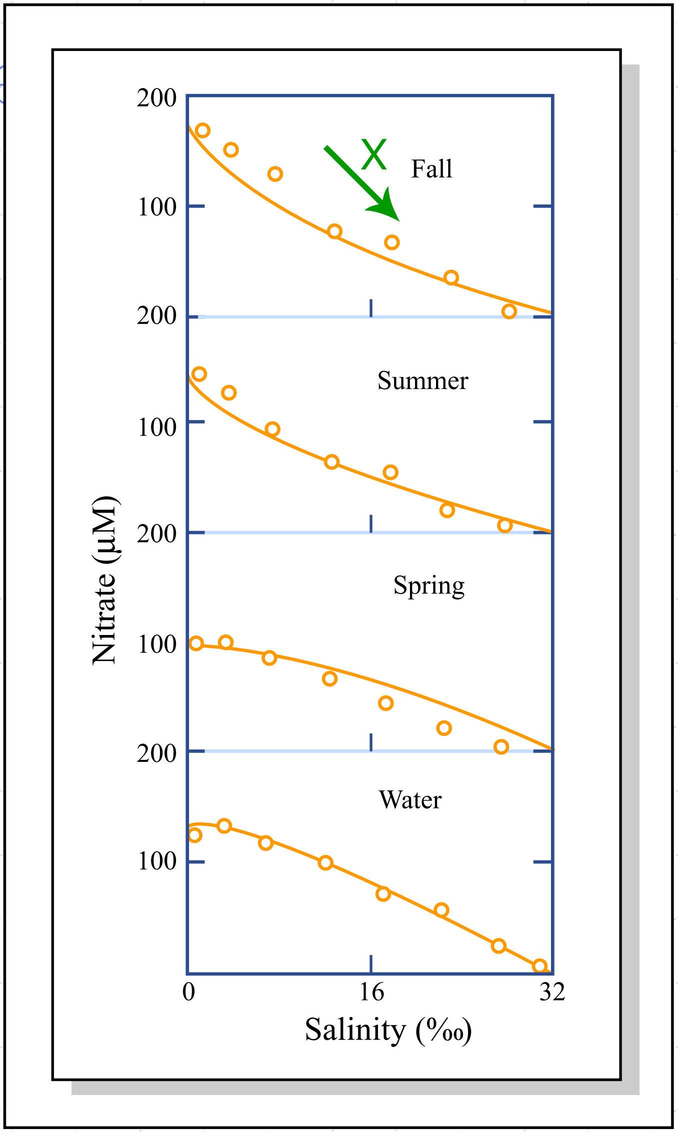


Figure by MIT OCW.

WE4-2 Nitrate-Salinity diagrams in Delaware R

Ciufuentes, et al. (1990)

Solid lines are predictions for conservative tracer & salinity at 4 times (not linear because river flow varies in space and time)

Symbols are data for nitrate & salinity

Why the discrepancy in fall, spring?

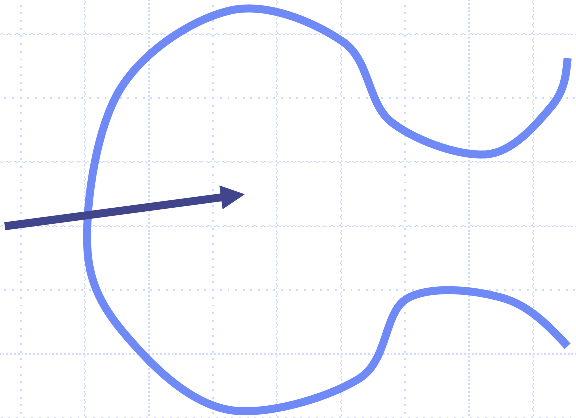
Residence times

- Why? Compare with k^{-1}
  - $t_{res} >> k^{-1}$ => reactions are important
  - $t_{res} << k^{-1}$ => reaction not important
- Also to determine if model has reached steady state
- Approaches
  - Continuous tracer
  - Instantaneous tracer
- Related time scales

Continuous tracer release; c(x,y,z) monitored after steady state

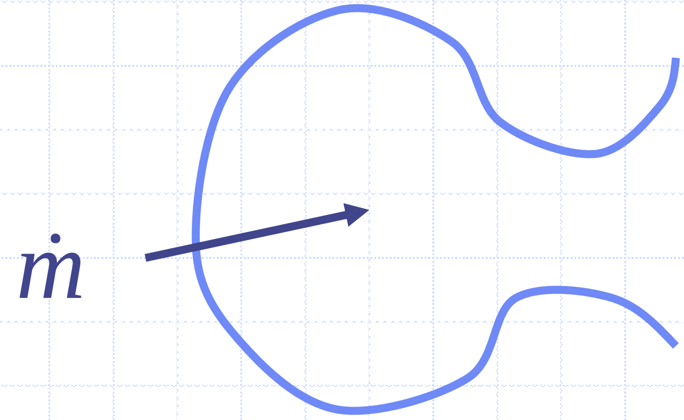
$$t_{res} = \frac{\int_0^V c\,dV}{\dot{m}} = \frac{M}{\dot{m}}$$

