MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Civil and Environmental Engineering
1.77 Water Quality Control

Problem Set 2 | Spring 2006 | Due March 2

1) Inert silt particles and organic particles (dead algae) settle to the bottom of a lake where they form a sediment layer of slowly increasing thickness. The bottom waters of the lake are aerobic. The chemical composition of the organic particles may be represented approximately by $C_6H_{12}O_6NH_3$. Except for a surface film of a few millimeters thickness at the sediment-water interface in which the sediment is aerobic, the remainder of the silt plus organic sediment (which may have a depth of the order of 50 cm) is completely anaerobic. Within this layer, anaerobic bacterial decomposition takes place so that the organic material is converted to methane (CH_4), carbon dioxide (CO_2) and ammonia (NH_3).

   $C_6H_{12}O_6NH_3 \Rightarrow 3CH_4 + 3CO_2 + NH_3$

   Assume that methane is produced continuously and uniformly within the anaerobic sediment layer at a constant rate S_M (mg/l-day), i.e., a "zero order" reaction.

   a) Make a sketch and write the governing PDE for the steady state diffusion of methane upward to the sediment-water interface. Assume that the concentration of methane at the sediment-water interface is zero, due to the rapid oxidation of methane at the interface, and that at a depth L, of the order 50 cm, the sediment is saturated with methane at concentration c_o.

   b) Solve for the steady state concentration distribution of methane in the sediment in the region z = 0, at the sediment interface, to z = L.

   c) Discuss how you would calculate the flux of methane at the sediment-water interface. What practical value would such a calculation have?

2) A paper manufacturing plant proposes to discharge its treated waste effluent into the epilimnion of a lake, through an outfall pipe. In order to obtain a license for the outfall, it well be necessary to characterize the plume that will result from the discharge. You have been hired to obtain and analyze the data necessary for licensing.

   You realize that before much can be accomplished, it is necessary to get a handle on the advection and diffusion characteristics of the lake, and therefore a small experiment has been designed. First, the temperature profile of the lake was measured in order to determine the depth of the thermocline (Figure 1). Next, 300 kg of Rhodamine WT, a fluorescent dye, was injected uniformly over the epilimnion in a time span of several minutes. Then a small launch was used to crisscross the patch of diffusing dye, which moved with the lake currents as shown in Figure 2. While traversing the plume, a small pump discharged lake water through a calibrated onboard fluorometer. Isoquants of dye (ppb) are show in Figure 2.

   The above procedure was followed three times after the dye release—at times of 7, 10 and 13 hours. Thus, three plots of dye concentration, similar to that shown in Figure 2, were obtained. Since we wish to get a quick estimate of the horizontal eddy diffusivity, the data were analyzed by converting the areas within the individual isoquants of the skewed patch into "equivalent circular areas" of radius r. The result is the data shown in Table I, which for the three different times shows concentration as a function of the square of the "equivalent radius" r^2.

   Perform the following analysis of the data:

