

Lecture Packet #7: Transient Systems and Groundwater Storage

How does the mass of water stored in an aquifer change?

Confined Aquifer – Compression of both the material and the water

Unconfined Aquifer – Water drains out pores at the water table when the water table drops and fills pores when the water table rises. The change in store from compression is negligible.

Return to the time derivative or change in storage term, which may be written as

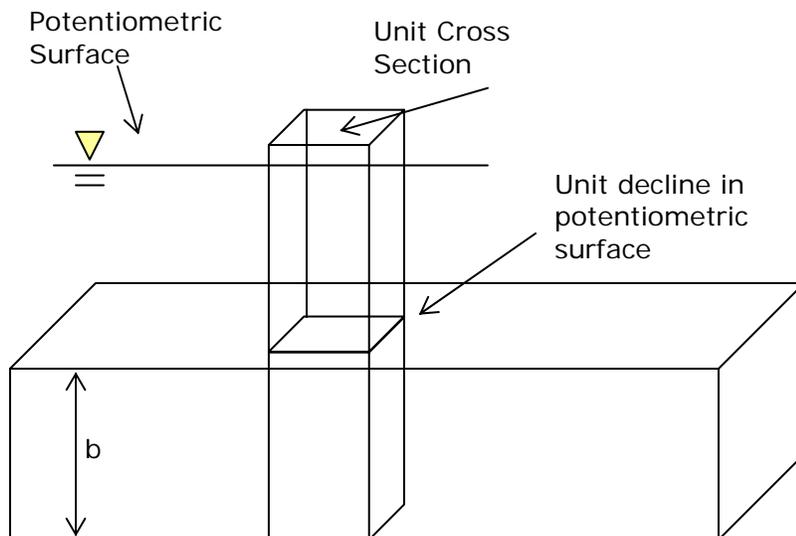
$$\frac{\partial \rho n}{\partial t} = n \frac{\partial \rho}{\partial t} + \rho \frac{\partial n}{\partial t}$$

Density changes
(compressibility)

Porosity changes

Overall definition: **S_s, specific storage**, (1/L) of a saturated aquifer is the volume that a unit volume of aquifer released from storage for a unit decline in head. (Volume per Volume per head change).

Confined Aquifers



For a confined aquifer we must consider **compressibility**.

How can a reduction in aquifer volume occur?

- Compression of the individual grains or rock skeleton (assumed negligible – individual grains are incompressible)
- Rearrangement of the grains – more compact
- Compression of the water in the pores

- We will get $S_s = \rho g \alpha + \rho g \beta n$

Stress at any depth is due to:

Total stress acting downward on a plane

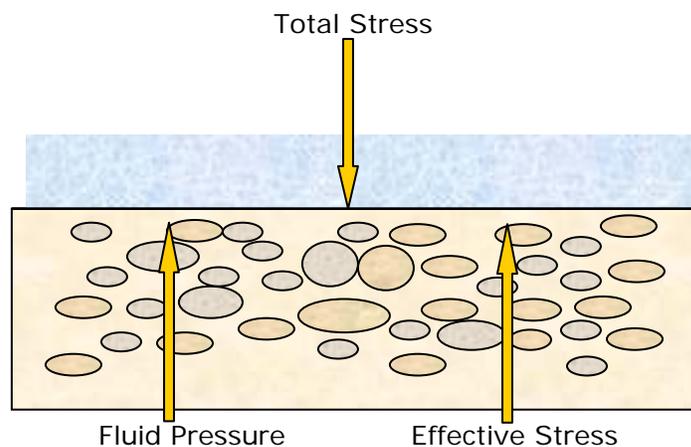
$\sigma_T =$ weight of rock and weight of water

Some of the stress is borne by rock skeleton; some by water.

$\sigma_T = \sigma_e + P \rightarrow$ effective stress (borne by rock) + fluid pressure

When pumping an aquifer the change in stress is:

$$d\sigma_T = d\sigma_e + dP$$



But the weight of overlying water and rock is essentially constant over time. So the change in total stress is zero.

$$d\sigma_T = d\sigma_e + dP = 0$$

$$d\sigma_e = -dP$$

Fluid pressure decreases, the stress on the grains becomes greater (imagine we took away the fluid).

Fluid pressure controls the volumetric deformation.

At a point, $P = \rho g h - \rho g z = \rho g(h - z)$ --- at a point, z is constant

$dP = d\rho g h$ and substituting ($d\rho g z = 0 =$ derivative of a constant)

$$d\sigma_e = -dP = -d\rho g h$$

If pumping increases, head goes down, and the effective stress goes up. Consider what happens when σ_e goes up.