$\dot{m}$



SS inventory over renewal rate;
heuristic interpretation

Types of Tracers

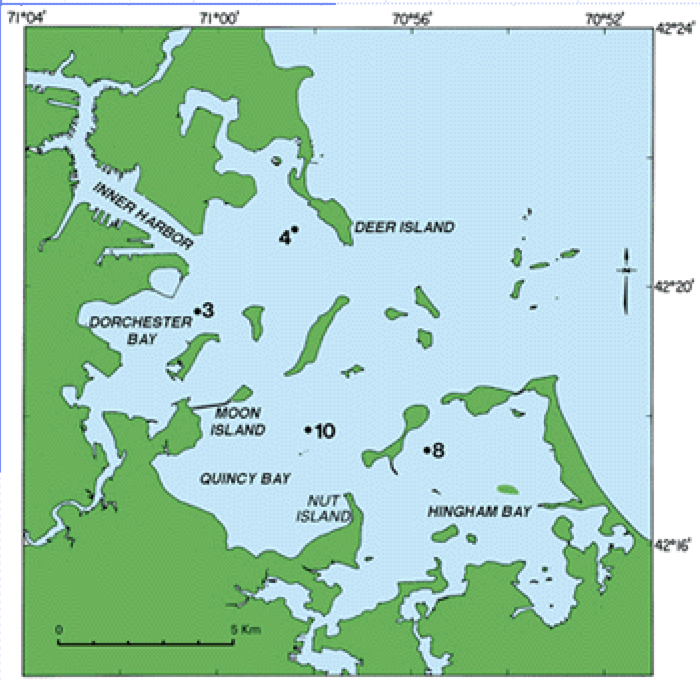


Advantages and Disadvantages of each

- Deliberate tracer (e.g., dye)
- Tracer of opportunity (e.g. trace metals from WWTP)
- Freshwater inflow (freshwater fraction approach; residence time sometimes called flushing time)