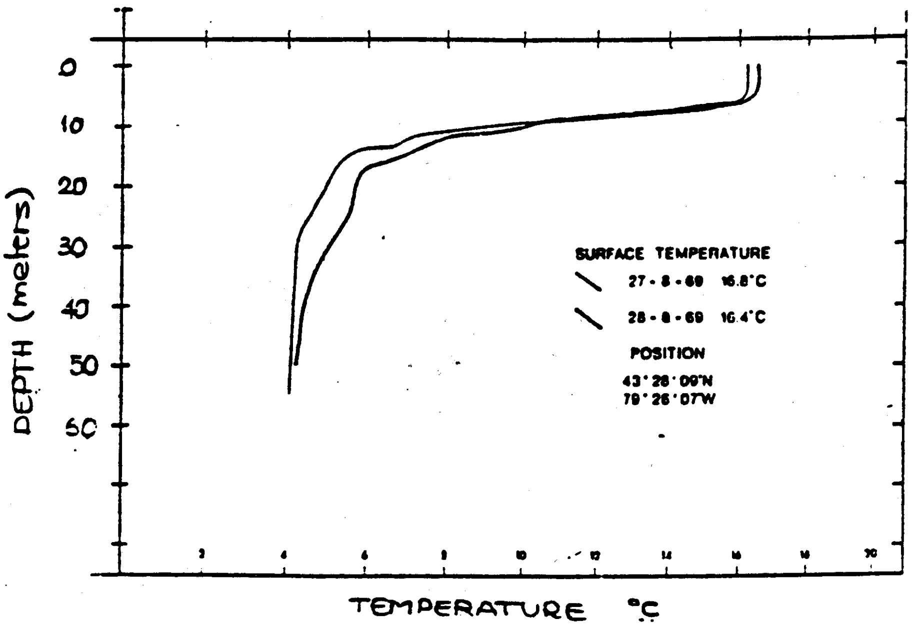
   a) Check to see if the dye is conserved

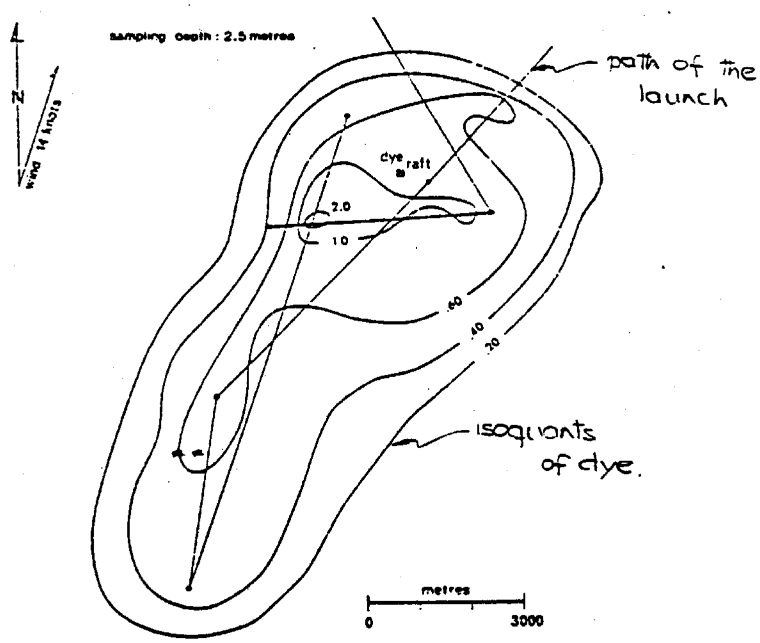
   b) Determine the radial diffusivity, E_r, as a function of time.

   c) Let c_T be a threshold diffusivity, i.e., a minimum detectable value of c (assume $c_T = 1$ ppb). Find an expression for the detectable area of the patch as a function of time. Also determine at which time the patch will no longer be detectable.

Table 1

Time (hours)	Concentration (ppb)	(Equivalent radius)^2 r^2 (m^2)
7	20	0.10×10^6
	10	0.30×10^6
	2.5	0.90×10^6
	1.0	1.10×10^6
10	10	0.10×10^6
	6.0	0.62×10^6
	1.5	1.24×10^6
	1.0	1.96×10^6
13	7.0	0.09×10^6
	5.0	0.51×10^6
	2.0	1.29×10^6
	1.0	2.40×10^6


DEPTH (meters)
0
10
20
30
40
50
60
SURFACE TEMPERATURE
27 - 8 - 69 16.8°C
28 - 8 - 69 16.4°C
POSITION
43° 28' 09"N
79° 26' 07"W
2
4
6
8
10
12
14
16
18
20
TEMPERATURE °C


sampling depth : 2.5 metres
N
wind 14 knots
path of the launch
dye raft
2.0
1.0
.60
.40
.20
isoquants of dye.
metres
0
3000