Water Produced from Aquifer Compaction

- Aquifer compressibility, α , [L^2/M] is defined as follows (corresponds to shifting of grains and reduction in porosity).

$$\alpha = \frac{-(dV_t)/V_t}{d\sigma_e}$$

V_t is the original volume (thickness)
 dV_t is the change in volume (thickness)
 $d\sigma_e$ is the change in effective stress

Aquifer gets smaller with increase in effective stress.

Consider a unit volume $V_t = 1$.

$$\alpha d\sigma_e = -dV_t$$

Volume of water pumped that comes from aquifer compaction:

We also know that the change in volume of water produced by aquifer compaction must equal the volume of water and sand accounting for the reduction.

$$dV_w = -dV_t \text{ substitution for the term on the right}$$

$$dV_w = \alpha d\sigma_e = -\alpha(d\rho gh) = -\alpha\rho g(dh)$$

For a unit decrease in head due to pumping $dh = -1$

$$dV_w = \alpha\rho g$$

because this is for a unit decrease in head, this is a component of our S_s , specific storage.

Water Produced by the Expansion of Water

We can also define fluid compressibility

$$\beta = \frac{-(dV_w)/V_w}{dP}$$

β is the fluid compressibility - compressibility of water
 n is the porosity (volume of water is total volume times n)
so, for a unit total volume, $nV_t = n(1) = n$

$$dV_w = -\beta n dP = -\beta n(d\rho gh - d\rho gz) = -\beta n \rho g dh$$

For a unit decline in head, $dh = -1$,

$$dV_w = +\beta n \rho g \quad \text{which is another component of } S_s$$

Specific Storage is sum:

$$S_s = \rho g(\alpha + \beta n)$$

It has units $[L^{-1}]$. It is volume produced per aquifer volume per head decline.

- First term is from aquifer compressibility.
- Second term is from water expansion.

Let's return to flow equation (time derivative term):

$$\frac{\partial \rho n}{\partial t} = n \frac{\partial \rho}{\partial t} + \rho \frac{\partial n}{\partial t}$$

The total change in mass with time is due to

- A change in density with time term associated with water compressibility.
- A change in porosity with time term associated with the compressibility of the aquifer.

Our analysis of "where water from storage comes from" gives a term involving $S_s = \rho g(\alpha + \beta n)$ to replace the change in mass with time term above.

Going back to the continuity equation:

$$\frac{\partial \rho n}{\partial t} = - \left[\frac{\partial [\rho q_x]}{\partial x} + \frac{\partial [\rho q_y]}{\partial y} + \frac{\partial [\rho q_z]}{\partial z} \right]$$

Rate of Change of Mass stored:

$$\frac{\partial \rho n}{\partial t} = n \frac{\partial \rho}{\partial t} + \rho \frac{\partial n}{\partial t} = - \left[\frac{\partial [\rho q_x]}{\partial x} + \frac{\partial [\rho q_y]}{\partial y} + \frac{\partial [\rho q_z]}{\partial z} \right]$$

Substituting in for Darcy's law, pulling out K (homogeneous, optional), and substituting for the storage terms, we obtain:

$$\rho S_s \frac{\partial h}{\partial t} = \rho \frac{\partial h}{\partial t} \beta n \rho g = \rho K \left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} \right]$$

Note storage term must convert a volume produced to a mass produced, so multiply by ρ .

Cancel first ρ , giving our final confined flow equation as:

$$S_s \frac{\partial h}{\partial t} = \rho g(\beta n + \alpha) \frac{\partial h}{\partial t} = K \left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} \right]$$

Summary of Storage Mechanisms – Confined Aquifer

Water is released from storage during a decrease in h by two mechanisms:

- Compaction of aquifer caused by increasing effective stress and rearrangement of grains.
- Expansion of water caused by decreasing fluid pressure.

This gives rise to a specific storage coefficient with two terms and a function of α and β .