$$t_{res} = \frac{\int_0^V f dV}{Q_f}$$

WE 4-4 Trace metals to calculate residences times for Boston Harbor



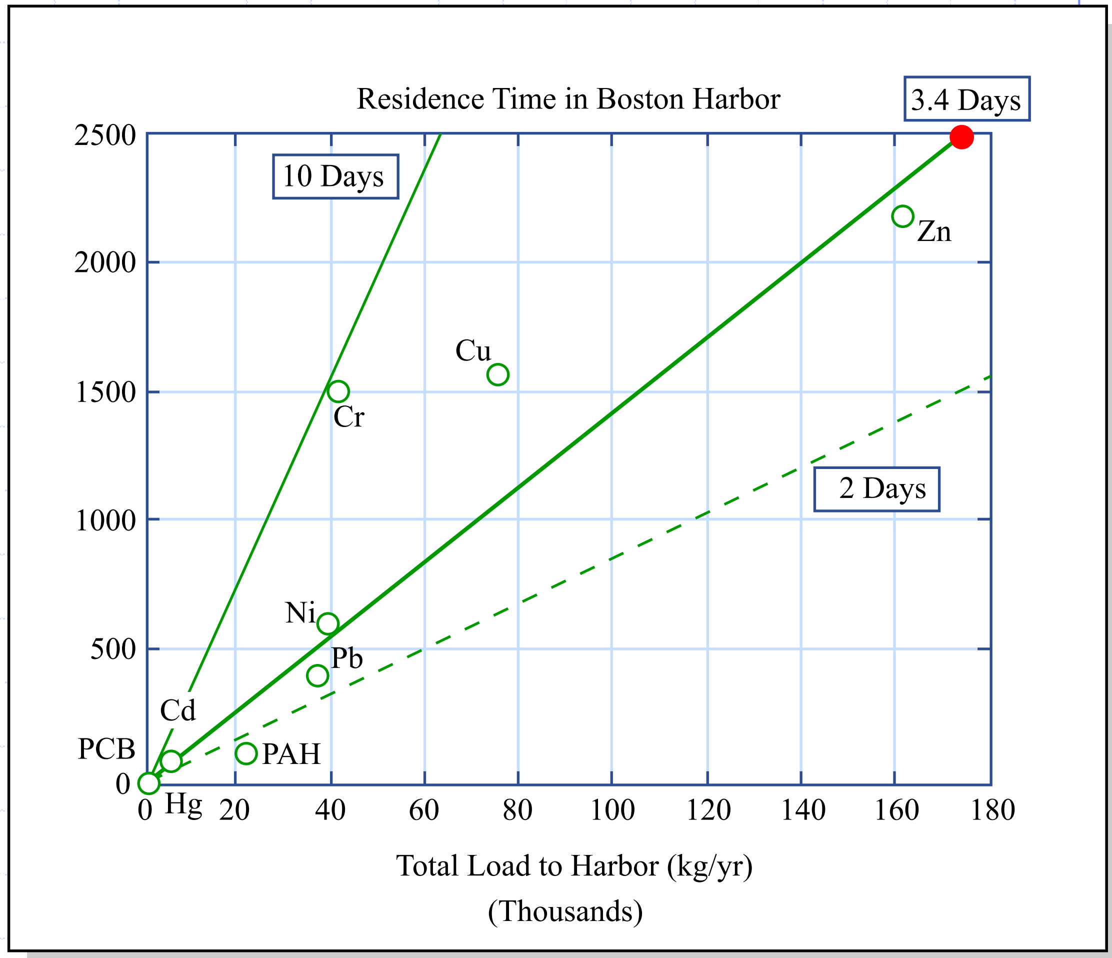


Figure by MIT OCW.

$$t_{res} = \frac{\bar{c}V}{\dot{m}}$$

$$= \frac{(2.5x10^{-6}\,kg/m^3)(6.3x10^2\,m^3)}{(1.7x10^5\,kg/yr)/(365d/yr)} = 3.4d$$

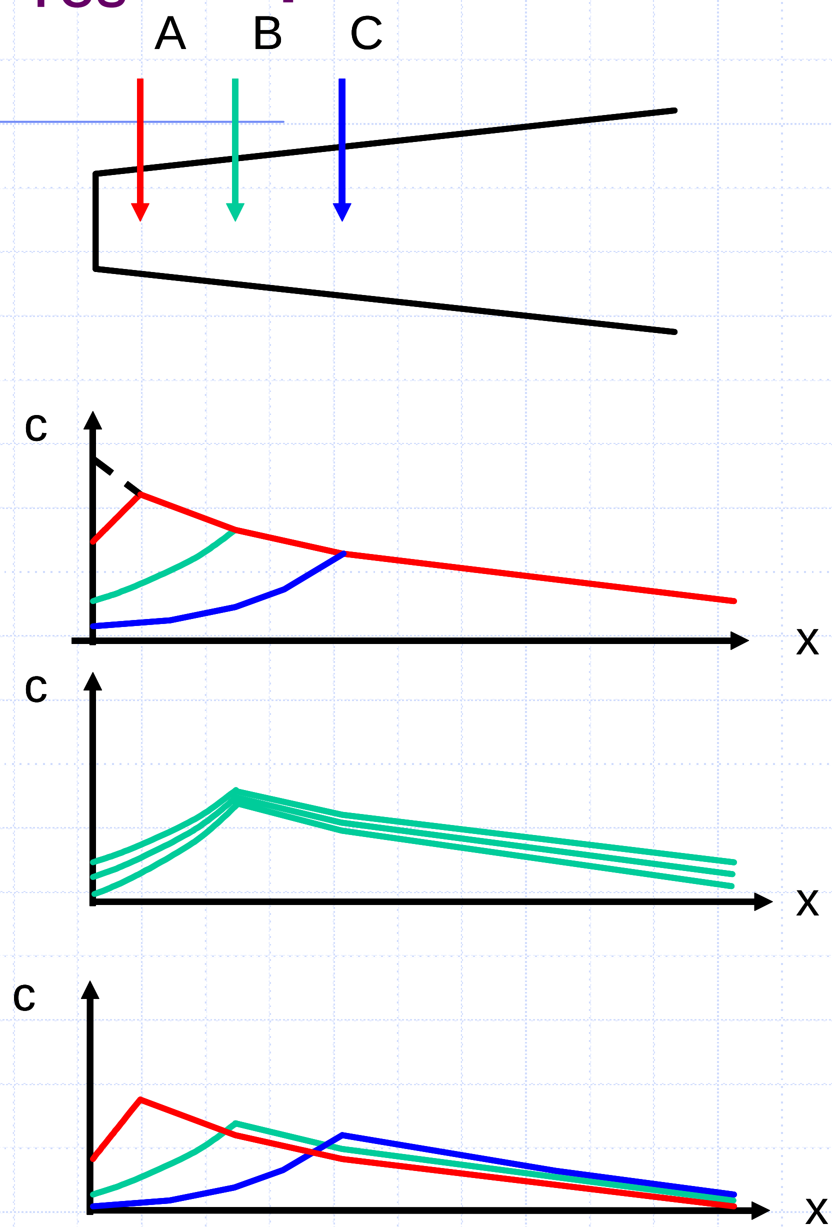
Shea and Kelly, 1992

Comments

- Ignore re-entries (by convention)
- If multiple sources, t_{res} is average time weighted by mass inflow rate
- Assumes steady-state, but “fix-ups” applicable to transient loading
- Residence time reflects injection location; not property of water body... unless well mixed, in which case:

$$c(x, y, z) = \bar{c} = const \qquad t_{res} = \frac{\bar{c}V}{\dot{m}}$$

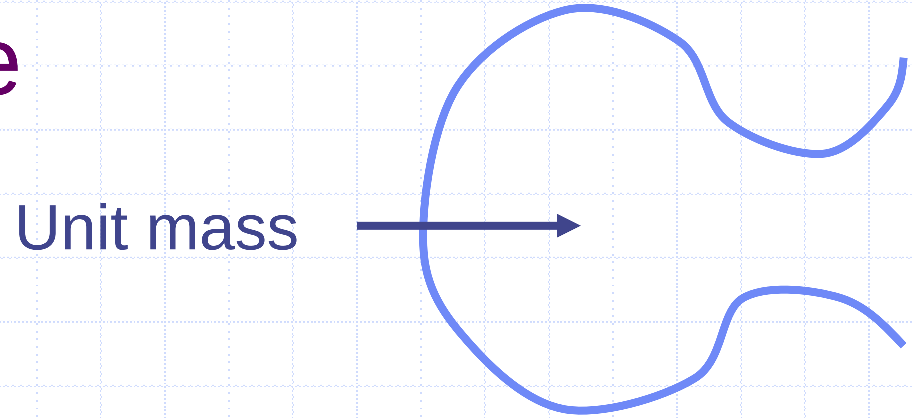
T_{res} depends on discharge location

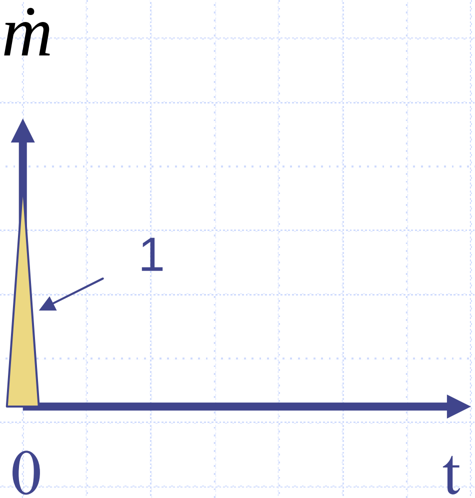


$$t_{res} = \frac{\int_0^V c\,dV}{\dot{m}} = \frac{M}{\dot{m}}$$

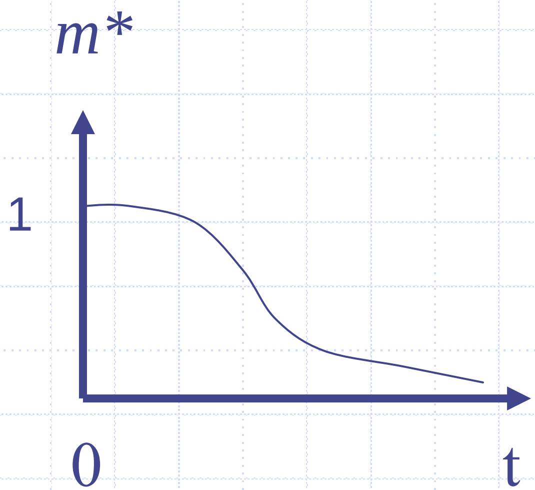
$t_{res\ A} > t_{res\ B} > t_{res\ C}$

Instantaneous Release; c(x,y,z,t) monitored over time

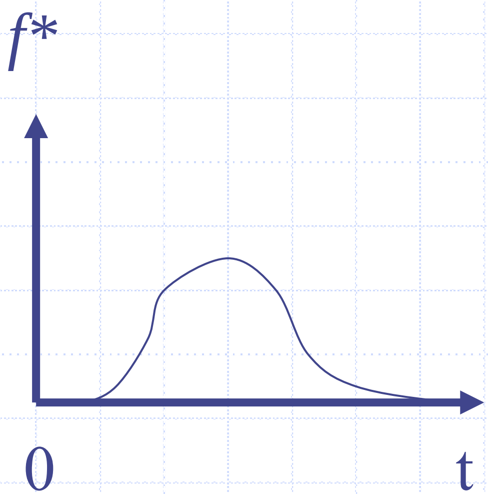




Rate of injection



Mass remaining in system



Mass leaving rate

$$f^*(t) = -\frac{dm^*}{dt}$$

$$\int_0^\infty f^*(t)\,dt = 1$$

Instantaneous release, cont'd

f* is also distribution of residence times (mass leaving no longer resides). By definition, t_{res} is mean (first temporal moment) of f*

$$t_{res} = \int_0^\infty f^* t\,dt - \int_0^\infty \frac{d\dot{m}}{dt} t\,dt = -m^* t\Big|_0^\infty + \int_0^\infty m^*(t)\,dt$$