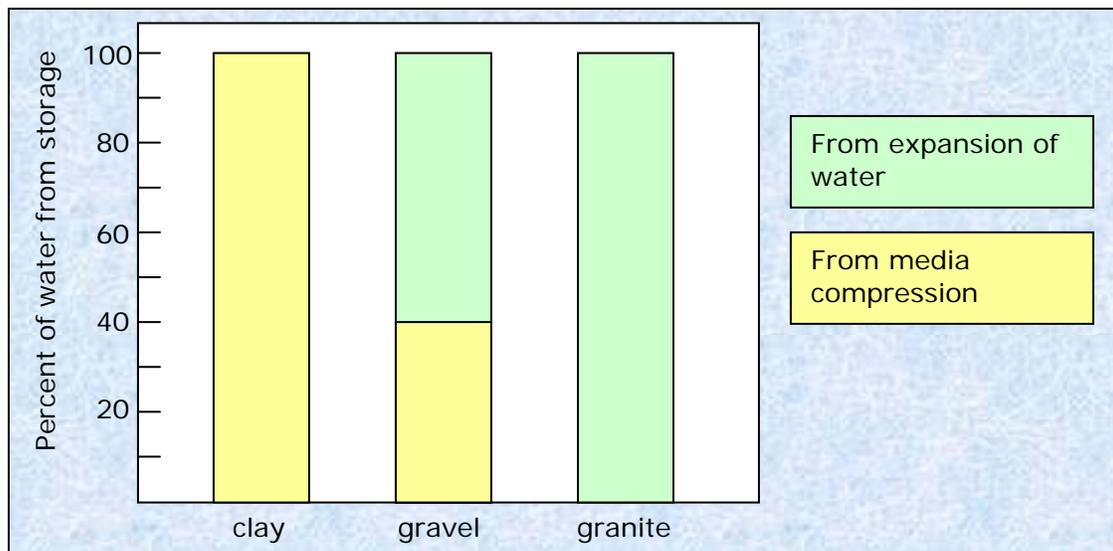
Material	Compressibility [m^2/N or $1/\text{Pa}$]
Clay	$10^{-6} - 10^{-8}$
Sand	$10^{-7} - 10^{-9}$
Gravel	$10^{-8} - 10^{-10}$
Sound Rock	$10^{-9} - 10^{-11}$
Water	4.4×10^{-10}

Where does stored water come from in confined aquifers?

Assume: $S_s = \rho g(\alpha + \beta n)$

- 10 ft of drawdown (reduction in head)
- Porosity of 30%

	Compressibility (m^2/N)	Water from Storage, $S_s \times \Delta H$
Clay	10^{-6}	0.0303
Sand	10^{-8}	0.000694
Granite	10^{-10}	0.000398
Water	4.4×10^{-10}	



With more realistic porosities, results are similar

	Porosity	Compression	Expansion	Total Water
Clay	45%	98.06%	1.94%	0.0305
Gravel	35%	39.37%	60.63%	0.00759
Granite	25%	0.90%	99.10%	0.00332

Key Equations for a confined aquifer

- 3D flow equation, Homogeneous Isotropic

$$S_s \frac{\partial h}{\partial t} = K \left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} \right] \quad \text{or} \quad S_s \frac{\partial h}{\partial t} = K \nabla^2 h$$

- 3D Heterogeneous, Anisotropic

$$S_s \frac{\partial h}{\partial t} = \left[\frac{\partial}{\partial x} K_x \frac{\partial h}{\partial x} + \frac{\partial}{\partial y} K_y \frac{\partial h}{\partial y} + \frac{\partial}{\partial z} K_z \frac{\partial h}{\partial z} \right]$$

- 3D Heterogeneous, Isotropic

$$S_s \frac{\partial h}{\partial t} = \left[\frac{\partial}{\partial x} \left(K \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(K \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left(K \frac{\partial h}{\partial z} \right) \right]$$

For **Steady State**, $S_s \frac{\partial h}{\partial t} = 0$ for any equation.

For steady state:

- No water from storage
- S values doesn't matter

2D Flow Equation

- Confined aquifer, homogeneous, isotropic.

$$S \frac{\partial h}{\partial t} = T \left[\frac{\partial^2 h}{dx^2} + \frac{\partial^2 h}{dy^2} \right]$$

$S = S_s b = \text{Storativity } [L^3 / L^3] \leftarrow \text{Values like } 10^{-2} \text{ to } 10^{-6}$
 $T = Kb = \text{Transmissivity } [L^2/T] \quad \text{Compare values to } S_y !$
 $B = \text{aquifer thickness } [L]$

- Confined aquifer, Heterogeneous, Anisotropic

$$S \frac{\partial h}{\partial t} = \left[\frac{\partial}{dx} T_x \frac{\partial h}{dx} + \frac{\partial}{dy} T_y \frac{\partial h}{dy} \right]$$

- Confined aquifer, Heterogeneous, Isotropic

$$S \frac{\partial h}{\partial t} = \left[\frac{\partial}{dx} \left(T \frac{\partial h}{dx} \right) + \frac{\partial}{dy} \left(T \frac{\partial h}{dy} \right) \right]$$

- Confined aquifer, Homogeneous, Anisotropic

$$S \frac{\partial h}{\partial t} = \left[T_x \frac{\partial^2 h}{dx^2} + T_y \frac{\partial^2 h}{dy^2} \right]$$