1st moment of f*

0th moment of m*

For mass of arbitrary loading M_o (not necessarily one)

$$t_{res} = \frac{\int_0^\infty f(t)\,t\,dt}{M_o} = \frac{\int_0^\infty M(t)\,dt}{M_o}$$

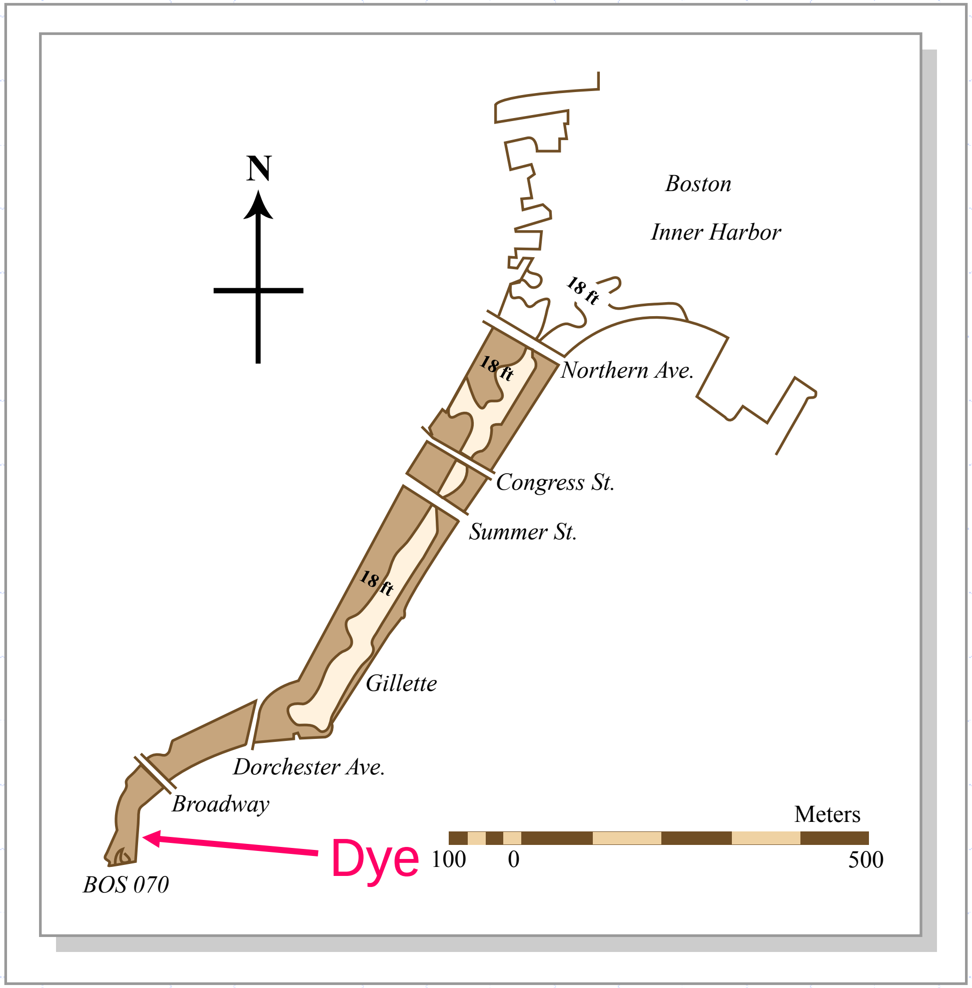
M(t) = mass remaining

f(t) is mass leaving rate

Thus two more operational definitions of residence time: 1st temporal moment of f(t) and 0th temporal moment of M(t)

WE 4-5 Residence time of CSO effluent in Fort Point Channel

Rhodamine WT injected instantaneously at channel head on three dates; results for one survey:



Adams, et al. (1998)

Figure by MIT OCW.

Fort Point Channel dye release, cont'd

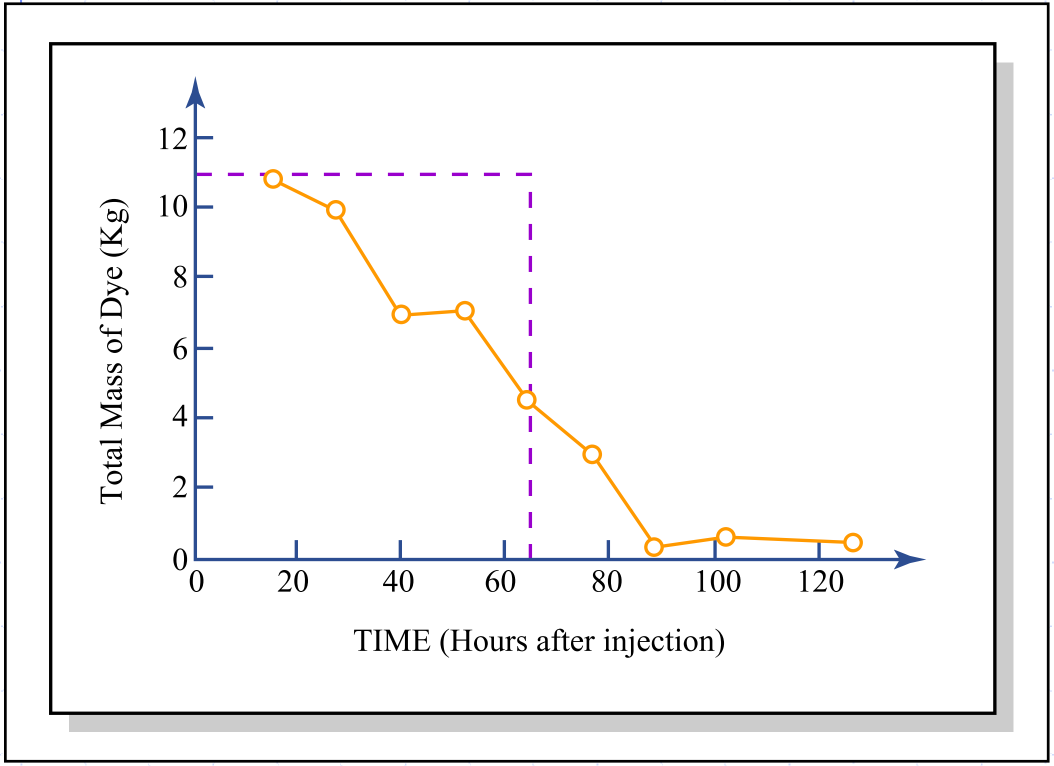


Figure by MIT OCW.

$$t_{res} = \frac{\int_0^\infty M(t)dt}{M_o} \cong 2.7 \text{ day}$$

t_{res}

Adams, et al. (1998)

Comments

- f(t) can be obtained from time rate of change of M(t); or from measurements of mass leaving (at mouth)
- Residence times for continuous and instantaneous releases are equivalent
- f(t) of f*(t) conveniently used to assess first order mass loss.

$$F = \int_0^\infty f^*(t) e^{-kt} dt$$

F = total fraction of mass that leaves

WE 4-6 Residence time of bacteria in CSO effluent in Fort Point Channel

(Adams et al., 1995)

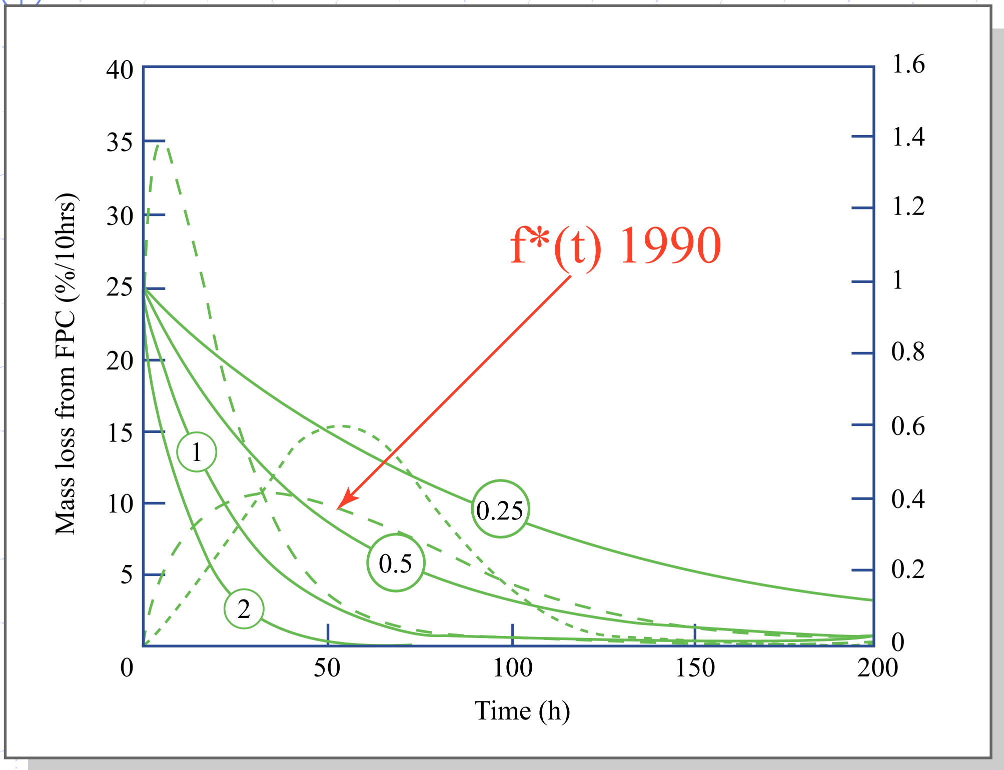


Figure by MIT OCW.

Residence time distributions f(t) determined from distributions of m(t).

Indicator bacteria “disappear” (die or settle) at rates of 0.25 to 2 d^{-1}

What fraction of bacteria would disappear for 1990 conditions?

$$F = \int_0^\infty f^*(t) e^{-kt} dt$$

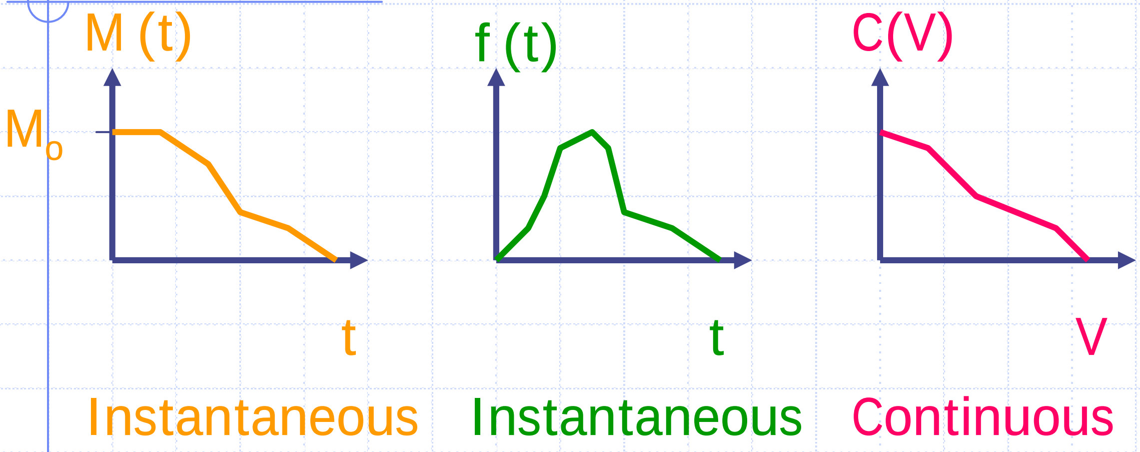
Fraction (of viable bacteria) that leave

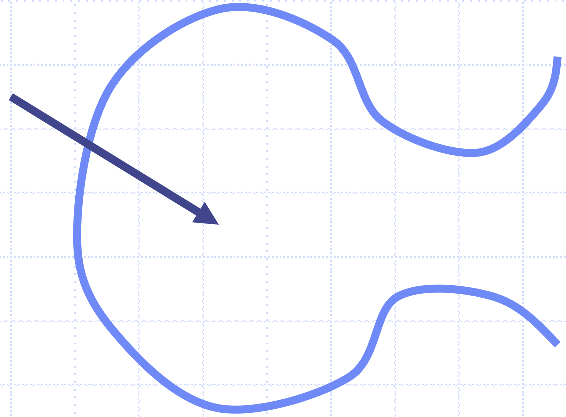
$$1 - F$$

Fraction that are removed within channel

k=2.0 d^{-1} => F=0.15 (85% removed); k=0.25 d^{-1}=> F=0.55 (45% removed)

Relative advantages of 3 approaches?





$$t_1 = \frac{\int_0^\infty M(t)dt}{M_o} \qquad t_2 = \frac{\int_0^\infty f(t)t dt}{M_o} \qquad t_3 = \frac{\int_0^V c dV}{\dot{m}}$$

- Amount of tracer (e.g., dye) required?
- Effort to dispense?
- Number of surveys and their spatial extent?
- Total duration of study?

Other related time scales

- Flushing time use to describe decay of initial concentration distribution (convenient for numerical models); used by EPA for WQ in marinas (see example)
- Age of water (oceanography): time since tracer entered ocean or was last at surface (complement of t_{res})
- Concepts often used interchangeably, but in general different; be careful

Dual Tracers

Used to empirically distinguish fate from transport: introduce two tracers (one conservative; one reactive) instantaneously. Applies to any time of water body, but consider well mixed tidal channel

$$\frac{dM_c}{dt} = -k_f M_c$$

Mass of conservative tracer declines due to tidal flushing

$$\frac{dM_{nc}}{dt} = -k_f M_{nc} - kM_{nc}$$

Mass of NC tracer declines due to tidal flushing and decay

$$\frac{d}{dt}\left(M_{nc} / M_c \right) = -k \left(M_{nc} / M_c \right)$$

Ratio of masses declines due to decay

$$M_{nc} / M_c = \left[M_{nc} / M_c \right]_o e^{-kt}$$

WE 4-7 Fort Point Channel again

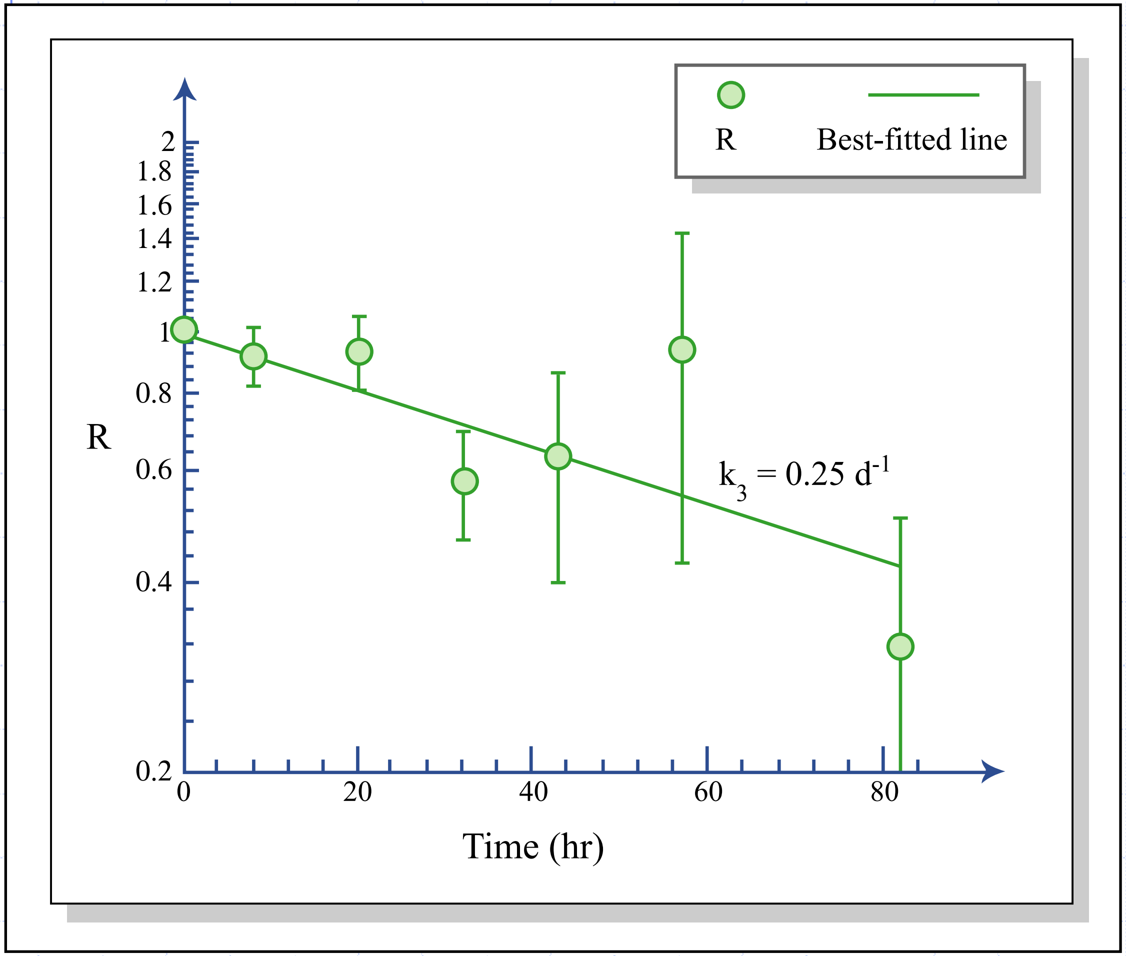


Figure by MIT OCW.

Fluorescent pigment particles (yellow DayGlo paint) were injected with dye. Pigment particles settle as well as flush.

$R = (M_p/M_{po})/(M_d/M_{do})$

$k = k_{settle} = 0.25\ d^{-1}$

$k = w_s/h$

$w_s = kh = (0.25d^{-1})(6m)$

$= 1.5\ m\ d^{-1}$

More in Chapter 9

Adams, et al. (1998)